Chapter 4

Development of a New Model for Slow Moving Products

4.1 INTRODUCTION

Intermittent demand patterns are very difficult to forecast and they are, most commonly, associated with spare parts' requirements. Croston proved the inappropriateness of single exponential smoothing (SES) in an intermittent demand context and he proposed a method that relies upon separate forecasts of the inter-demand intervals and demand sizes, when demand occurs.

Intermittent demand appears at random, with some time periods having no demand at all. Moreover, demand, when it occurs, is not necessarily for a single unit or a constant demand size. Intermittent demand is often referred to as lumpy, sporadic or erratic demand.

Key issues remaining in this area relate to the further development of robust operational definitions of intermittent demand for forecasting and stock control purposes and a better modelling of the underlying demand characteristics for the purpose of proposing more powerful estimators useful in stock control.

Two new models have been developed for the slow moving items .Enhanced Probabilistic Demand Model (EPDM) and Demand Size Based Model (DSBM) have been developed in this work which is described below in the following sections.

The organization of this chapter consists of three sections. Section 4.2 discusses about the formulation of Enhanced Probabilistic Demand Model (EPDM). Section 4.3 deals with the formation of Demand Size Based Model (DSBM) and Section 4.4 discusses in detail about the results and analysis of the two models.

4.2 ENHANCED PROBABILISTIC DEMAND MODEL (EPDM)

The Enhanced Probabilistic Demand Model (EPDM) has been derived and compared with the basic models like Simple Exponential Smoothing (SES), Croston method

(CRO), Syntetos Boylan Approximation (SBA) and Teunter Syntetos Babai (TSB) and the results are observed to be in close proximity.

The croston's method has been modified in such a way that the forecast was updated only when there was a positive demand in the previous period. The previous period's positive demand has been used. The demand interval was replaced by the probability with modifications to TSB. For finding out the probability, the periods having positive demand were divided with the total number of periods without using any smoothing constant as in TSB, which helped to find out the actual value of probability.

It has been assumed that if the demand at any period is 0, forecast of that period will be the forecast value for the next period. For normal demands, forecast is calculated as shown below.

For example if demand at period $D_{t-1} = 0$, the forecast of the period F_{t-1} is the forecast value for the period F_t .

If $D_{t-1} = 0$,

$$\boldsymbol{F_t = F_{t-1}}$$

If $D_{t-1} > 0$,

$$\boldsymbol{F_t = F_{t-1} + \alpha (D_{t-1} - F_{t-1})}$$

$$\boldsymbol{P_t = \frac{Number\ of\ periods\ having\ positive\ demand\ up\ to\ t-1}{Total\ number\ of\ periods\ (t-1)}}$$

Where,

F_t - Forecast made at time t

D_t- Demand in the period t

α - Smoothing constant ($0<\alpha<1$)

P_t- Probability of positive demand

Table 4.1 Forecast using Enhanced Probabilistic Demand Model (EPDM)

Period	Demand/Probability		Forecasted Demand	Forecasted Probability	Forecast
1	D_1	P_{t1}	-	-	-
2	D_2	P_{t2}	$F_2 = D_1$	$P_2 = P_{t1}$	$F_2 \times P_2$
3	$D_3=0$	P_{t3}	$F_3 = F_2 + \alpha (D_2 - F_2)$	$P_3 = P_{t2}$	$F_3 \times P_3$
4	D_4	P_{t4}	$F_4 = F_3$	$P_4 = P_{t3}$	$F_4 \times P_4$
5			$F_5 = F_4 + \alpha (D_4 - F_4)$	$P_5 = P_{t4}$	$F_5 \times P_5$

4.3 DEMAND SIZE BASED MODEL (DSBM)

In this new model, demand size has been updated with the forecasted time interval.

Forecast for first period

$$f_t = D_1$$

$$T_t = T_{act1}$$

T_{act1} is the number of periods after which the demand occurs for the first time.

It has been assumed that if the demand at any period is 0, the forecast value of that respective period will be the forecast for the next period. For normal demands, forecast will be calculated as shown below.

If actual demand at time t is zero ($D_{t-1}= 0$),

$$f_t = f_{t-1}$$

$$T_t = T_{t-1}$$

If actual demand at time t is not zero ($D_{t-1} \neq 0$),

$$f_t = f_{t-1} + \alpha (D_{t-1} - f_{t-1})$$

$$T_t = T_{t-1} + \alpha (T_{act(t-1)} - T_{t-1})$$

If $1<T_t<1.5$ T_t is considered as 1 and F_t is shown as forecast for next period

If $1.5<T_t<2.5$ T_t is considered as 2 and F_t is shown as zero for next period and for the next to next period calculated value of F_t is shown as forecasted demand size because the T_t is demand interval between two non-zero demands.

Where,

D_t- Actual demand size

T_{act}- actual time interval

f_t - Forecast of demand size made at time t

T_t- Forecast of time interval made at time t

F_t- Forecast for next period after updating the demand size with the time interval

α - smoothing constant ($0<\alpha<1$)

Table 4.2 - Forecast using Demand Size Based Model (DSBM)

Period	Actual Demand/Time		Forecasted Demand	Forecasted Time interval	Forecast
1	D_1	T_{act1}	-	-	-
2	D_2	T_{act2}	$f_2 = D_1$	$T_2 = T_{act1}$	F_2
3	D_3=0	T_{act3}	$f_3 = f_2 + \alpha (D_2 - f_2)$	$T_3 = T_2 + \alpha (T_{act2} - T_2)$	F_3
4	D_4	T_{act4}=2	$f_4 = f_3$	$T_4 = T_3$	F_4
5			$f_5 = f_4 + \alpha (D_4 - f_4)$	$T_5 = T_4 + \alpha (T_{act4} - T_4)$	F_5

4.4 RESULTS AND ANALYSIS OF DEMAND FORECASTING MODEL FOR SLOW MOVING PRODUCTS

Two new models for forecasting the intermittent sales are devised. The various existing models like, Croston, Syntetos Boylan Approximation, Single exponential smoothing and TSB along with the Enhanced Probabilistic Demand Model (EPDM) were compared against a data of products having intermittent demand. Demand size based model's forecast values have been ensured by the forecast accuracy.

Slow moving data of products were chosen out of which 12 products have short-demand histories and 3 products have long-demand histories. Results are shown in the tables and graphs respectively.

For analysis of new models proposed for slow moving data of 15 products were chosen out of which 12 products having short-demand histories and three products have long-demand histories. The following tables show the actual sales of products.

4.4.1 FORECASTED VALUES FOR PRODUCTS HAVING SHORT-DEMAND HISTORIES USING ENHANCED PROBABILISTIC DEMAND MODEL (EPDM)

The data is collected from a leading Biscuits Company and the sales data of some of the biscuits are taken up in this work and has been worked upon for the forecasts. Table 4.3 shows the sales of the biscuits that have been recorded for 4 months.

Table 4. 3- Actual data for slow moving products

Month	**Dec'12**	**Jan'13**	**Feb'13**	**Mar'13**
Product 1	3	3	0	0
Product 2	3	1	1	0
Product 3	2	0	1	1
Product 4	3	4	0	0
Product 5	11	10	3	0
Product 6	15	1	6	0
Product 7	5	0	1	1
Product 8	0	30	88	114
Product 9	540	396	22	0
Product 10	204	14	2	0
Product 11	91	4	5	0
Product 12	3	0	1	0

The following tables from Tables 4.4 -4.7 show the forecasted values of products having short-demand histories using SES,CRO,SBA,TSB and Enhanced Probabilistic Demand

Model (EPDM) by varying the smoothing constant α from 0.05 to 0.2 and the graphs show the comparison of forecasts between existing models and new model on each individual product at different values of smoothing constant

Table 4.4-Comparison of forecasted values for products with α=0.05

	α =0.05				
Forecast	SES	CRO	SBA	TSB	EPDM
Product 1	2.71	3	2.93	2.71	1.5
Product 2	2.66	2.81	2.73	2.66	2.1
Product 3	1.81	1.82	1.77	1.82	1.43
Product 4	2.75	3.05	2.97	2.75	1.53
Product 5	10.02	10.55	10.29	10.02	7.91
Product 6	13.19	13.89	13.54	13.19	10.41
Product 7	4.38	4.4	4.29	4.4	3.46
Product 8	11.23	19.42	18.94	5.27	27.72
Product 9	481.9	507.26	494.58	481.9	380.45
Product 10	175.63	184.88	180.25	175.63	138.66
Product 11	78.44	82.57	80.5	78.44	61.93
Product 12	2.62	2.76	2.69	2.62	1.45

Table 4.5-Comparison of forecasted values for products with α=0.1

	α =0.1				
Forecast	SES	CRO	SBA	TSB	EPDM
Product 1	2.43	3	2.93	2.43	1.5
Product 2	2.36	2.62	2.55	2.36	1.97
Product 3	1.65	1.66	1.62	1.66	1.36
Product 4	2.51	3.1	3.02	2.51	1.55
Product 5	9.1	10.11	9.86	9.1	7.58
Product 6	11.56	12.84	12.52	11.56	9.63
Product 7	3.84	3.89	3.79	3.9	3.18
Product 8	21.75	24.1	23.5	11.82	32.72
Product 9	427.72	475.24	463.36	427.72	356.43
Product 10	150.03	166.7	162.53	150.03	125.03
Product 11	67.11	74.57	72.71	67.11	55.93
Product 12	2.28	2.55	2.48	2.29	1.4

Table 4.6-Comparison of forecasted values for product with α=0.15

	α =0.15				
Forecast	SES	CRO	SBA	TSB	EPDM
Product 1	2.17	3	2.93	2.17	1.5
Product 2	2.08	2.45	2.38	2.08	1.83
Product 3	1.51	1.53	1.49	1.54	1.29
Product 4	2.28	3.15	3.07	2.28	1.58
Product 5	8.22	9.67	9.43	8.22	7.25
Product 6	10.09	11.87	11.57	10.09	8.9
Product 7	3.35	3.45	3.36	3.47	2.92
Product 8	31.57	29.02	28.3	19.29	37.5
Product 9	377.35	443.94	432.84	377.35	332.96
Product 10	127.05	149.48	145.74	127.05	112.11
Product 11	56.96	67.01	65.33	56.96	50.26
Product 12	1.97	2.35	2.29	2	1.35

Table 4.7-Comparison of forecasted values for products with α=0.2

	α =0.2				
Forecast	SES	CRO	SBA	TSB	EPDM
Product 1	1.92	3	2.93	1.92	1.5
Product 2	1.82	2.28	2.22	1.82	1.71
Product 3	1.38	1.41	1.38	1.43	1.23
Product 4	2.05	3.2	3.12	2.05	1.6
Product 5	7.39	9.24	9.01	7.39	6.93
Product 6	8.77	10.96	10.69	8.77	8.22
Product 7	2.92	3.07	2.99	3.1	2.67
Product 8	40.72	34.2	33.34	27.37	42.06
Product 9	330.69	413.36	403.03	330.69	310.02
Product 10	106.56	133.2	129.87	106.56	99.9
Product 11	47.9	59.88	58.38	47.9	44.91
Product 12	1.7	2.17	2.11	1.75	1.3

The forecast values are calculated by the following methods: Single Exponential Smoothing, Croston method, Syntetos-Boylan approximation, Teunter Syntetos Babai and the Enhanced Probabilistic Demand Model. The smoothing constant values of 0.05, 0.1, 0.15 and 0.2 have been used in order to determine the best possible values. The graphs are shown in Figs 4.1 – 4.12. The forecasted values for all the products infers that lower the Smoothing constant, higher the forecasted values. As smoothing constant goes high, forecasted values falls close to accuracy. The EPDM model gives closer values than the Croston and Syntetos-Boylan since the only the positive demand of the previous period is considered for the forecast calculations along with the respective period. Since no average of the previous values is used, forecast values are better with the Enhanced Probabilistic Demand Model.

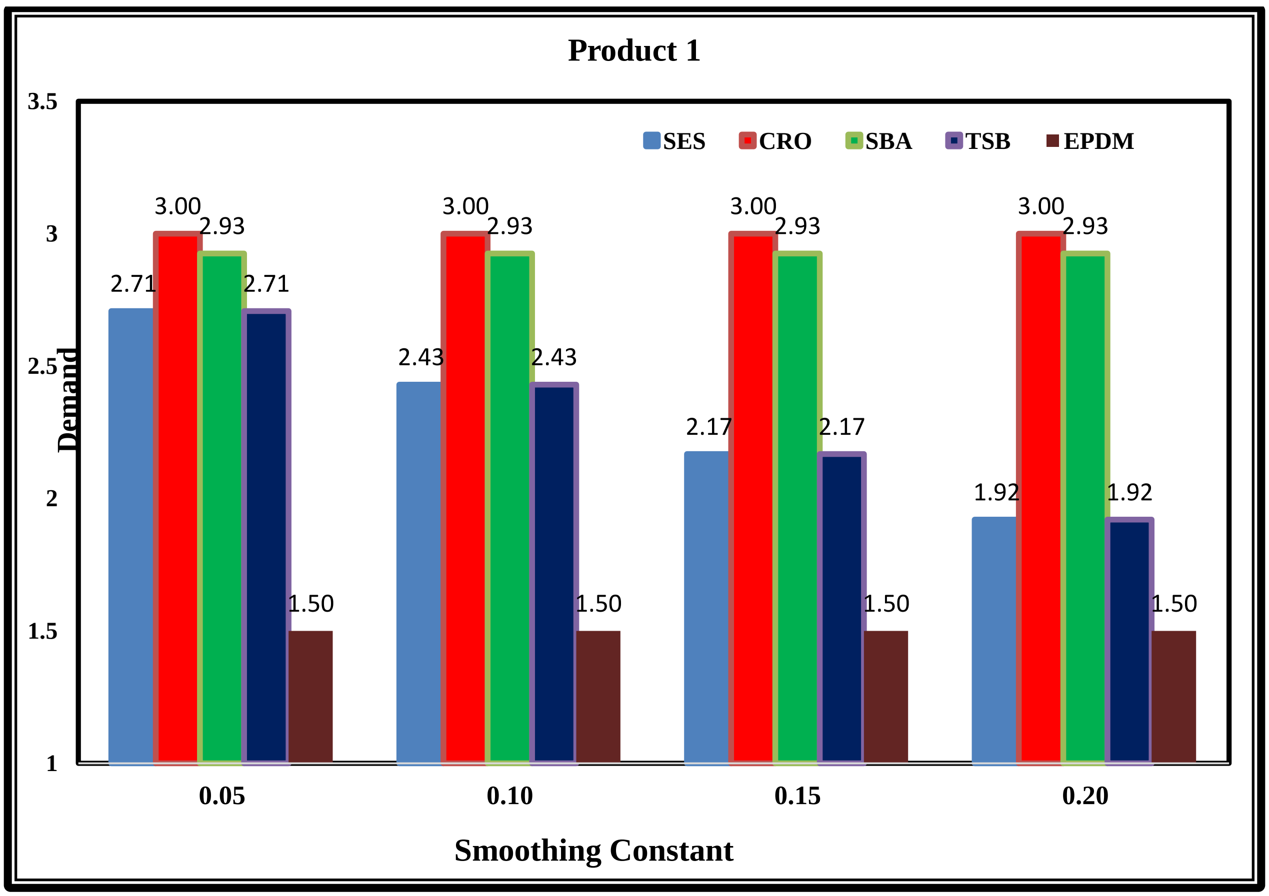

Figure 4.1-Comparison of forecasts for product 1.

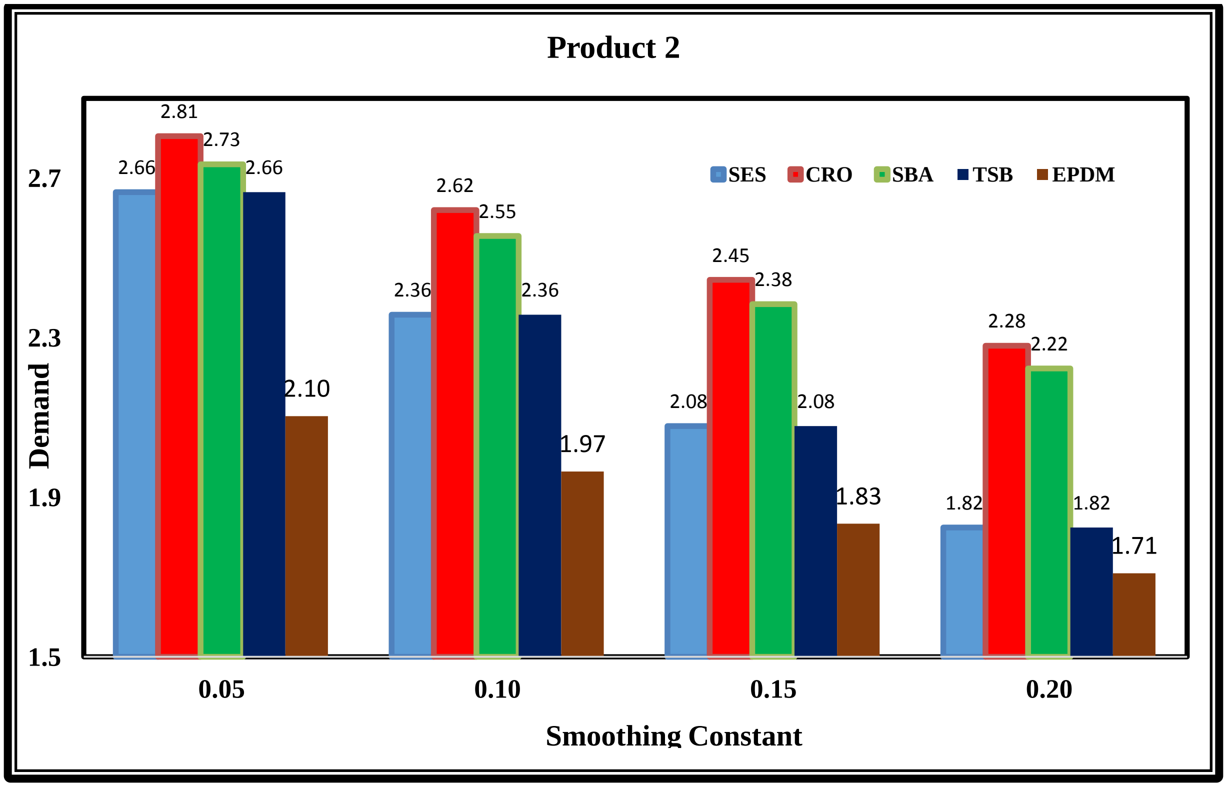

Figure 4.2-Comparison of forecasts for product 2.

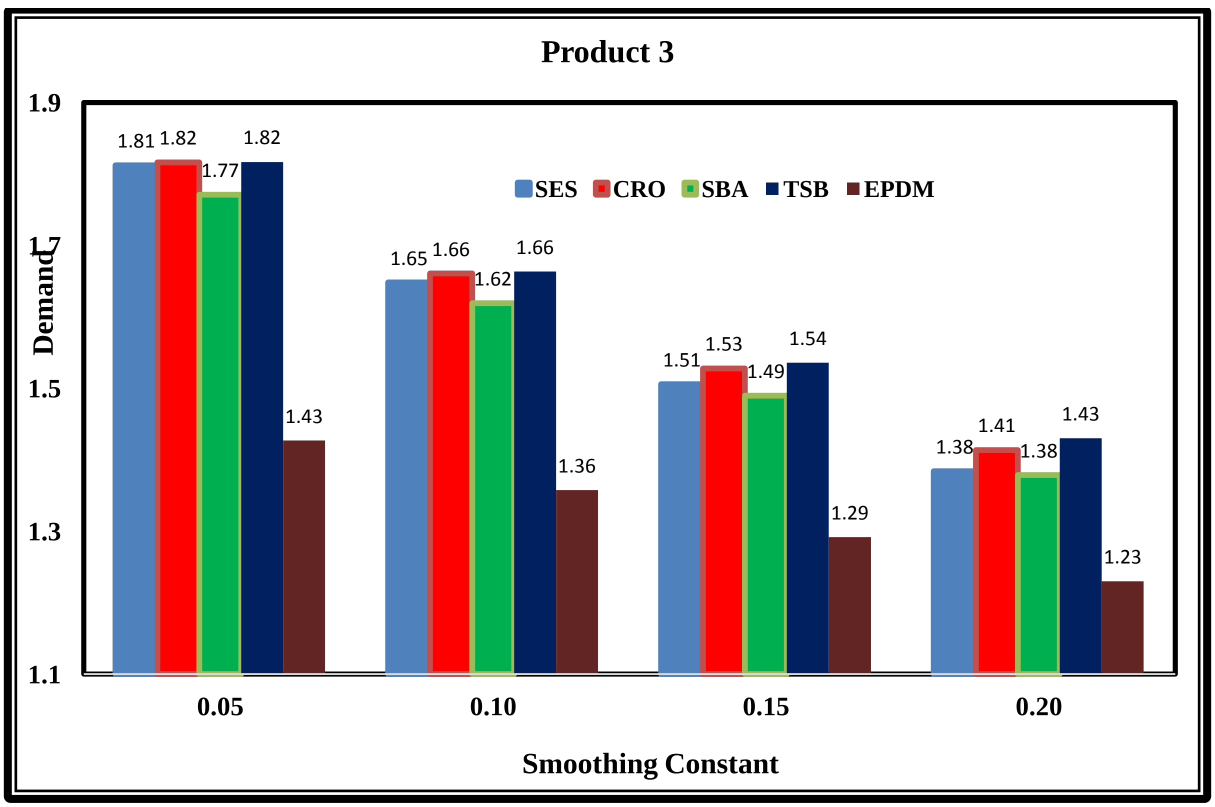

Figure 4.3-Comparison of forecasts for product 3.

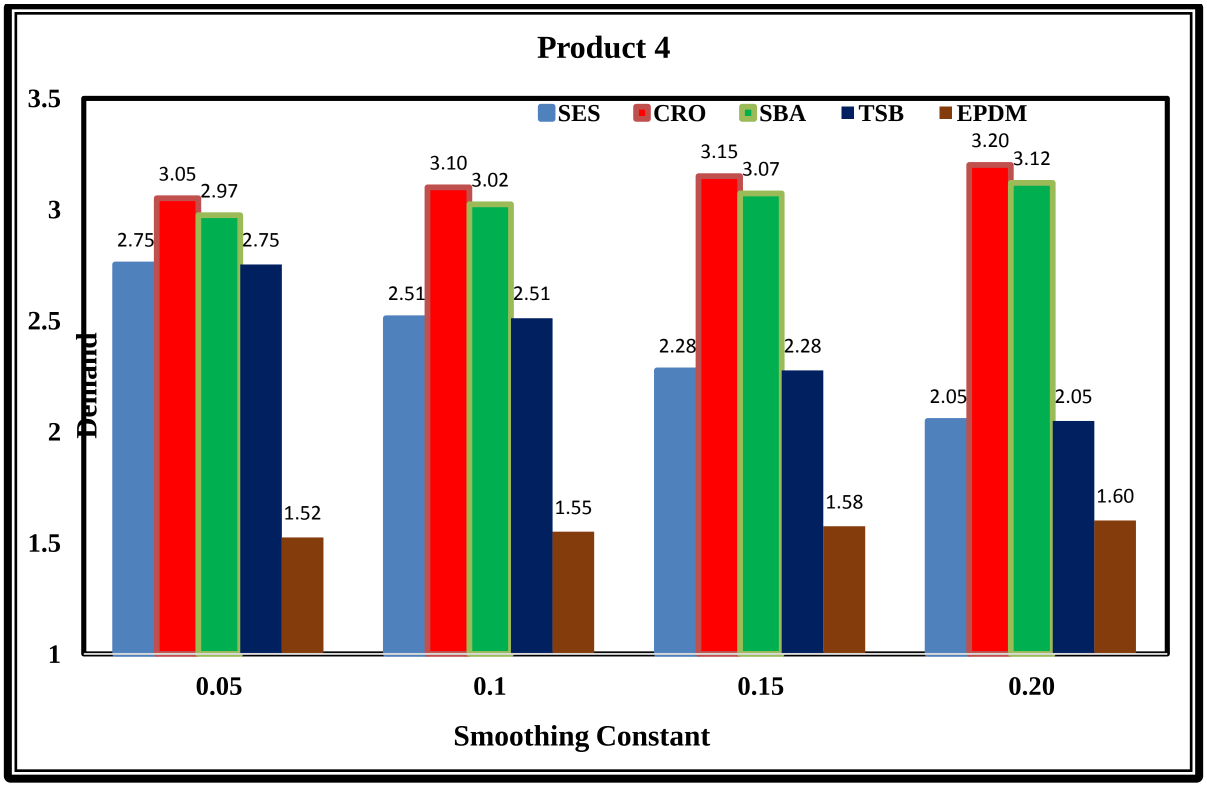

Figure 4.4-Comparison of forecasts for product 4.

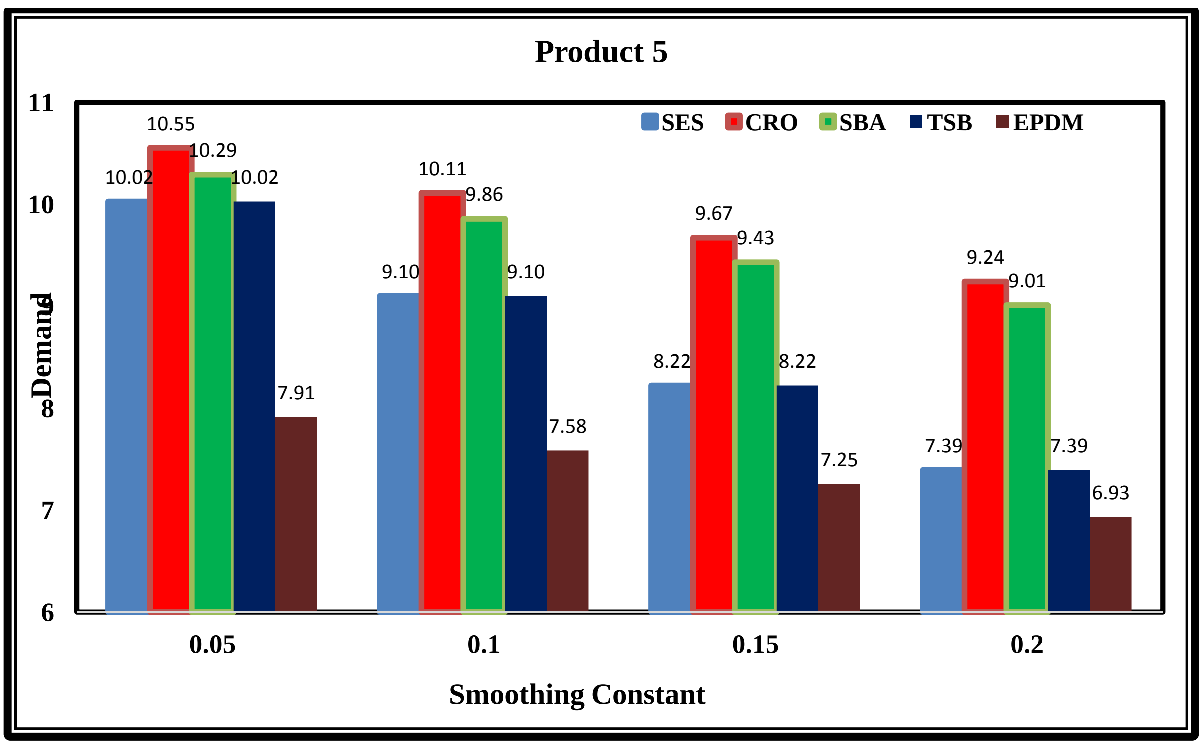

Figure 4.5-Comparison of forecasts for product 5.

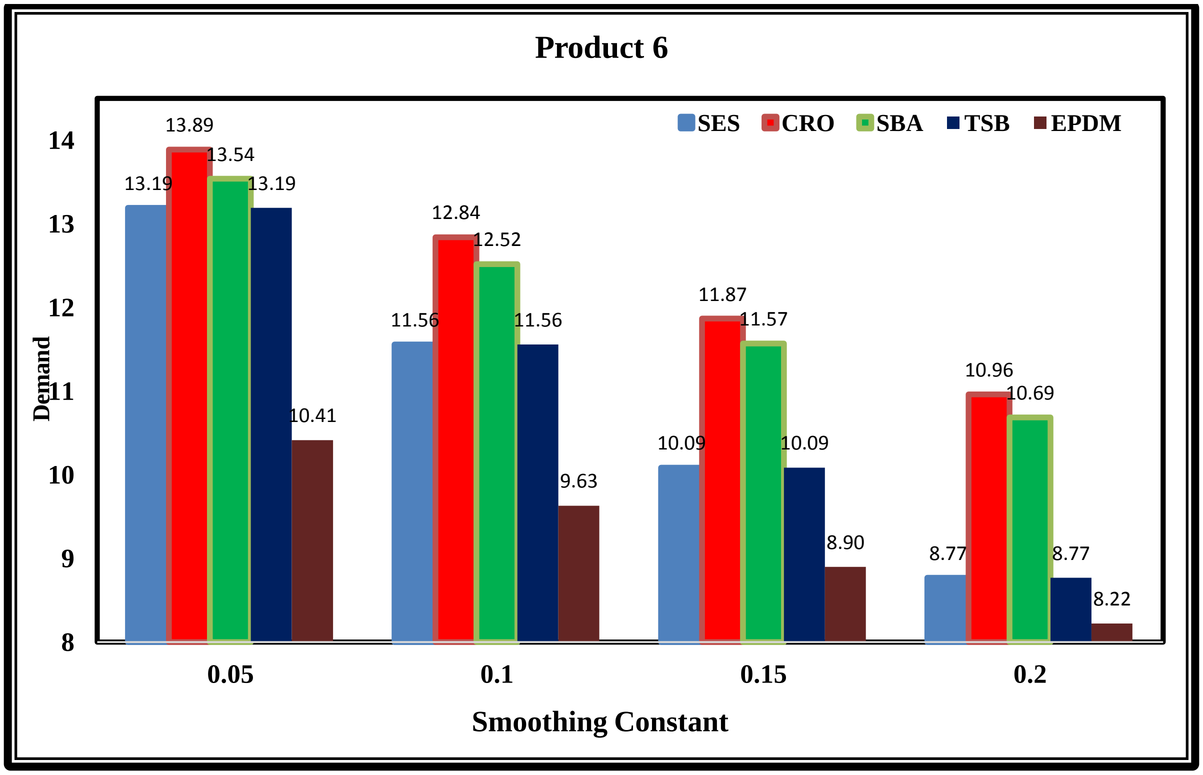

Figure 4.6-Comparison of forecasts for product 6.

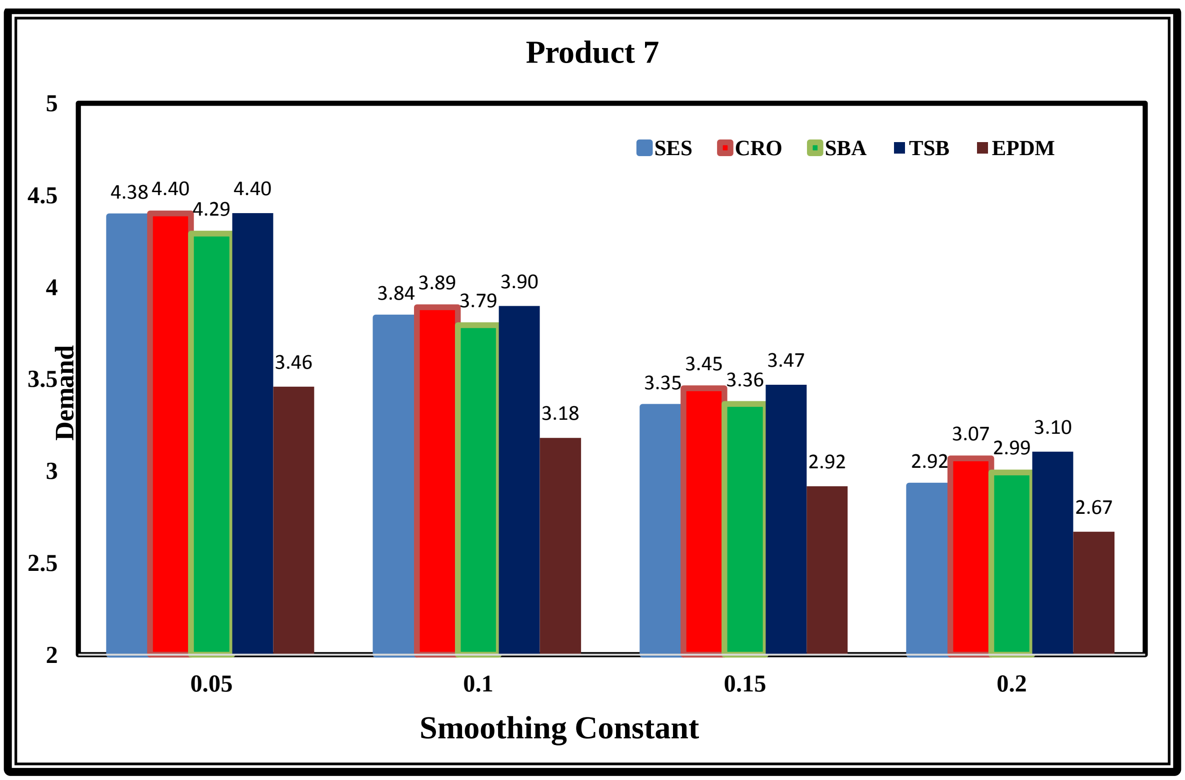

Figure 4.7 -Comparison of forecasts for product 7.

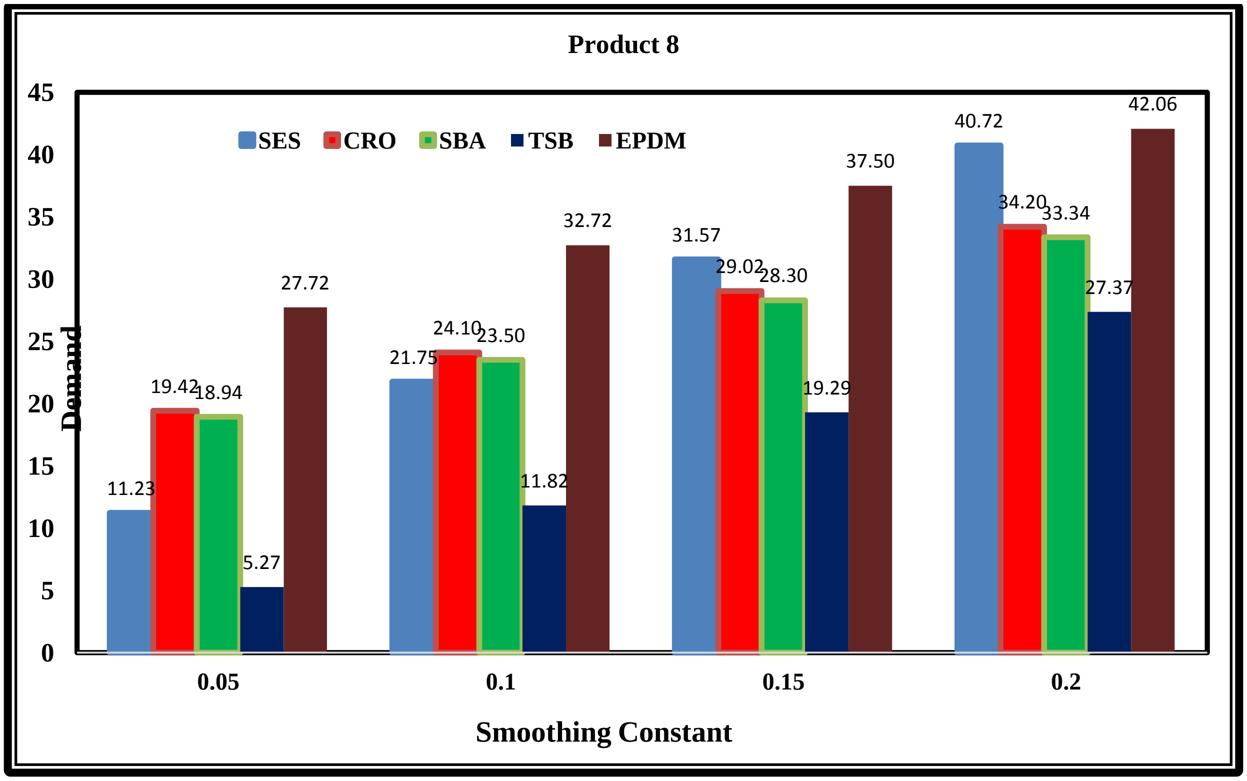

Figure 4.8-Comparison of forecasts for product 8.

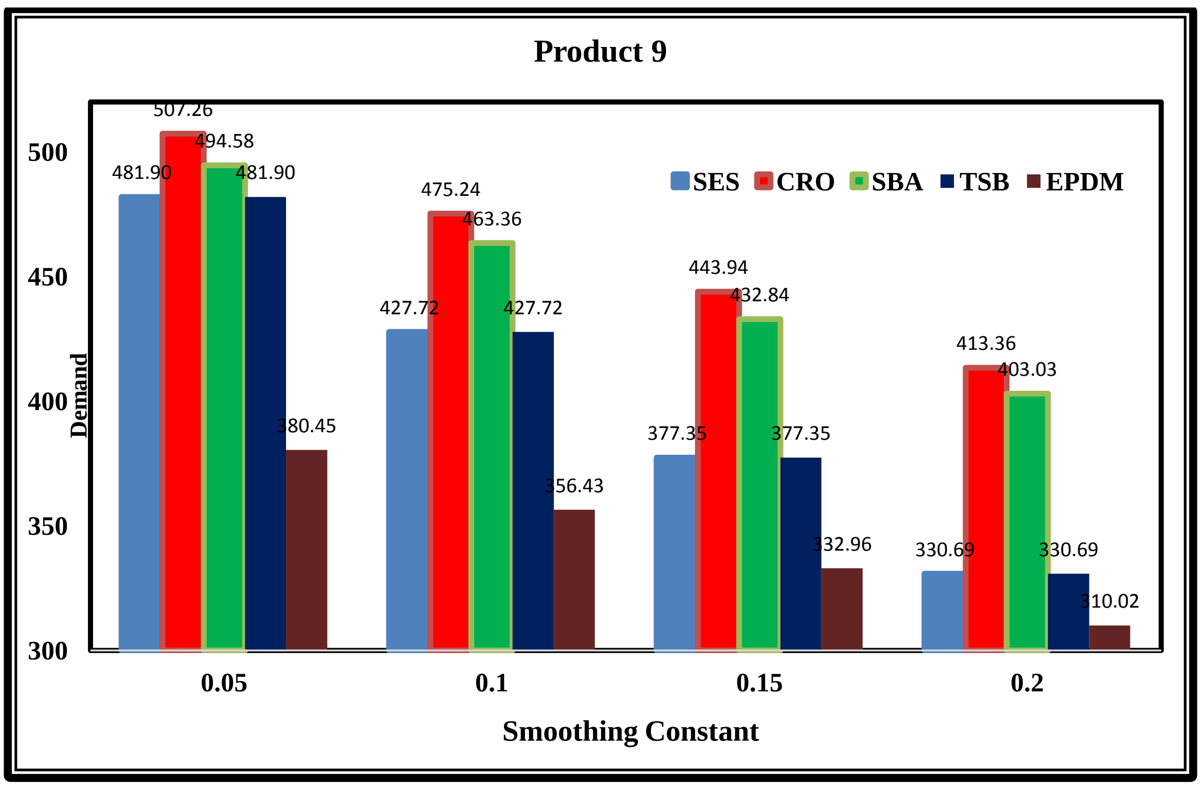

Figure 4.9-Comparison of forecasts for product 9.

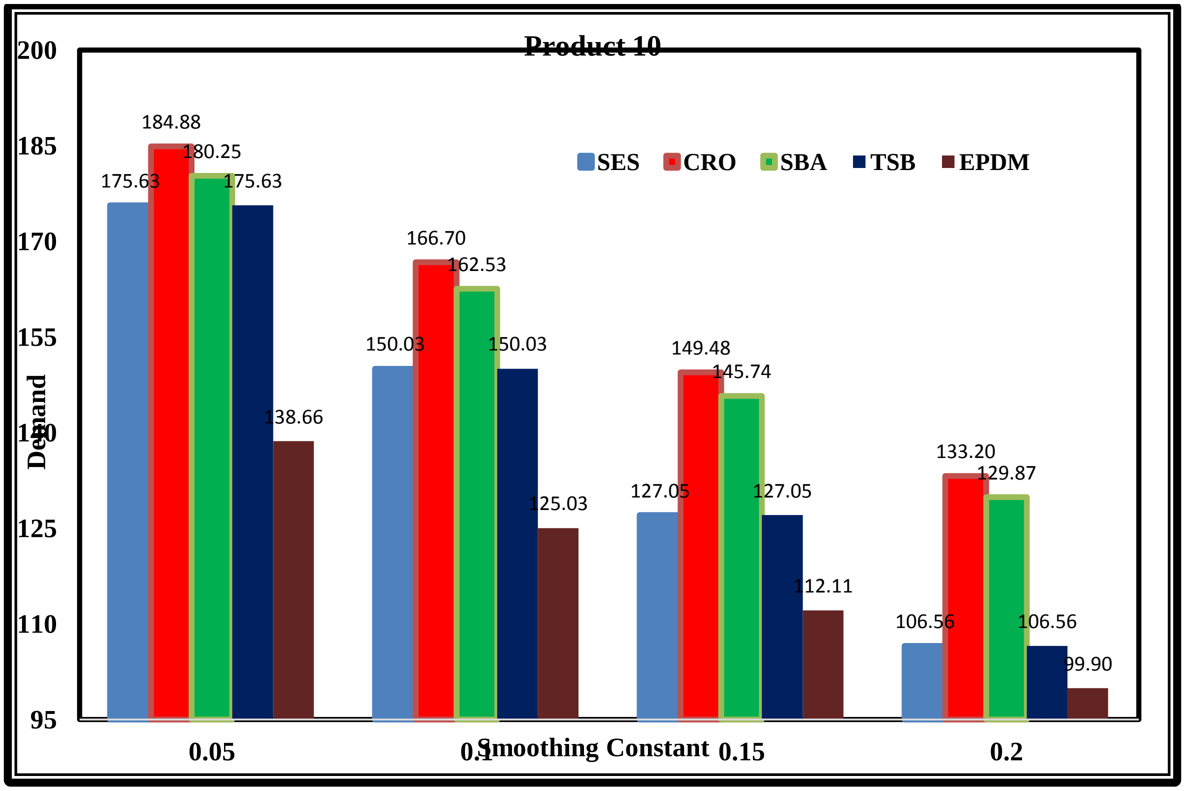

Figure 4.10-Comparison of forecasts for product 10.

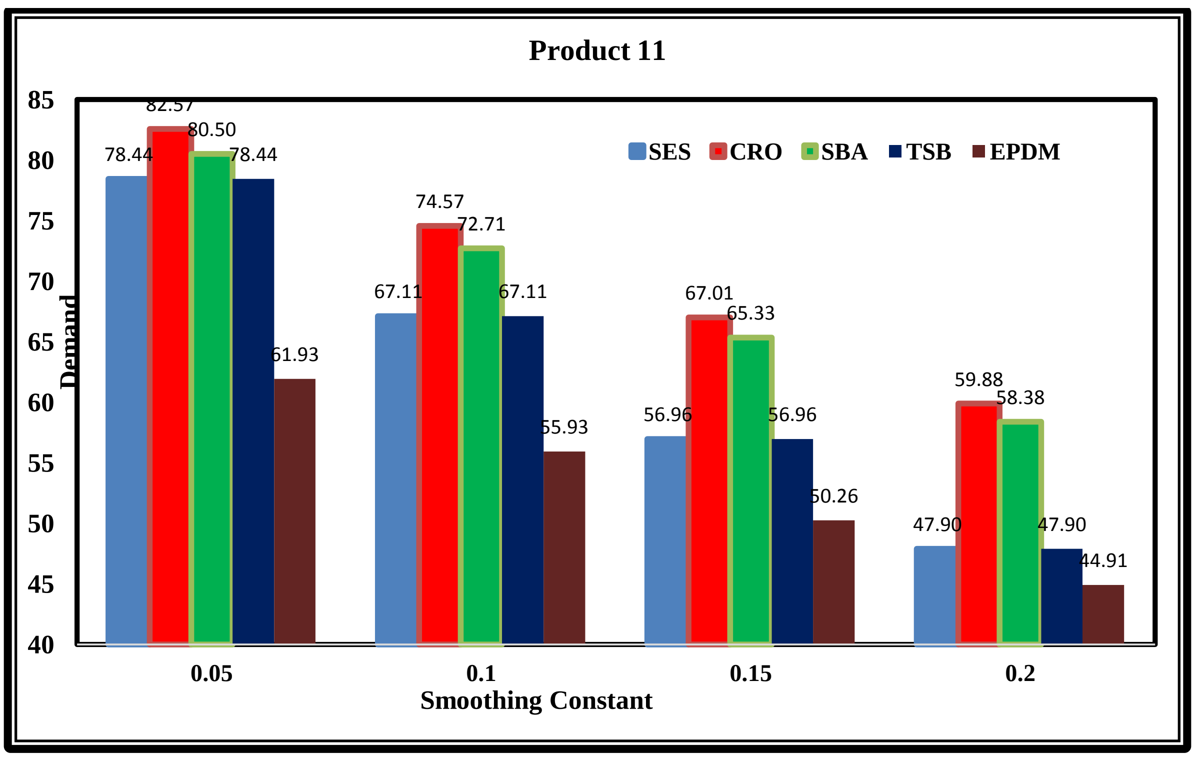

Figure 4.11-Comparison of forecasts for product 11.

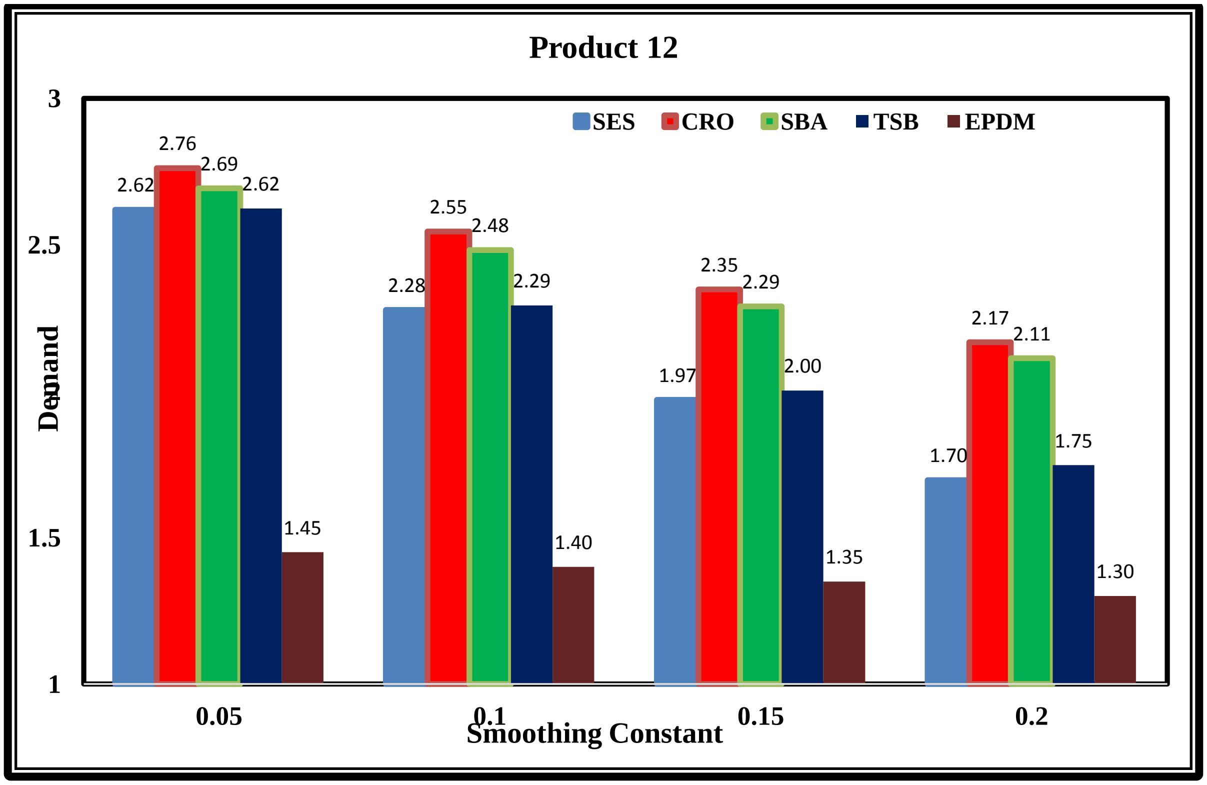

Figure 4.12-Comparison of forecasts for product 12.

4.4.2 FORECASTED VALUES FOR PRODUCTS HAVING LONG-DEMAND HISTORIES USING ENHANCED PROBABILISTIC DEMAND MODEL (EPDM)

The consumable used in the biscuits company whose demand is generally low is taken for the analysis over here and the data is shown in Table 4.8.The data shows that the products does not having continuous demand. Periods with no demands appear frequently.

Table 4.8- Actual data for slow moving products A, B, C

Period	Product A	Product B	Product C
1	0	0	0
2	5	0	0
3	114	13	0
4	664	22	1
5	0	23	99
6	6	1	100
7	56	40	100

8	0	18	100
9	32	3	0
10	0	7	0
11	54	7	0
12	0	5	0
13	18	15	0
14	0	20	100
15	40	6	0
16	20	20	0
17	8	0	0
18	24	0	200
19	0	11	0
20	20	42	0
21	4	5	0
22	9	9	100
23	38	5	0
24	0	4	100
25	22	4	0
26	4	28	0
27	0	9	100
28	20	11	0
29	0	8	0
30	19	0	0
31	320	32	100
32	81	0	0
33	28	0	0
34	6	10	0
35	8	10	0
36	0	10	100
37	120	50	0
38	8	12	0
39	0	53	0

40	14	16	100
41	44	10	0
42	0	13	100
43	0	0	0
44	9	35	0
45	28	34	0
46	0	1	100
47	22	3	0
48	0	8	0
49	0	10	0
50	0	0	100
51	0	8	0
52	30	0	0
53	16	0	0

The following tables 4.9-4.12 show the forecasted values of products having long-demand histories using SES, CRO, SBA, TSB and Enhanced Probabilistic Demand Model (EPDM) by varying the smoothing constant α from 0.05 to 0.2. The graphs of the same are shown in Fig 4.13-4.15.

Table 4.9-Comparison of forecasted values for products with α=0.05

	α=0.05				
Forecast	SES	CRO	SBA	TSB	EPDM
Product A	22.28	22.03	21.48	20.78	24.28
Product B	10.87	10.80	10.53	10.61	12.18
Product C	24.55	15.96	15.56	13.81	17.29

Table 4.10-Comparison of forecasted values for products with α=0.1

	α=0.1				
Forecast	SES	CRO	SBA	TSB	EPDM
Product A	17.11	20.22	19.72	19.51	23.34
Product B	9.90	12.09	11.79	10.00	11.99
Product C	22.87	23.84	23.25	18.93	25.20

Table 4.11-Comparison of forecasted values for products with α=0.15

	α=0.15				
Forecast	SES	CRO	SBA	TSB	EPDM
Product A	13.90	16.70	16.28	16.72	20.67
Product B	7.97	11.22	10.94	8.21	11.11
Product C	20.32	26.87	26.20	19.22	28.62

Table 4.12-Comparison of forecasted values for products with α=0.2

	α=0.2				
Forecast	SES	CRO	SBA	TSB	EPDM
Product A	12.63	14.10	13.75	15.16	18.48
Product B	6.22	9.99	9.74	6.50	10.08
Product C	17.98	27.70	27.01	17.83	29.94

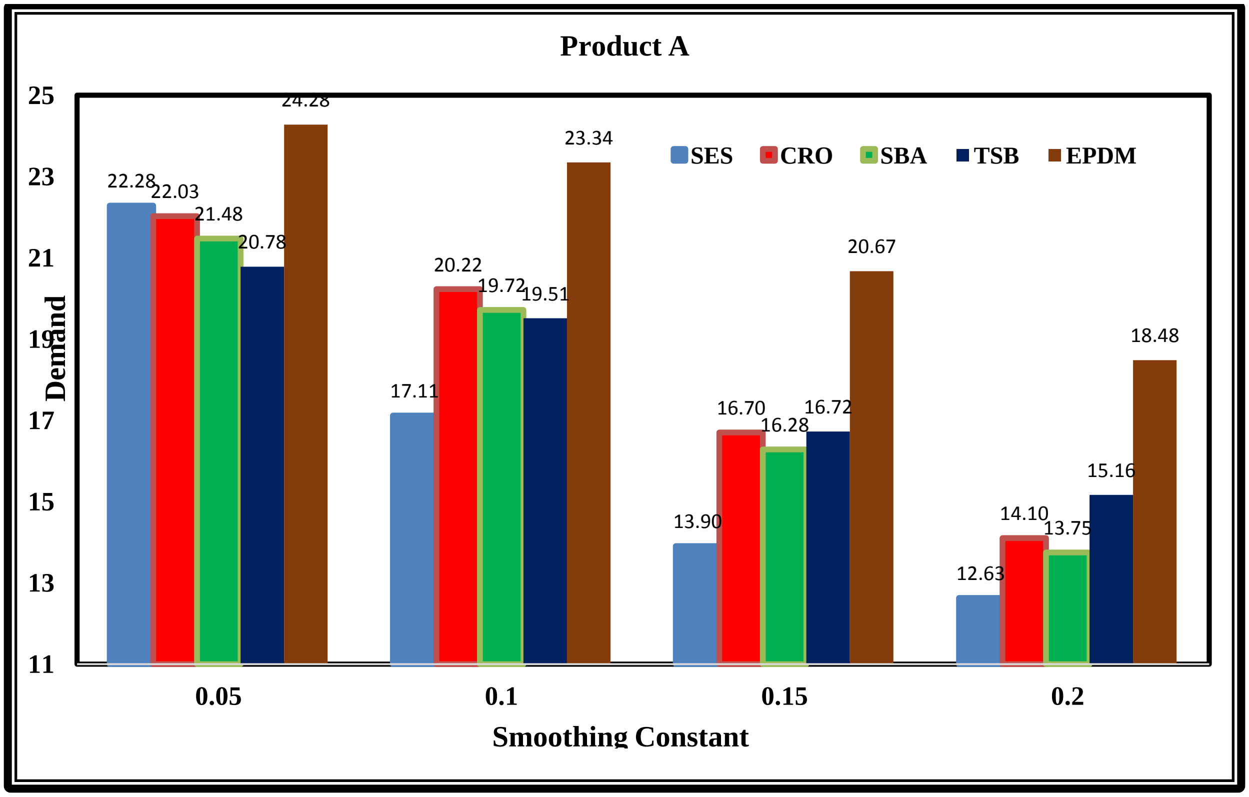

Figure 4.13- Comparison of forecasts for Product A.

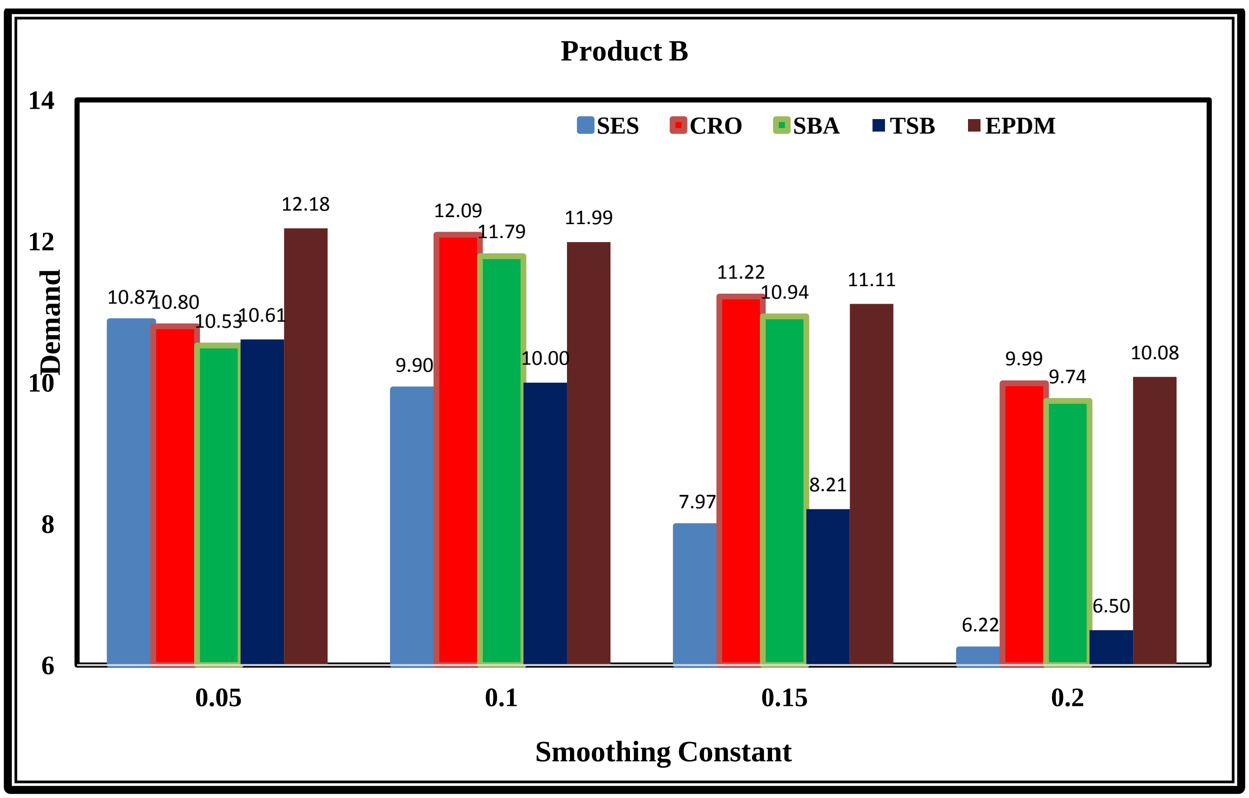

Figure 4.14-Comparison of forecasts for Product B.

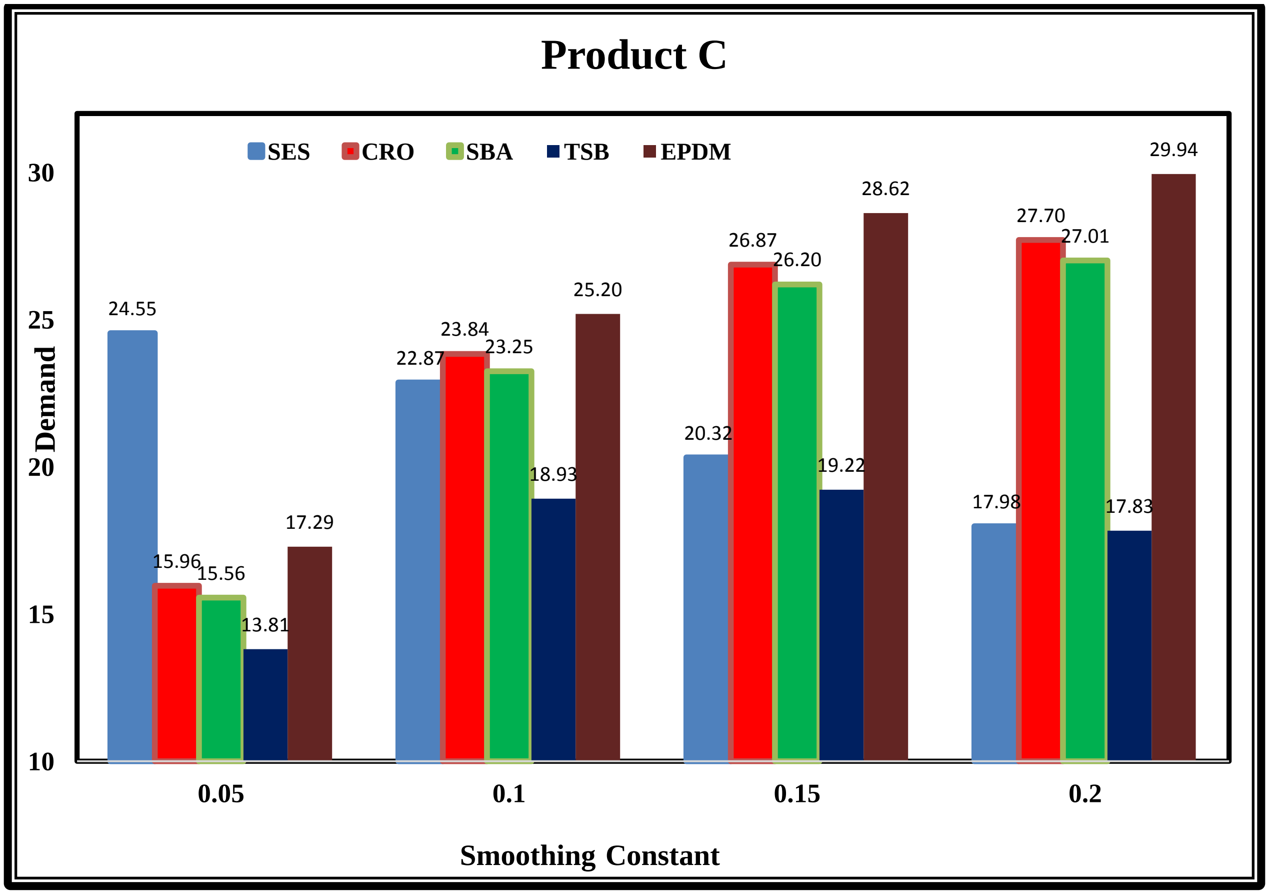

Figure 4.15-Comparison of forecasts for Product C.

4.4.3 RESULTS FOR DEMAND SIZE BASED MODEL (DSBM)

The forecast values are calculated using the Demand Size Based Model. Various values of smoothing constant from 0.1 to 0.6 are used to calculate the forecast so that the forecast values could be determined.

For analysis of Demand Size Based Model (DSBM), products A, B and Product C having long-demand histories are chosen whose actual values are shown in Table 4.8. The following tables 4.13, 4.15 and 4.17 show the forecasted values from period 36 at different values of smoothing constant from 0.1 to 0.6. To evaluate the performance of model, forecast accuracies Modified MAPE 1(M-MAPE), ME and MAD are calculated to find out variation from actual values which were shown in Tables 4.14, 4.16 and 4.18 and the Fig 4.16, 4.17 and 4.18 show the variation of accuracies at different values of smoothing constant.

Table 4.13- Forecasted Values for Product A.

PERIOD	ACTUAL	FORECAST					
		α=0.1	**α=0.2**	**α=0.3**	**α=0.4**	**α=0.5**	**α=0.6**
36	0	48.49	47.75	42.25	33.64	24.63	17.13
37	120	55.64	62.2	65.57	68.18	0	0
38	8	50.87	51.36	48.3	44.12	40.16	36.34
39	0	50.87	51.36	48.3	44.12	40.16	36.34
40	14	47.19	43.89	38.01	0	0	0
41	44	46.87	43.91	39.8	36.84	35.53	35.5
42	0	46.87	43.91	39.8	36.84	35.53	35.57
43	0	46.87	43.91	39.8	36.84	35.53	35.57
44	9	0	0	0	0	0	0
45	28	41.58	0	0	0	0	0
46	0	41.58	35.14	29.76	26.62	25.13	24.65
47	22	0	0	0	0	0	0
48	0	39.62	32.51	27.46	24.77	23.56	23.06
49	0	39.62	32.51	27.46	24.77	23.56	23.06
50	0	39.62	32.51	27.46	24.77	23.56	23.06
51	0	39.62	32.51	27.46	24.77	23.56	23.06
52	30	0	0	0	0	0	0
53	16	0	0	0	0	0	0

Table 4.14- Accuracy of forecast for Product A

Accuracy	Smoothing constant					
	0.1	0.2	0.3	0.4	0.5	0.6
ME	20.25	15.44	12.38	7.96	2.35	1.32
MAD	33.84	29.13	27.70	29.09	25.62	25.53
M-MAPE	2.09	1.80	1.71	1.80	1.58	1.58

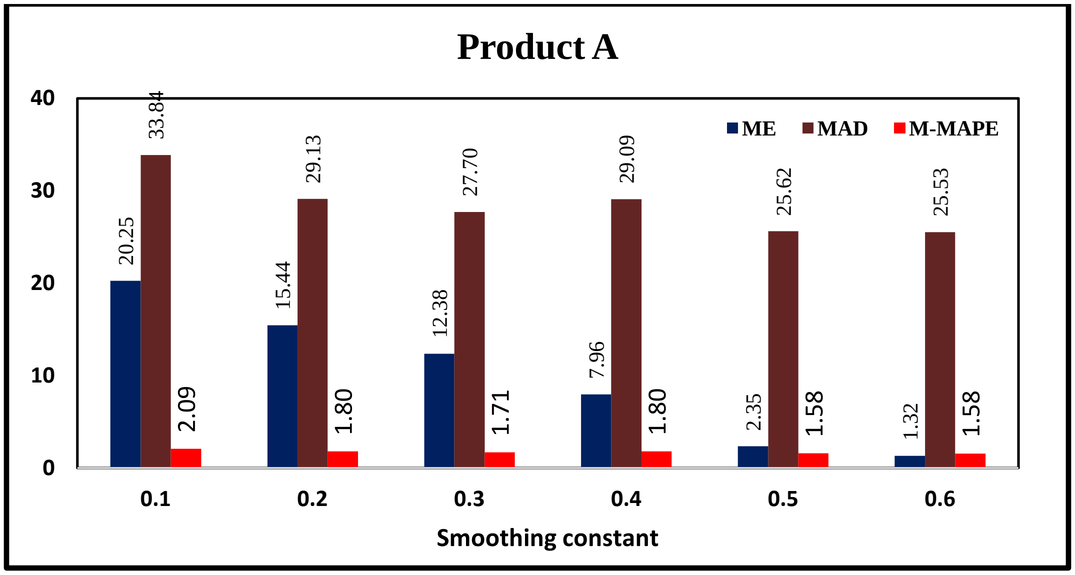

Figure 4.16-Comparison of accuracies with varying α.

Table 4.15- Forecasted Values for Product B.

PERIOD	ACTUAL	FORECAST					
		α=0.1	α=0.2	α=0.3	α=0.4	α=0.5	α=0.6
36	10	13.06	12.79	12.45	11.95	11.37	10.83
37	50	16.76	20.23	23.71	27.17	30.69	34.33
38	12	16.28	18.58	20.2	21.1	21.34	20.93
39	53	19.95	25.47	30.04	33.86	37.17	40.17
40	16	19.56	23.57	25.83	26.72	26.59	25.67
41	10	18.6	20.86	21.08	20.03	18.29	16.27
42	13	18.04	19.29	18.66	17.22	15.65	14.31
43	0	18.04	19.29	18.66	17.22	15.65	14.31
44	35	19.74	22.43	23.56	24.33	0	0
45	34	21.16	24.74	26.69	28.19	29.66	31.08
46	1	19.15	19.99	18.98	17.32	15.33	13.04
47	3	17.53	16.6	14.19	11.59	9.17	7.01
48	8	16.58	14.88	12.33	10.15	8.58	7.61
49	10	15.92	13.9	11.63	10.09	9.29	9.04
50	0	15.92	13.9	11.63	10.09	9.29	9.04
51	8	15.13	12.72	10.54	9.26	0	0
52	0	15.13	12.72	10.54	9.26	8.65	8.42
53	0	15.13	12.72	10.54	9.26	8.65	8.42

Table 4.16- Accuracy of forecast for Product B

ACCURACY	Smoothing constant					
	α=0.1	α=0.2	α=0.3	α=0.4	α=0.5	α=0.6
ME	2.56	3.47	3.40	3.09	0.81	0.53
MAD	14.90	15.10	14.82	14.55	15.52	15.75
M-MAPE	1.00	1.01	1.00	0.98	1.04	1.06

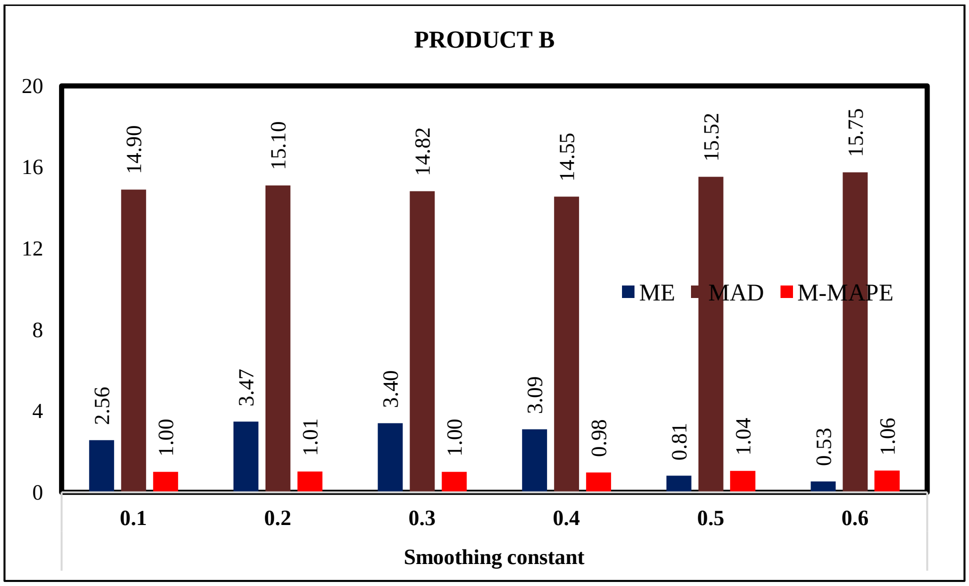

Figure 4.17-Comparison of accuracies with varying α.

Table 4.17- Forecasted Values for product C

PERIOD	ACTUAL	FORECAST					
		α=0.1	α=0.2	α=0.3	α=0.4	α=0.5	α=0.6
36	100	0	0	0	0	0	0
37	0	0	0	0	0	0	0
38	0	74.8	0	0	0	0	0
39	0	74.8	98.03	103.06	102.75	101.51	100.61
40	100	0	0	0	0	0	0
41	0	0	0	0	0	0	0
42	100	0	0	0	0	0	0
43	0	0	0	0	0	0	0
44	0	79.59	98.74	101.51	100.99	100.38	100.1
45	0	79.59	98.74	101.51	100.99	100.38	100.1
46	100	0	0	0	0	0	0

47	0	0	0	0	0	0	0
48	0	81.63	98.99	0	0	0	0
49	0	81.63	98.99	101.06	100.59	100.19	100.04
50	100	0	0	0	0	0	0
51	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0
53	0	83.47	99.19	100.74	100.36	100.09	100.02

Table 4.18- Accuracy of forecast for Product C

ACCURACY	Smoothing constant					
	α=0.1	α=0.2	α=0.3	α=0.4	α=0.5	α=0.6
ME	4.24	5.50	0.42	0.31	0.14	0.05
MAD	23.53	17.76	12.18	12.08	11.91	11.81
M-MAPE	1.00	0.75	0.52	0.51	0.51	0.50

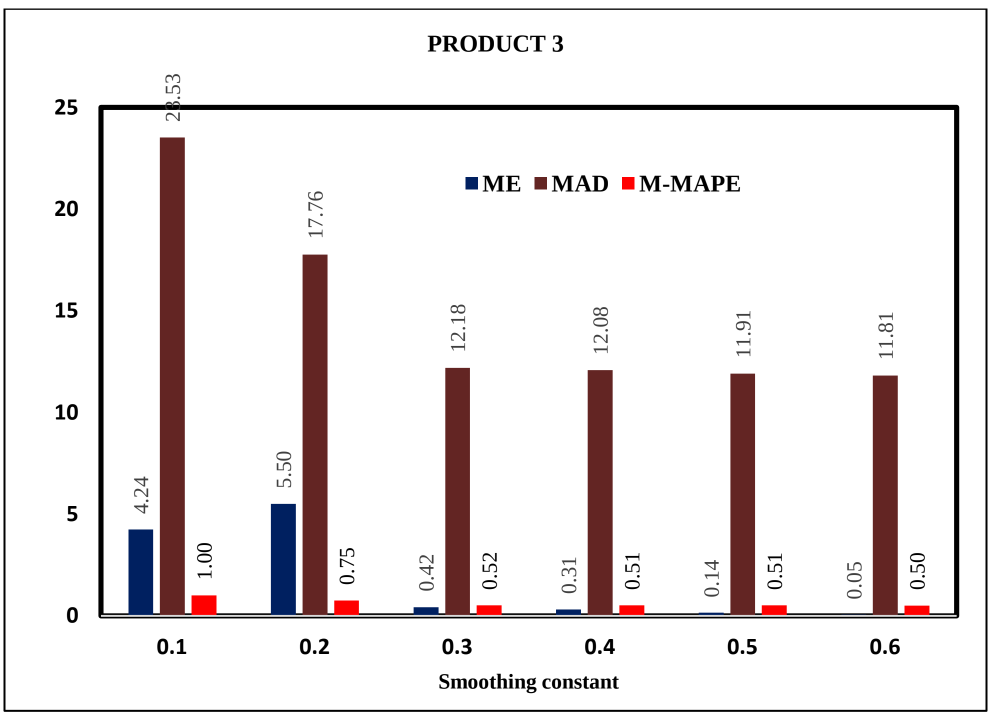

Figure 4.18-Comparison of accuracies with varying α.

New models EPDM and DSBM, by modifying the Croston's method have been developed by considering actual probabilities in the demand size per period and directly estimating the demand size in the second model without any average per period considerations.

The new models developed have been compared with the commonly used demand forecasting models. The results show that the models developed have close approximations with them .They also appear to be versatile for different values of the smoothening constant.i.e. Good forecast values are captured for different values of alpha. It is understood clearly form the graphs that the Enhanced Probabilistic Demand Model (EPDM) works fine for slow moving products with long and short demand histories. They give better results compared to Croston and Syntetos Boylan Approximation at selected values of smoothing constants .It is comparatively easier to differentiate the data based on the probability values without the average demand interval and covariance. Demand Size Based Model (DSBM), which is the modified Croston method, paves way for simple method for forecasting of zero demand sizes or the interval between the non-zero demand sizes. It is inferred from the calculated values of mean error and mean absolute deviation, that the model results in the accurate values at higher values of smoothing constant α=0.5 and 0.6 as compared to other models .Since more preference is given to the actual demand sizes or time intervals, estimation of the forecast accurately with respect to varying demand sizes is easier. Irrespective of data, M-MAPE is very less which is approximately equal to one and resembles a better forecast than the other models used for intermittent demand forecast which vary randomly with respect to data. Thus Demand Size Based Model (DSBM) can be used to forecast slow moving products having long demand histories whereas for short demand histories it can't be used.