Chapter 6

Transport of Heavy Quark-Parton Cascade Model

Relativistic heavy ion collision at RHIC and the LHC have given rise to a new phase of matter. When two heavy ion collide, the region of their collision consists of deconfined gluons and quarks within a very small region of space. This state of matter, as we know today, is called quark gluon plasma (QGP) [1]–[4].

One of the signals coming out of QGP is quenching of heavy quarks. On other hand high momentum hadron spectra are observed to be highly suppressed relative to those in proton on proton collisions [5, 6], suggesting a quenching effect due to the QGP medium. A similar effect is observed for high $p_T$ charm or beauty quarks with most recent results showing suppression of D or B mesons to same order as that of light partons [7], the focus is now shifted to the accurate theoretical calculations from experimental observations. However, calculations from hydrodynamics give a rough estimate of the ratio of thermalization time for heavy quarks, $\tau_Q$, and light partons, $\tau'_q$, [8], $\frac{\tau_Q}{\tau'_q} \sim \frac{M_Q}{T}$. For $M_Q = 1.35–4.5 \text{ GeV}$ and $T = 300 \text{ MeV}$, this ratio is found to be $\sim 5$ and suggests that relaxation time for heavy quarks is larger than that of light quarks and gluons. If thermalization time, $\tau_{q/g}$, is taken to be $\mathcal{O}(1\text{fm}/c)$, and if equilibrium temperature, $T_i$ and freeze-out temperature, $T_f$ are taken as 300 MeV and 170 MeV [9] respectively, then lifetime of QGP can be approximately shown to be
This might imply that heavy quark relaxation time for $T=300$ MeV is comparable to QGP lifetime at this condition. Even if heavy quark is subjected to large suppression [7], it may not fully thermalize in QGP.

In last chapter, we discussed the calculations of charm quark energy loss by the mechanism of multiple scattering with medium particles. Other theoretical calculations and phenomenological models of heavy quarks energy loss have also been developed in recent years [10]–[14]. In these literatures, both elastic scattering and inelastic gluon emission off heavy quarks have been suggested as the major mechanisms by which a heavy quark may lose energy in the presence of a thermal medium. In most of these earlier works, collisional energy loss seems to dominate in the lower momentum region while radiative energy loss emerges as the chief mechanism for higher momenta charms.

Transport models can attempt to fully describe a heavy ion collision and the ensuing dynamics. The Parton Cascade Model is one such transport calculations [15, 16, 17]. It is based on Boltzman transport equation and does not include any equilibration assumptions. However the calculations must be well calibrated and validated under controlled conditions before utilizing them fully for heavy quarks.

In this chapter, we will discuss in detail the evolution of charm quark using Parton Cascade Model. In section 6.2, we show our results calculated for a fixed temperature, followed by discussions. In the final section we summarize our results and suggest future lines of inquiry.

### 6.1 Parton Cascade Model

The Parton Cascade Model VNI/BMS [18, 19, 20] forms the basis for our present study. This model can be used to study the full time evolution of hard probes and a thermal QCD medium. The PCM has been used to study gluons and lighter quarks as hard probes of the QGP. In the current work we use VNI/BMS to study charm quark evolution in an infinite QGP medium for the first time. It is necessary to check the consistency of our calculations within a controlled environment which we will discuss next.
The QGP like medium effect is modeled by taking a box of finite volume with periodic boundary conditions. This provides a system of infinite matter at fixed temperature. The matter inside the box is thermalized quarks and gluons (QGP) and their thermal distributions are used to generate partons at a given temperature and zero chemical potential. We insert a charm with the four momentum \( p^\mu = \{0, 0, p_z, E = \sqrt{p_z^2 + M_c^2} \} \) into the box and let it evolve according to the Relativistic Boltzmann Equation given by,

\[
p^\mu \frac{\partial F_j(x, \vec{p})}{\partial x^\mu} = \sum_{\text{processes } i} C_i[F_j],
\]

where \( F_j(x, \vec{p}) \) is the charm single particle phase space distribution and the collision term on r.h.s. is a non-linear functional of phase space distribution terms inside an integral and can be expressed as:

\[
C_i(F) = (\pm) \frac{1}{2E_i S_i} \int \prod d\Gamma_j |\mathcal{M}|^2 (2\pi)^4 \delta^4(P_{\text{in}} - P_{\text{out}}) D(F_j(x, \vec{p})),
\]

\[
D(F_j) = \prod_{\text{in}} F_j \prod_{\text{out}} [1 \pm F_i] - \prod_{\text{in}} [1 \pm F_i] \prod_{\text{out}} F_j,
\]

\[
\prod d\Gamma_j = \prod_{i \neq j} \frac{d^3 p_j}{(2\pi)^3 (2E_j)}
\]

Here \( D(F_j) \) is the quantum statistical distribution factor and \( S_i \) is another factor required for averaging over all the particle species excluding ‘ith’ parton and defined as,

\[
S_i = \prod_{j \neq i} K^{\text{in } a}_i K^{\text{out } a}_i,
\]

with \( K^{\text{in, out } a}_i \) being the identical partons of species ‘a’ in the initial or the final process \( j \).

We have included the matrix elements for all \( 2 \rightarrow 2 \) binary elastic scattering processes for charm interaction with gluons or light quarks \((u, d, s)\) and \( 2 \rightarrow n \) process for radiative (brehmsstrahlung) corrections after each scattering. Mathematically \( 2 \rightarrow n \) process can be shown to be \( 2 \rightarrow 2 \) elements multiplied by a radiative factor,

\[
|\mathcal{M}|^2 = |\mathcal{M}_{ab \rightarrow cd}(\hat{s}, \hat{t}, \hat{u}, Q^2)|^2 [T_c(Q^2, \mu_0^2)T_d(Q^2, \mu_0^2)],
\]

where \( T(Q^2, \mu_0^2) \) is the time-like branching or ‘Sudakov’ radiative factor after each scattering. We will return to this shortly afterwards.
6.1.1 Elastic scattering of charm quark

The matrix elements for elastic \((ab \rightarrow cd)\) processes in Eqn. 6.4 are
\[
gc \rightarrow gc, \tag{6.5}
\]
\[
q(\bar{q})c \rightarrow q(\bar{q})c.
\]

The corresponding differential scattering cross section is defined to be,
\[
\frac{d\hat{\sigma}}{dQ^2} = \frac{1}{16\pi(\hat{s} - M^2_{cd})^2} \sum |M|_{ab \rightarrow cd}^2. \tag{6.6}
\]

The total cross section is also calculated and used in the calculations to select interacting pairs. The total cross section can be shown to be,
\[
\hat{\sigma}_{\text{tot}} = \sum_{c,d} \int_{\mu^2_{\text{min}}}^{\hat{s}} \left( \frac{d\hat{\sigma}}{dQ^2} \right)_{ab \rightarrow cd} dQ^2. \tag{6.7}
\]

The invariant transition amplitude, \(|M|^2\) for elastic scattering which can be calculated or obtained from [21], are shown below for \(q(\bar{q})c \rightarrow q(\bar{q})c\),
\[
\sum |M|^2 = \frac{64\pi^2\alpha_s^2}{9} \frac{(M^2_c - \hat{u})^2 + (\hat{s} - M^2_c)^2 + 2M^2_c\hat{t}}{\hat{t} - \mu^2_D}. \tag{6.8}
\]

While, for \(gc \rightarrow gc\),
\[
\sum |M|^2 = \pi^2\alpha_s^2[g1 + g2 + g3 + g4 + g5 + g6],
\]

where,
\[
g1 = 32\frac{(\hat{s} - M^2_c)(M^2_c - \hat{u})}{(\hat{t} - \mu^2_D)^2},
g2 = \frac{64}{9} \frac{(\hat{s} - M^2_c)(M^2_c - \hat{u}) + 2M^2_c(\hat{s} + M^2_c)}{(\hat{s} - M^2_c)^2},
g3 = \frac{64}{9} \frac{(\hat{s} - M^2_c)(M^2_c - \hat{u}) + 2M^2_c(\hat{t} + \hat{u})}{(M^2_c - \hat{u})^2},
g4 = \frac{16}{9} \frac{M^2_c(4M^2_c - \hat{t})}{(\hat{s} - M^2_c)(M^2_c - \hat{u})},
g5 = \frac{16}{9} \frac{(\hat{s} - M^2_c)(M^2_c - \hat{u}) + M^2_c(\hat{s} - \hat{u})}{(\hat{t} - \mu^2_D)(\hat{s} - M^2_c)},
g6 = -16\frac{(\hat{s} - M^2_c)(M^2_c - \hat{u}) - M^2_c(\hat{s} - \hat{u})}{(\hat{t} - \mu^2_D)(M^2_c - \hat{u})}. \tag{6.9}
\]
In order to regularize the cross sections we have used the thermal mass of QGP medium which is defined as 
\[ \mu_D = \sqrt{(2N_c + N_f)/6gT}, \]
where \( g = \sqrt{4\pi\alpha_s} \). \( N_c \), no. of colours and \( N_f \), no. of flavours are taken 4 and 3 respectively. We have kept strong coupling, \( \alpha_s = 0.3 \) for the entire calculation.

The Boltzmann transport equation is then solved numerically via Monte Carlo algorithms, a geometric interpretation of the cross section is used to select which collisions will occur.

### 6.1.2 Charm Quark Radiation

It is known that collisional loss alone is unable to explain the data showing suppression of non-photonic electrons at RHIC or D mesons at LHC [22]. On one hand hard thermal loop (HTL) approximation [23, 24] predicts a large drag on heavy quark which is much bigger than what experimental data has suggested, while the radiative corrections to heavy quark energy loss when combined with elastic scattering is able to explain the results agreeably [22].

In our calculations, radiative corrections are included in form of time-like branching of the probe charm into a final charm and a shower of radiated partons. The basic idea is that during a binary scattering the outgoing partons may acquire some virtuality. These partons are allowed to radiate a shower of partons until their virtuality decreases to some preassigned cutoff value, \( \mu_0^2 \) (\( \approx M_C^2 \) for charm quarks). The probability for time-like branching \( (b \to cd) \), where a parton of time-like virtual mass, \( M_b^2 \), decays into partons \( c, d \) with \( M_{c,d}^2 < M_b^2 \) and having fractions \( z \) and \( (1 - z) \) respectively, of momentum, \( 'p_b' \), of the parent parton and requires to follow the kinematics,

\[
M_b^2 = \frac{M_c^2}{z} + \frac{M_d^2}{(1-z)} \quad \frac{k^2}{z(1-z)}, \quad \quad (6.10)
\]

The time-like branching probability are associated with Altarelli-Parisi splitting function, \( P_{b\to cd}(z) \), which is defined as

\[
P_{b\to cd}(z) \equiv P_{Q\to Qg}(z) = \frac{4}{3} \frac{1 + z^2}{1 - z}, \quad \quad (6.11)
\]
Finally the Sudakov time-like radiative factor is given by,

\[ T_a(Q^2, \mu_0^2) = \exp \left[ - \int_t^{Q_{\text{max}}^2} dQ^2 \frac{\alpha_s(Q^2)}{2\pi} \int dz P_{b\rightarrow c+d}(z) \right] \]  

(6.12)

which is then put into Eqn. 6.4 to get the final matrix elements.

Let us now move to another aspect of gluon radiation off heavy quarks. Any quark subjected to multiple collision may radiate a shower of partons as we discussed earlier. However emission of multiple partons within a certain length scale may lead to a reduction of the bremsstrahlung cross-sections which we can briefly discuss here. This reduction in emitted gluon spectrum is known as Landau Pomeranchuk Migdal (LPM) effect [26]. This arises from the fact that if the formation time of an emitted gluon, \( \tau_f \), after a \( Qg(Qq) \) scattering is larger than the typical mean free path, \( \lambda \), of the heavy quark itself, then a gluon emitted from the next scattering centre may interact coherently with the initial gluon. This interference of emitted gluons may continue if there are a number of scattering centres before the shower of gluons dissociates itself completely from the emitting parton. This is different from Bethe-Heitler (BH) [27] case where all the emitted gluons are assumed to be incoherent and by construction are independent of each other.

The formation time of an emitted parton is \( \tau_f \),

\[ \tau_f = \frac{2\omega}{k_\perp^2}. \]  

(6.13)

If \( \tau_f > \lambda \), then the number of coherent scattering centres is found by Baier et al (BDMPS) to be [28]

\[ N_{\text{coh}} = \frac{\tau_f}{\lambda} \sim \sqrt{\frac{\omega}{\mu^2 \lambda}}, \mu^2 = \langle k_\perp^2 \rangle. \]  

(6.14)

For \( \omega \gg \mu^2 \lambda, N_{\text{coh}} > 1 \), it can be shown that

\[ \omega \frac{dI_{\text{LPM}}}{d\omega dz} = \omega \frac{dI_{\text{BH}}}{d\omega dz} \frac{1}{N_{\text{coh}}}. \]  

(6.15)

This implies a typical suppression of emitted gluon spectrum in case of coherent(LPM) emission when compared to incoherent(BH) gluon distribution. The incoherent gluon distribution is approximately given by,

\[ \omega \frac{dI_{\text{BH}}}{d\omega dz} \sim \frac{\alpha_s}{\lambda}. \]  

(6.16)
It has been found that there exists a certain critical length, \( L_{cr} = \lambda \sqrt{\frac{E}{\mu^2}} \), where \( E \) is the probe charm energy. For a distance, \( L > L_{cr} \), traveled by the charm, the average energy loss by charm can be shown as:

\[
-\Delta E_{LPM} = \int dz \int d\omega \frac{dI_{LPM}}{d\omega dz},
\]

\[
\approx \frac{\alpha_s L}{\lambda} \sqrt{\frac{\mu^2}{\lambda} E},
\]

\[
\propto \sqrt{E}. \tag{6.17}
\]

This implies that the average energy loss by a heavy quark in the case of LPM radiative loss is less than that of BH where \( \Delta E_{BH} \propto E \) instead.

However for \( L < L_{cr} \), the energy loss by the probe is found to be

\[
-\Delta E_{LPM} \approx \int dz \sqrt{\frac{\mu^2}{\lambda} \omega_f}, \quad \omega_f = \frac{\mu^2 L^2}{\lambda}
\]

\[
\propto L^2 \tag{6.18}
\]

which shows a strong quadratic dependence of \( \Delta E \) on length traversed by probe or system size.

Gluon bremsstrahlung from heavy quarks differ from light quark. Emission of gluons by heavy quarks at very small angles is suppressed compared to light quarks. This phenomenon is commonly called the Dead Cone effect [29]. Mathematically this is given by a certain dead cone factor, which can be obtained from emitted gluon distributions from light(\( q \)) and heavy quarks(\( Q \)) as:

\[
\omega \frac{dI_q}{d\omega} = \frac{\alpha_s C_F}{\pi} \frac{dk^2}{k^2} \tag{6.19}
\]

\[
\omega \frac{dI_Q}{d\omega} = \frac{\alpha_s C_F}{\pi} \frac{k^2 dk^2}{(k^2 + \omega^2 \theta_0^2)^2}, \text{ where } \theta_0^2 = \frac{M_Q}{E_Q}
\]

Now from ratio of the above two gluon distributions, it can be shown for small angle approximation, \( k_\perp \approx \omega \theta \) that

\[
dI_Q = dI_q (1 + \frac{\theta_0^2}{\theta^2})^{-2} \tag{6.20}
\]
where we may define,

$$D = (1 + \frac{\theta_0^2}{\theta^2})^{-2}$$

(6.21)

to be the dead cone factor. The dead cone term is intrinsically present in all $2 \rightarrow 3$ processes namely $Qg \rightarrow Qgg$, $Qq \rightarrow Qqg$ etc., matrix elements as shown earlier by [30].

Radiative energy loss via LPM effect has been calculated earlier for heavy quarks by [30, 31]. The LPM effect in radiative corrections to charm quark energy loss has been utilized to describe the observed suppression of single non-photonic electrons [12].

The Higher-Twist matrix approach for gluon radiations can give LPM effect, too. The HT matrix elements integrate over all the multiple collision interference effects up to the order of process used in the calculations. In VNI/BMS calculations, a leading order approximation of this effect has been assumed and is being treated dynamically. The LPM mechanism has been formulated in terms of Monte Carlo simulation by [32] and has been used by [33] for gluons and light quarks. In the present work similar techniques are used to calculate the energy loss of charm quark via radiation(bremsstrahlung) and added to the collisional(elastic) loss to obtain the average energy loss, momentum broadening per unit length, $\hat{q}$ and energy loss per unit length traversed.
6.2 Results and Discussions

In our calculations we have set the strong coupling constant to a fixed value of $\alpha_s = 0.3$ to allow comparison with analytical calculations and other transport models. The temperature is set to $T=350$ MeV, which is roughly the average temperature of the QGP phase attained at RHIC energies. The mass of charm is taken as $M_c = 1.35$ GeV.

In 6.1, we show energy loss, $\Delta E$, of charm quark over a given path length ($\tau \approx L \sim 5 \text{fm}/c, c \sim 1$ in this case). For discussions on path length dependence of energy loss evolution, other figures in this paper will be referred soon.

Now let us return to 6.1 for specific discussions. The loss due to elastic scattering, gluon radiation and total loss due to both are shown separately in the same figure. We find that collisional loss dominates over radiation up to $7-12$ GeV, and beyond this elastic loss shows a tendency to decrease while radiation enhances and finally dominates the picture. However the radiation too appears to decrease for very high energy charms. We feel that as momentum of charm increases, the no. of elastic scattering tends to saturate so that average collisional energy loss becomes almost constant for all higher energy charms. Also as kinetic energy
of charm is increased, medium induced gluon radiation increases, making it the dominant energy loss mechanism at high energies. But let us recall that in our case, radiation takes place only after elastic scattering, and as the no. of scattering saturates ultimately, so does the radiative loss for very high energy charm quarks.

In [8], it has been discussed that for small coupling, $\alpha_s$, collisional loss tends to dominate for low and intermediate energy charm (for $\gamma v_{Q} \sim 1$, $\gamma = (1 - \beta^2)^{-1/2}$) while for higher energetic heavy quarks we have bremsstrahlung (for $\gamma v_{Q} \sim 1/g$, $g = \sqrt{4\pi\alpha_s}$) dominating over collisional energy-loss. Other discussions on the topic are given in [34].

In 6.2 we show the energy profile of a 16 GeV charm after several time intervals of propagation through the thermal medium. Here $P(E)$ can be defined as $P(E) = \frac{1}{N} \frac{dN}{dE}$. The energy loss due to collisional and collisional+radiative processes is shown separately in the same figure. The collisional loss (upper panel) shows a shift in the position of the peak with long tail like structure extending towards the low energy regions. A recent study of charm quark energy profile using a Langevin equation along with a hydrodynamical background has instead shown a more Gaussian like distribution [35]. Some other discussions on the differences between Boltzmann and Langevin equations for heavy quark dynamics are also given in [35]. Additionally we find that inclusion of radiative corrections brings about a significant change
Figure 6.4: Energy of probe charm with distance traveled for radiative energy loss only

in the profile and indicates that for high energy charm quarks the effect of radiative loss is much greater than collisional loss, with the bulk of 16.0 GeV charm quarks ultimately shifting to very low energy (< 2.0 GeV) regions after 10 fm.

Next we study the evolution of charm quark energy as a function of distance traveled through the medium in 6.3 and 6.4. The calculation uses two different initial energies (16 GeV and 50 GeV respectively) for charm. Collisional loss and radiative loss are shown in these two figures separately – the radiative energy-loss figure was obtained by subtracting the elastic energy-loss calculation from the full calculation that included collisional+radiative energy-loss. We would like to elucidate the fact that these two diagrams show energy of charm quark after each ‘fm’ of path length traversed and shows the path length behaviour of charm quark. These plots also give the total energy loss of charm and shouldn’t be confused with average behaviour shown in 6.1.

Now let us discuss Figs. 6.3 and 6.2 in detail. The curves for the 50 GeV charm quarks show a clear distinction between the radiative and collisional energy-loss mechanisms: whereas the collisional energy-loss shows initially a linear behavior, the radiative energy-loss leads to a much stronger, near quadratic, fall-off in the energy for the first 20 fm/c. For the charm quarks with an initial energy of 16 GeV the differences are far less pronounced, but even
here a ratio between the two curves would yield interesting differences. For both cases, we compare our results to analytical calculations of \( \frac{dE}{dx} \). For collisional loss we have used an analytical form calculated by Peshier and Peigne [36] which can be written as:

\[
\frac{dE}{dx} = \frac{4\pi\alpha_s^2 T^2}{3} \left[ \left( 1 + \frac{N_f}{6} \right) \ln \frac{E_p(x) T}{\mu_D^2} + \frac{2}{9} \ln \frac{E_p(x) T}{M_c^2} + c(N_f) \right]
\]

(6.22)

Both for our PCM calculation as well as for the analytical expression we have used the following values for the parameters in order to compare the two. They are: a medium temperature of \( T = 350 \) MeV (applicable for RHIC-QGP system), a charm mass of \( M_c=1.35 \) GeV, no. of flavours and colours, \( N_f=4 \), \( N_c=3 \), a fixed coupling strength of \( \alpha_s = 0.3 \), and a screening mass \( \mu_D = \sqrt{(2N_c + N_f)/6gT} \). We find that for the above set of parameters, the PCM results show good agreement to the predictions from the analytical expression, validating our computational setup and approach.

Next we move over to results on charm quark radiative energy loss 6.4. The radiative energy loss is compared to an analytical calculation by R. Abir et al [37] shown below:

\[
\frac{dE}{dx} = 24\alpha_s^3 \left( \rho_g + \frac{9}{4}\rho_g \right) \frac{1}{\mu_g} (1 - \beta_1) \\
\times \left( \frac{1}{\sqrt{(1 - \beta_1)}} [\log (\beta_1)^{-1}]^{1/2} - 1 \right) \mathcal{F}(\delta)
\]
where

\[ F(\delta) = 2\delta - 2 \log \left( \frac{1 + M_c^2 e^{2\delta} / s}{1 + M_c^2 e^{-2\delta} / s} \right) \]

\[ - \frac{M_c^2 \cosh \delta / s}{1 + 2M_c^2 \cosh \delta / s + M_c^4 / s^2}, \]

\[ \delta = \frac{1}{2} \log \left[ \frac{\log \beta^{-1}}{(1 - \beta)} \left( 1 + \sqrt{1 - \frac{(1 - \beta)^{1/2}}{\log \beta^{-1} 1^{1/2}}} \right) \right], \]

\[ s = E^2(1 + \beta_0)^2, \beta_1 = \frac{g^2 T}{C E}, \beta_0 = (1 - M_c^2 / E^2)^{1/2}, \]

\[ C = \frac{3}{2} - \frac{M_c^2}{4ET} + \frac{M_c^4}{48E^2T^2 \beta_0} \log \left[ \frac{M_c^2 + 6ET(1 + \beta_0)}{M_c^2 + 6ET(1 - \beta_0)} \right] \] (6.23)

As in the elastic energy-loss case, we have used identical values for parameters in the PCM calculation and in the analytic case, such as \( T = 350 \text{ MeV}, M_c = 1.35 \text{ GeV}, \alpha_s = 0.3, N_f = 4, N_c = 3 \) and \( \mu_D = \sqrt{(2N_c + N_f)/6gT} \).

Note, however, that the calculations of [37] is carried out in the Bethe-Heitler limit of radiative energy loss with the effects of the dead-cone formalism being explicitly included in the calculation. The authors of [37] state that the LPM effect if added would only affect a marginal change in the final gluon emission spectrum which is clearly not what our results suggest. The PCM simulation explicitly takes the LPM effect into account as discussed in the previous sections. We do find that our simulation results for coherent gluon emission of charm quarks agrees reasonably well to that of the analytical calculation upto \( x = 5-6 \text{ fm} \), supporting the claim that modifications to the heavy-quark emission spectrum due to the LPM effect for this particular medium length, are modest. For \( x > 6 \text{ fm} \), however, the simulation result involving LPM effect and analytical curve in the BH limit move apart from each other. When we change the energy of charm probe, \( E_c \), from 16 GeV to 50 GeV, the differences between BH and LPM radiative mechanisms increase and become more profound and visible. This may be indicative of the rising importance of the coherent gluon emission effects at higher charm quark energies.
Overall we are confident that the comparison and agreement between PCM and the analytical calculations validates the PCM approach to heavy-quark energy loss and allows us to utilize the PCM for observables and calculations that are beyond the scope of analytical approaches, e.g. in the rapidly evolving non-equilibrium domain of ultra-relativistic heavy-ion collisions.

Next let us move over to our calculation of transverse momentum broadening per unit length of charm quarks also known as the transport coefficient \( \hat{q} \) [38, 39]. In other words \( \langle \hat{q} \rangle \) is a jet-quenching parameter calculated as a measure of momentum broadening within various energy loss models. Also the term 'transverse' refers to the direction perpendicular to the original direction of propagation and consequently for a jet of partons in the medium, the average or mean momentum of the jet remains unchanged while the momenta of each parton show broadening resulting in the redistribution of the transverse momentum spectrum of the jet partons. Some recent calculations have suggested values of this coefficient ranging from 0.5–20 GeV\(^2\)/fm [40] for light quarks. For heavy quarks, it was calculated in [41] which showed the value of \( \hat{q} \sim 0.3–0.7 \) GeV\(^2\)/fm. More detailed discussions and recent results on \( \hat{q} \) of partons and heavy quarks can be found in [42, 43].

Generally, the transport coefficient \( \hat{q} \) can be defined as:

\[
\left( \frac{d}{dx} \Delta p_T^2 \right) = \hat{q} = \rho \int d^2q_\perp \frac{d\sigma}{d^2q_\perp} \left( \frac{d^2\sigma}{d^2q_\perp} \right) \tag{6.24}
\]

where \( \frac{d\sigma}{d^2q_\perp} \) is the differential scattering cross-section of \( Q \) with medium quarks and gluons. In case of a Monte Carlo simulation this definition can be rewritten as:

\[
\hat{q} = \frac{1}{l_x} \sum_{i=1}^{N_{coll}} (\Delta p_{T,i})^2 \tag{6.25}
\]

For \( T = 350 \) MeV and the probe charm energy of 16 GeV, we find \( \hat{q} \) to be 1.2 GeV\(^2\)/fm with an uncertainty of \( \pm 0.2 \) GeV\(^2\)/fm, while for charm energy of 50 GeV \( \hat{q} \) is calculated to be 1.1 GeV\(^2\)/fm with \( \pm 0.3 \) GeV\(^2\)/fm uncertainty. Due to the rather large statistical uncertainty in our \( \hat{q} \) extraction, we cannot make any statements regarding the energy-dependence of \( \hat{q} \) at this time. Our results do suggest a range of values for \( \hat{q} \) somewhere between 1–1.5 GeV\(^2\)/fm for the RHIC system.
6.3 Summary

The present work aims to validate the applicability of Parton Cascade Model in case of heavy quark. A benchmark for heavy quark using microscopic Boltzmann transport equation has been set under controlled conditions. However we have to check the consistency of our calculations under stringent conditions before utilizing the model fully for heavy quarks. In the present work we also find that our calculations of Parton Cascade Model give us results which can be statistically improved, for charm quark energy evolution in an infinite QGP like medium at a fixed temperature. The energy loss spectra as calculated from the simulation agree considerably to some recent analytical results. Henceforth we will set out to study charm and bottom quark evolution as well as QGP medium responses at different collider energies. As a next series of calculations, we can take a set of different temperatures, to calculate the effect of heavy quark energy loss on its $p_T$ spectra, correlations, transport coefficients etc. These aspects will be addressed in our future publications.