Chapter 6

Conclusions

We now provide an overall summary to the primary results of this thesis.

In Chapter 2, we studied the role played by initial correlations of bipartite quantum systems on the subsystem dynamics. Working within the Shabani-Lidar framework, the principal result that emerges as a result of our analysis is that, for the system dynamics to be a completely positive map, or in other words, a physical evolution, the only allowed initial system bath states are (tensor) product states. This brings us back to the well known Stinespring dilation for realization of completely positive maps. Our analysis solely rested on two very reasonable assumptions of the set of initial system bath states. This demonstrated robustness of the folklore scheme could be of much importance in the study of open quantum systems.

In Chapter 3, we studied the computation of correlations for two-qubit $X$-states, namely, classical correlation, quantum discord, and mutual information. We exploit the geometric flavour of the problem and obtain the optimal measurement scheme for computing correlations. The optimal measurement turned out to be an optimization problem over a single real variable and this gave rise to a three-element POVM. We studied the region in the parameter space where the optimal measurement requires three elements and the region where the optimal measurement is a von Neumann measurement along $x$ or $z$-axis.
We further bring out clearly the role played by the larger invariance group (beyond local unitaries) in respect of the correlation ellipsoid and exploit this notion for simplifying the computation of correlations. We then immediately draw many new insights regarding the problem of computation of correlations and provide numerous concrete examples to detail the same.

In Chapter 4, we studied the robustness of non-Gaussian entanglement. The setup involved the evolution of Gaussian and non-Gaussian states under symmetric local noisy channel action. The noisy channels we are concerned with are the noisy attenuator and the noisy amplifier channels. This problem has consequences for protocols in quantum networks involving continuous variable systems. In this physical setting it was recently conjectured that Gaussian states are more robust than non-Gaussian states with regard to robustness of entanglement against these noisy environments. This conjecture is along the lines of other well established extremality properties enjoyed by Gaussian states. We demonstrate simple examples of non-Gaussian states with 1 ebit of entanglement which are more robust than Gaussian states with arbitrary large entanglement. Thereby proving that the conjecture is too strong to be true. The result will add to the growing list of plausible uses of non-Gaussian quantum information alongside the Gaussian-only toolbox.

In Chapter 5, we explore the connection between nonclassicality and entanglement in continuous variable systems and in particular the Gaussian setting. The nonclassicality of a state is inferred from its Sudarshan diagonal function. Motivated by the definition of entanglement breaking channels, we define nonclassicality breaking channels as those channels which guarantee that the output is classical for any input state. We classify Gaussian channels that are nonclassicalitybreaking under the restricted double cosetting appropriate for the situation on hand. We show that all nonclassicalitybreaking channels are entanglementbreaking channels. This is a surprising result in light of the fact that a nonclassicalitybreaking channel requires only one mode whereas the very definition of an entanglementbreaking channel requires two modes. We further show that the nonclassi-
ality of the output states of an entanglement-breaking channel are of a weak type. In the sense that a suitable squeeze transformation, independent of the input state, can take all these output states to classical states. The study reveals another close connection between nonclassicality and entanglement.

A natural future direction to explore from this study is that of the role played by these channels as a resource in quantum communication, namely, the capacity problem. The capacity of a channel is the rate at which information can be reliably sent across many uses of the channel. In the quantum setting, there are many variants of capacities depending on the available resources and the tasks to be accomplished. These quantities are well understood only for a handful of channels and is thus an interesting scope for further studies in both the finite and the continuous variable systems.