List of Figures

1.1 Showing the unitary realization of any quantum channel. The initial state is appended with a fixed environment state denoted $|0\rangle\langle0|$ and the composite is evolved through a joint unitary $U_{SR}$. Then the environment degrees are ignored to obtain the evolved system state. ........................................ 58

1.2 Showing a schematic diagram for the various ways in which the three representations of CPTP maps, namely, the unitary representation, the operator sum representation and the Choi-Jamiolkowski representation are related. ............................................................ 66

2.1 Showing the folklore scheme. In the folklore scheme, initial system states $\rho_S$ are elevated to product states of the composite, for a fixed fiducial bath state $\rho_B^{\text{fid}}$, through the assignment map $\rho_S \rightarrow \rho_S \otimes \rho_B^{\text{fid}}$. These uncorrelated system-bath states are evolved under a joint unitary $U_{SB}(t)$ to $U_{SB}(t) \rho_S \otimes \rho_B^{\text{fid}} U_{SB}(t)\dagger$ and, finally, the bath degrees of freedom are traced out to obtain the time-evolved states $\rho_S(t) = \text{Tr}_B \left[ U_{SB}(t) \rho_S \otimes \rho_B^{\text{fid}} U_{SB}(t)\dagger \right]$ of the system. The resulting quantum dynamical process (QDP) $\rho_S \rightarrow \rho_S(t)$, parametrized by $\rho_B^{\text{fid}}$ and $U_{SB}(t)$, is completely positive by construction. Initial system states are identified by the blue region and the final states by the red. ................................................................. 105
2.2 Showing the SL scheme. In sharp contrast to the folklore scheme, there is no assignment map in the SL scheme. The distinguished bath state $\rho^{\text{fid}}_B$ is replaced by a collection $\Omega^{SB}$ of (possibly correlated) system-bath initial states $\rho_{SB}(0)$. The dynamics gets defined through $\rho_{SB}(0) \rightarrow \rho_{SB}(t) = U_{SB}(t) \rho_{SB}(0) U_{SB}(t)^\dagger$ for all $\rho_{SB}(0) \in \Omega^{SB}$. With reduced system states $\rho_S(0)$ and $\rho_S(t)$ defined through the imaging or projection map $\rho_S(0) = \text{Tr}_B \rho_{SB}(0)$ and $\rho_S(t) = \text{Tr}_B \left[U_{SB}(t) \rho_{SB}(0) U_{SB}(t)^\dagger\right]$, this unitary dynamics of the composite induces on the system the QDP $\rho_S(0) \rightarrow \rho_S(t)$. As before, initial system states are identified by the blue region and the final states by the red.

2.3 Depicting, for the case $d_S = 3$ (qutrit), the image of $\Omega^{SB}$ under $\text{Tr}_B(\cdot)$ in the plane spanned by the commuting (diagonal) $\lambda$-matrices ($\lambda_3, \lambda_8$).

3.1 Showing the x-z cross-section of the correlation ellipsoid associated with a general $X$-state. The point I is the image of input state as identity, $C$ the center of the ellipsoid, and $E$ the image of the equatorial plane of the input Bloch sphere.

3.2 Showing the conditional states corresponding to the 3-element measurement scheme of (3.30).
3.3 Showing $S^A(z)$ for various values of $a_x$ labelled in increasing order. The line marked $z_I$ denotes $z = z_I$. A three-element POVM scheme results for values of $a_x \in (a^V_x(a_z, z_c), a^H_x(a_z, z_c))$ [the curves (2) and (4)]. For values of $a_x \leq a^V_x(a_z, z_c)$ [example of curve (1)], the von Neumann projection along the $z$-axis is the optimal and for values of $a_x \geq a^H_x(a_z, z_c)$ [example of curve (5)], the von Neumann projection along the $x$-axis is the optimal. The optimal $z = z_0$ is obtained by minimizing $S^A(z)$ (marked with a dot). $S^A(z)$ for $z > z_I$ is not meaningful and this region of the curves is shown with dashed lines. In this example $(a_z, z_c) = (0.58, 0.4)$. For $z_I = 0.6$, a three-element POVM results for (red) curves between (2) and (3).

3.4 Showing $dS^A(z)/dz$ for various values of $a_x$ labelled in increasing order. A root exits for values of $a_x \in (a^V_x(a_z, z_c), a^H_x(a_z, z_c))$ [between curves (2) and (4)]. For values of $a_x \leq a^V_x(a_z, z_c)$ and $a_x \geq a^H_x(a_z, z_c)$, there is no $z_0$ [examples are curves (1) and (5)].

3.5 Showing the various measurement schemes across a slice (the $a_x - a_z$ plane) of parameter space (of correlation ellipsoids) with $z_c = 0.4$. We see that only for a tiny wedge-shaped region marked $\Pi^{(3)}_\theta$, the region between $a^V_x(a_z, z_c)$ (1) and $a^H_x(a_z, z_c)$ (2), one can expect a 3-element POVM. Region above (2) corresponds to a von Neumann measurement along the $x$-axis and the region below curve (1) corresponds to a von Neumann measurement along the $z$-axis. Curves marked (4) depict the boundary of allowed values for $a_z, a_x$. The curve (3) is the line $a_z = a_x$. The inset shows curves (1), (2) and (3) for $a_x \in [0.2, 0.21]$.

3.6 Showing $\delta(a_z)$ for decreasing values of $z_c$ from left to right. The first curve corresponds to $z_c = 0.95$ and the last curve to $z_c = 0.1$. We see that size of the ‘wedge’-shaped region (Fig. 3.5) first increases and then decreases with increasing $z_c$. 

21
3.7 Showing $S^A_{\text{min}}$ [curve (1)] and quantum discord [curve (4)] for a one-parameter family of states $\hat{\rho}(a)$. The ellipsoid corresponding to $\hat{\rho}(a)$ has parameters $(a_x, a_y, a_z, z_c, z_I) = (a, 0.59, 0.58, 0.4, 0.5)$ where $a \in [0.59, 0.7]$. Point E denotes the change of the optimal measurement from a von Neumann measurement along the z-axis to a three-element POVM, while point F denotes the change of the optimal measurement from a three-element POVM to a von Neumann measurement along the x-axis, with increasing values of the parameter $a$. The curve (3) (or (2)) denotes the over(under)-estimation of quantum discord (or $S^A_{\text{min}}$) by restricting the measurement scheme to a von Neumann measurement along the z or x-axis. This aspect is clearly brought out in the following Fig. 3.8.

3.8 Showing quantum discord [curve (4)] with increasing $a_x$ when there is a transition of the measurement scheme from the von Neumann measurement along the z-axis to the three-element scheme (E) and finally to the von Neumann measurement along the x-axis (F). The over-estimation of quantum discord is depicted in curve (3) where the measurement scheme is restricted to one of von Neumann measurements along the z or x-axis. We see a gradual change in quantum discord in contrast to the sharp change in the restricted case.

3.9 Showing mutual information (1), quantum discord (2) and classical correlation (3) as a function of $z_I$ for the ellipsoid parameters $(a_z, z_c, a_x) = (0.58, 0.4, 0.65)$ and $z_0 = 0.305919$. For $z_I \leq z_0$, the optimal measurement is a von Neumann measurement along the x-axis, and for $z_I > z_0$ the optimal measurement is a three-element POVM.

3.10 Showing the x-z cross-section of the ellipsoid associated with the Mueller matrix in (3.59).
3.11 Showing the conditional entropy $S_{vN}^A(\theta)$ resulting from von Neumann measurements for the example in (3.59).

3.12 Showing the entropy variation with respect to variation of $a_x$ in $[0.78032, 0.781553]$, with $(a_y, a_z, z_c)$ fixed at $(0.616528, 0.77183, 0.122479)$. Curve (1) depicts $S_{vN}^A$ for von Neumann measurement along $\theta = \pi/2$, the constant line [curve (2)] to a von Neumann measurement along $\theta = 0$, and curve (3) to $S_{min}^A$ resulting from the three element POVM scheme. The example in ((3.59)) corresponds to $a_z = 0.780936$. The inset compared the various schemes of the example in (3.59). D refers to the a measurement restricted only to the von Neumann projection along $z$ or $x$-axis, B to the best von Neumann projection, and F to the optimal three-element measurement.

3.13 Showing the optimal $\theta = \theta_{opt}$ of $\Pi_\theta^{(3)}$ [curve(1)] resulting in $S_{min}^A$ depicted as curve (3) in Fig. 3.12. Curve (2) shows the probability (scaled by a factor of 2) $2p_0(\theta_{opt})$ of the conditional state corresponding to input POVM element $(1, 0, 0, 1)^T$.

4.1 Comparison of the robustness of the entanglement of a NOON state with that of two-mode Gaussian states under the two-sided action of symmetric noisy attenuator.

4.2 Comparison of the robustness of the entanglement of a PNES state with that of two-mode Gaussian states under the action of two-sided symmetric noisy attenuator.

4.3 Comparison of the robustness of the entanglement of a NOON state with that of all two-mode Gaussian states under the action of two-sided symmetric noisy amplifier.
4.4 Comparison of the robustness of the entanglement of a PNES state with that of all two-mode Gaussian states under the action of two-sided symmetric noisy amplifier. .......................... 187

5.1 A schematic diagram depicting the notion of entanglement breaking channels. .......................... 192

5.2 Showing the notion of nonclassicality breaking channels. .......................... 192
5.3 Showing a pictorial comparison of the nonclassicality breaking condition, the entanglement breaking condition, and the complete positivity condition in the channel parameter space \((a, b)\), for fixed \(\det X\). Curves (1), (2), and (3) correspond to saturation of these conditions in that order. Curve (3) thus corresponds to quantum-limited channels. Frame (a) refers to the first canonical form \((\kappa \mathbb{I}, \text{diag}(a, b))\), frame (c) to the second canonical form \((\kappa \sigma_3, \text{diag}(a, b))\), and frame (d) to the third canonical form, singular \(X\). Frame (b) refers to the limiting case \(\kappa = 1\), classical noise channel. In all the four frames, the region to the right of (above) curve (1) corresponds to nonclassicality breaking channels; the region to the right of (above) curve (2) corresponds to entanglement breaking channels; curve (3) depicts the CP condition, so the region to the right of (above) it alone corresponds to physical channels. The region to the left (below) curve (3) is unphysical as channels. In frames (c) and (d), curves (2) and (3) coincide. In frame (b), curve (3) of (a) reduces to the \(a\) and \(b\) axis shown in bold. In frames (a) and (c), curves (1) and (2) meet at the point \((1 + \kappa^2, 1 + \kappa^2)\), in frame (b) they meet at \((2, 2)\), and in frame (d) at \((1, 1)\). The region between (2) and (3) corresponds to the set of channels which are not entanglement breaking. That in frame (c) and (d) the two curves coincide proves that this set is vacuous for the second and third canonical forms. That in every frame the nonclassicality breaking region is properly contained in the entanglement breaking region proves that a nonclassicality breaking channel is certainly an entanglement breaking channel. The dotted curve in each frame indicates the orbit of a generic entanglement breaking Gaussian channel under the action of a local unitary squeezing after the channel action. That the orbit of every entanglement breaking channel passes through the nonclassicality breaking region, proves that the nonclassicality in all the output states of an entanglement breaking channel can be removed by a fixed unitary squeezing, thus showing that every entanglement breaking channel is ‘essentially’ a nonclassicality breaking channel.
5.4 Showing the relationship between nonclassicality breaking and entangle-
ment breaking channels established in the present Chapter. The output
state corresponding to any input to an entanglement breaking channel is
rendered classical by a single squeeze transformation that depends only
on the channel parameters and independent of the input states. In other
words, an entanglement breaking channel followed by a given squeeze
transformation renders the original channel nonclassicality breaking. In
contrast, every nonclassicality breaking channel is also entanglement break-
ing.