

3.1 Introduction

* The flow of Newtonian fluids is governed by the Navier–Stokes equations obey the Newton’s law of viscosity. There are many fluids such as biological fluids, polymeric liquids, liquid crystals, lubricating oils, etc., that are not obey the Newton’s law of viscosity are known as Non-Newtonian fluids. However a good number of fluid rheology are already in existence, particularly for most of those fluids used as lubricants and having Non-Newtonian behavior, the flow can be analyzed with the help of a power-law model. Furthermore, a power-law fluid model characterizes both pseudoplastic and dilatant fluids - two important classes of Non-Newtonian lubricants and again can characterize Newtonian fluid as a special case. It is because of such wide coverage in the analysis of lubricants together with its mathematical simplicity that the Sisko fluid model (Sisko, 1958) has been preferred for application in the present problem. Some recent studies dealing with the flows of Non-Newtonian fluids are mentioned in Hayat et. al. (2004), Fetecau and Fetecau (2003; 2005), Asghar et. al. (2002), Hayat and Kara (2006), Chen et al. (2003; 2004)

Recently the flows through porous media are prevailed in nature. No doubt, the study of such flows has acquired high importance in many scientific and engineering applications. Specifically the interest in viscoelastic flows through porous media has grown considerably due to large demands of such diverse areas as bio-rheology, geophysics, and chemical and petroleum industries. With these facts in

view, Tan and Masuoka (2005a) discussed the Stokes first problem for a second grade fluid in a porous half-space with heated boundary. In another paper, Tan and Masuoka (2005b) discussed the Stokes first problem for an Oldroyd-B fluid in a porous half space.

The present work concentrates on the similarity analysis and numerical solutions for Magnetohydrodynamics (MHD) flow of a Non-Newtonian fluid filling the porous half-space. The study the flow of an electrically conducting Sisko fluid through a porous medium based on modified Darcy’s law is made. The similarity analysis of governing equation is discussed using the deductive group symmetry method as we discussed in previous chapter.

### 3.2 Governing Equations

The fundamental equations governing the motion of steady incompressible boundary layer flow of electrically conducting fluid through porous medium are given by: (see chapter 1)

\[
div \vec{V} = 0
\]  

\[
\rho \frac{D\vec{V}}{Dt} = -\nabla p + div \vec{\tau} - \sigma \mu H^2(t) \vec{V} + \vec{R}
\]

where \( \vec{V} \) is the velocity field, \( \rho \) is density, \( p \) is the pressure, \( \vec{\tau} \) the extra stress tensor, \( \sigma \) the electrical conductivity, \( \mu \) is the magnetic permeability, \( H(t) \) is the magnetic field strength, a function \( t \) and \( \vec{R} \) represents the Darcy’s (Scheidegger, 1974) resistance in the porous medium. In this chapter, we consider a Sisko fluid and its extra stress tensor is (Sisko 1958)

\[
\vec{\tau} = \left[ a + b \sqrt{\frac{1}{2} tr(\vec{\Lambda}^2)} \right]^{n-1} \vec{\Lambda}
\]

Where \( a, b \) and \( n \) are constants defined differently for different fluids and \( \vec{\Lambda} \) is the rate of deformation tensor defined by

\[
\vec{\Lambda} = \vec{L} + \vec{L}^T; \quad \text{Where } \vec{L} = \nabla \vec{V}
\]

For the two-dimensional flow problem, we define the velocity and the stress fields of the following form

\[
\vec{V} = (x(t), y(t), z(t))
\]

\[
\vec{\tau} = (\tau_{xx}, \tau_{xy}, \tau_{xz}, \tau_{yy}, \tau_{yz}, \tau_{zz})
\]
\[ \nabla = [u(x, y, t), v(x, y, t), 0], \quad \tau = \tau(x, y, t) \]  

(3.5)

where \( u \) and \( v \) are the component of velocity in \( X \)-direction and \( Y \)-direction respectively.

Now substituting equation (3.5) into equation (3.3) along with equation (3.4) we get

\[ \tau_{xx} = 2 \left( \frac{\partial u}{\partial x} \right) \left[ a + b \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right] \]  

(3.6)

\[ \tau_{xy} = \tau_{yx} = \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \left[ a + b \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right] \]  

(3.7)

\[ \tau_{yy} = 2 \left( \frac{\partial v}{\partial y} \right) \left[ a + b \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right] \]  

(3.8)

\[ \tau_{zz} = \tau_{zy} = \tau_{xz} = \tau_{zx} = 0 \]  

(3.9)

Substituting all the values in the momentum equation (3.2), we get

\[ \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \]

\[ = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left[ a + b \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right] \]

\[ + \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \left[ a + b \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right] \]

\[ -\sigma \mu^2 H^2 u + R_x \]  

(3.10)
And

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial x} \right)$$

$$= -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \left( a + b \left| \frac{\partial u}{\partial x} \right|^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \frac{n-1}{2} \right)$$

$$+ \frac{\partial}{\partial y} \left| \frac{\partial v}{\partial y} \right|^2 \left( a + b \left| \frac{\partial u}{\partial x} \right|^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 \right)$$

$$- \sigma \mu^2 H^2 (t) v + R_y$$

(3.11)

where \( R_x \) and \( R_y \) are the components of \( \mathbf{R} \) in the \( X \) and \( Y \) directions, respectively.

For the problem consider here, we define the velocity and the stress fields of the following form

$$\mathbf{V} = \left[ u(y,t), 0, 0 \right], \quad \tau = \tau(y,t)$$

(3.12)

According to Tan and Masuoka (2005a, 2005b), the constitutive relationship between the pressure drop and the velocity for the unidirectional flow of a Sisko fluid is

$$\frac{\partial p}{\partial x} = -\phi \left( a + b \left| \frac{\partial u}{\partial y} \right| \right) \frac{\partial^2 u}{\partial y^2},$$

(3.13)

Where \( k > 0 \) and \( \phi \) (0 < \( \phi \) < 1) are the permeability and porosity.

The pressure gradient in equation (3.13) is regarded as a measure of the flow resistance in the bulk of the porous medium. If \( R_x \) is a measure of the flow resistance offered by the solid matrix in the \( X \)-direction then \( R_x \) through equation (3.13) is inferred by

$$R_x = -\frac{\phi}{k} \left( a + b \left| \frac{\partial u}{\partial y} \right| \right) \frac{\partial^2 u}{\partial y^2},$$

(3.14)

Under (3.12), the continuity equation (3.1) is satisfied identically and from equations (3.2), (3.3), (3.13), and (3.14) we have
\begin{align}
\rho \frac{\partial u}{\partial t} &= -\nabla p + \frac{\partial}{\partial y}\left[\left(a + b \left|\frac{\partial u}{\partial y}\right|^n\right)\frac{\partial u}{\partial y}\right] - \frac{\phi}{k}\left(a + b \left|\frac{\partial u}{\partial y}\right|^n\right)\frac{\partial^2 u}{\partial y^2} - \sigma \mu^2 H^2 u, \tag{3.15}\end{align}

It is interesting to observe that, for \(a = 0\) \((b \neq 0, n \neq 1)\), the above equation will be equation of power-law fluids and for \(b = 0\) \((a \neq 0, n = 1)\) it will be equation of Newtonian fluid.

### 3.3 Formulation of the Problem

We consider a Cartesian coordinate system with \(Y\)-axis in the vertical upward direction and \(X\)-axis parallel to the rigid plate at \(y = 0\). An infinite rigid plate bound the flow of an incompressible and electrically conducting Sisko fluid. The Sisko fluid occupies the porous half-space \(y > 0\). The motion of the plate produces the flow with the time dependent velocity (free stream velocity) \(U_0 V(t)\). For zero pressure gradient, the resulting problem from (3.8), yields

\begin{align}
\rho \frac{\partial u}{\partial t} &= \frac{\partial}{\partial y}\left[\left(a + b \left|\frac{\partial u}{\partial y}\right|^n\right)\frac{\partial u}{\partial y}\right] - \frac{\phi}{k}\left(a + b \left|\frac{\partial u}{\partial y}\right|^n\right)\frac{\partial^2 u}{\partial y^2} - \sigma \mu^2 H^2 u, \tag{3.16}\end{align}

\begin{align}
\begin{cases}
u(0,t) = U_0 V(t), & u(\infty,t) = 0, \\
u(y,0) = g(y), & y > 0 \end{cases} \tag{3.17}
\end{align}

in which \(U_0\) is the characteristic velocity.

The above equations can be made dimensionless using the following variables,

\begin{align}
u^* &= \frac{u}{U_0}, & y^* &= \frac{y U_0}{v}, & t^* &= \frac{t U_0^2}{v}, \tag{3.18}
\end{align}

\begin{align}
b^* &= \frac{b U_0^2}{a v^{n-1}}, & \frac{1}{K} &= \frac{\phi v^2}{k U_0^2}, & M &= \frac{\sigma \mu^2 v}{\rho U_0^2}. \tag{3.19}
\end{align}

Accordingly, the above boundary value problem after dropping the asterisks (for simplicity) becomes

\begin{align}
\frac{\partial u}{\partial t} &= \frac{\partial}{\partial y}\left[\left(1 + b \left|\frac{\partial u}{\partial y}\right|^n\right)\frac{\partial u}{\partial y}\right] - \frac{1}{K}\left[1 + b \left|\frac{\partial u}{\partial y}\right|^n\right]\frac{\partial^2 u}{\partial y^2} - MH^2(t) u, \tag{3.20}
\end{align}
\begin{align*}
\begin{cases}
  u(0,t) &= V(t), \quad u(\infty,t) = 0, \quad t > 0 \\
  u(y,0) &= h(y), \quad y > 0 \quad \text{Where } h(y) = g(y)U_0^{-1}
\end{cases}
\end{align*}

(3.21)

Without loss of generality, we assume that the velocity gradient is positive then equation (3.13) can be written as

\[
\frac{\partial u}{\partial t} = \left(1 - \frac{1}{K}\right) \frac{\partial^2 u}{\partial y^2} + \left(nb - \frac{b}{K}\right) \left(\frac{\partial u}{\partial y}\right)^{n-1} \frac{\partial^2 u}{\partial y^2} - MH^2(t)u
\]

(3.22)

3.4 Application of Deductive Group Symmetry Technique

Our method of solution depends on the application of the class of a one-parameter continuous deductive group of transformations to the partial differential equation (3.22). Under this class, first, we search the group transformations, through which one will reduce the two independent variables and the differential equation (3.22) will transforms into an ordinary differential equation. Finally, the reduced differential equation is solved numerically by using MATLAB bvp4c coding.

3.4.1 The Group Systematic Formulation

The procedure is initiated with the group \( C_G \), a class of transformation of one-parameter \('a'\) of the form:

\[
C_G : \quad \bar{Q} = c^Q(a)Q + h^Q(a), \quad Q = y,t,u,H
\]

(3.23)

Where \( c \)'s and \( h \)'s are real-valued and at least differentiable in the real argument \( a \).

To transform the differential equation, transformations of the derivatives of \( u \) are obtained from \( C_G \) via chain-rule operations:

\[
\begin{align*}
  \bar{Q}_T &= \left(\frac{c^Q}{c^T}\right) Q_i \\
  \bar{Q}_{TJ} &= \left(\frac{c^Q}{c^T c^J}\right) Q_{ij}
\end{align*}
\]

(3.24)

\( Q = u,H; \quad i,j = y,t \)
Equation (3.15) is said to be transformed invariantly (See. chapter 2), for some function $A(a)$ whenever:

$$
\left[ \frac{\partial \bar{u}}{\partial t} - \left( 1 - \frac{1}{K} \right) \frac{\partial^2 \bar{u}}{\partial y^2} - \left( nb - \frac{b}{K} \right) \left( \frac{\partial \bar{u}}{\partial y} \right)^{n-1} \frac{\partial^2 \bar{u}}{\partial y^2} + MH^2 \bar{u} \right] = A(a) \left[ \frac{\partial u}{\partial t} - \left( 1 - \frac{1}{K} \right) \frac{\partial^2 u}{\partial y^2} - \left( nb - \frac{b}{K} \right) \left( \frac{\partial u}{\partial y} \right)^{n-1} \frac{\partial^2 u}{\partial y^2} + MH^2 u \right]
$$

Substituting the values from the equation (3.23) and (3.24) in above equation, yields

$$
\frac{c^u}{c^t} \frac{\partial u}{\partial t} - \left( 1 - \frac{1}{K} \right) \frac{c^u}{(c^y)^2} \frac{\partial^2 u}{\partial y^2} - \left( nb - \frac{b}{K} \right) \left( \frac{c^u}{(c^y)^2} \right)^{n+1} \left( \frac{\partial u}{\partial y} \right)^{n-1} \frac{\partial^2 u}{\partial y^2} + M \left( c^H H + h^H \right)^2 \left( c^u u + h^u \right) \quad (3.25)
$$

$$
= A(a) \left[ \frac{\partial u}{\partial t} - \left( 1 - \frac{1}{K} \right) \frac{\partial^2 u}{\partial y^2} - \left( nb - \frac{b}{K} \right) \left( \frac{\partial u}{\partial y} \right)^{n-1} \frac{\partial^2 u}{\partial y^2} + MH^2 u \right]
$$

The invariance of equation (3.18), implies that

$$
h^H = h^u = 0; \quad \frac{c^u}{c^t} = \frac{c^u}{(c^y)^2} = \left( \frac{c^u}{(c^y)^2} \right)^{n+1} = \left( c^H \right)^2 c^u = A(a) \quad (3.26)
$$

Also for the absolute invariance of the auxiliary conditions, implies that $h^y = 0$

These yields,

$$
c^t = (c^y)^2, \quad c^u = c^y, \quad c^H = \frac{1}{c^y}, \quad A(a) = \frac{1}{c^y} \quad (3.27)
$$

Finally, we get the one-parameter group $G$, which transforms invariantly the differential equation (3.22) and the auxiliary conditions (3.21).

Thus, the subgroup $G$ of the group $C_G$ is of the form:
Chapter 3: MHD Transient Flow of a Sisko Fluid in a Porous Medium

\[
S: \begin{cases} 
\bar{t} = \left[ c_y(a) \right]^2 t + h'(a) \\
\bar{y} = c_y(a) y 
\end{cases}
\]

\[
G: \begin{cases} 
\bar{u} = c_y(a) u \\
\bar{H} = \left[ c_y(a) \right]^{-1} H 
\end{cases}
\]

(3.28)

3.4.2 Complete Set of the Absolute Invariants

Our aim is to make use of group methods to represent the problem in the form of an ordinary differential equation. Then we have to proceed in our analysis to obtain a complete set of absolute invariants. If \( \eta = \eta(y,t) \) is the absolute invariant of the independent variables then, the two absolute invariants corresponding to the dependent variables \( u(y,t), H(t) \) are given by (Eisenhart, 1961)

\[
g_j(y,t,u,H) = F_j(\eta); \quad j = 1,2
\]

(3.29)

The application of a basic theorem in group theory states that: (See. chapter 2)

A function \( g(y,t,u,H) \) is an absolute invariant of a one-parameter group if it satisfies the following first-order linear partial differential equation,

\[
\sum_{i=1}^{4} (\alpha_{i} Q_i + \beta_{i}) \frac{\partial g}{\partial Q_i} = 0, \quad Q_i = t, y, u, H
\]

(3.30)

Where

\[
\alpha_{i} = \left. \frac{\partial c^i}{\partial a} \right|_{a=a^0} \quad \text{and} \quad \beta_{i} = \left. \frac{\partial h^i}{\partial a} \right|_{a=a^0} \quad i = 1,2,3,4
\]

(3.31)

and \( a^0 \) denotes the value of parameter \( a \) which yields the identity element of the group \( G \).

Since \( h^y = h^u = h^H = 0 \) implies that \( \beta_2 = \beta_3 = \beta_4 = 0 \) and from (3.31) we get

\[
\alpha_1 = 2\alpha_2 = \alpha_3 = -\alpha_4.
\]
Hence,

\[
(\alpha_1 t + \beta_1) \frac{\partial g}{\partial t} + \alpha_2 y \frac{\partial g}{\partial y} + \alpha_3 u \frac{\partial g}{\partial u} + \alpha_4 H \frac{\partial g}{\partial H} = 0.
\]  

(3.32)

The absolute invariant of independent variable owing the equation (3.32) is \( \eta = \eta(y,t) \) if it will satisfies the first order linear partial differential equation

\[
(\alpha_1 t + \beta_1) \frac{\partial \eta}{\partial t} + \alpha_2 y \frac{\partial \eta}{\partial y} = 0.
\]  

(3.33)

Using the definition of \( \alpha 's \), we get

\[
2(t + \beta) \frac{\partial \eta}{\partial t} + y \frac{\partial \eta}{\partial y} = 0, \quad \text{where} \quad \beta = \frac{\beta_1}{\alpha_1}
\]  

(3.34)

For the solution of equation (3.34), consider the auxiliary equation

\[
\frac{dt}{2(t + \beta)} = \frac{dy}{y} = \frac{d\eta}{0}
\]  

(3.35)

Applying the variable separable method one can obtain the general solution of (3.35) in as

\[
\eta = \phi\left(\frac{1}{y} \left(1 + \frac{\beta}{2}\right)^{-1/2}\right)
\]

where \( \phi \) is an arbitrary function. Without loss of generality, if we assume the the arbitrary function \( \phi \) as an identity function, the absolute invariant of independent variables is given by

\[
\eta = y \left(1 + \frac{\beta}{2}\right)^{-1/2}
\]  

(3.36)

Further, for the absolute invariants of dependent variables owing the equation (3.32), the general solution of equation (3.32) is obtained from the auxiliary equation

\[
\frac{dt}{2(t + \beta)} = \frac{dy}{y} = \frac{du}{u} = \frac{dH}{H} = \frac{dg}{0}
\]  

(3.37)

Solving equation (3.37), using variable separable method, we get

\[
g(t, y, u, H) = \Gamma\left(\frac{1}{y} \left(1 + \frac{\beta}{2}\right)^{-1/2}, u \left(1 + \frac{\beta}{2}\right)^{-1/2}, H \left(1 + \frac{\beta}{2}\right)^{1/2}\right)
\]  

(3.38)

where \( \Gamma \) is an arbitrary function.
Now without loss of generality select the various $\Gamma$, so that

$$
g_1(t, y, u, H) = u(t + \beta)^{-1/2}
g_2(t, y, u, H) = H(t + \beta)^{1/2}
$$

(3.39)

These give the absolute invariant of dependent variables, with respect to under considered subgroup $G$, as

$$
g_1(t, y, u, H) = u(t + \beta)^{-1/2} = F_1(\eta)
g_2(t, y, u, H) = H(t + \beta)^{1/2} = F_2(\eta)
$$

(3.40)

Since $H(t)$ is function of $t$ only and $\eta$ dependents on $y$ and $t$, $F_2(\eta)$ must be constant, say $H_0$. Thus, finally we get the complete set of absolute invariants for the group $G$, that transforms the partial differential equation (3.22) into ordinary differential equation together with auxiliary conditions (3.21), as

$$
u(t + \beta)^{-1/2} = F_1(\eta), \quad H(t + \beta)^{1/2} = H_0
$$

OR,

$$
u(y, t) = (t + \beta)^{1/2} F(\eta), \quad H(t) = H_0(t + \beta)^{-1/2},
$$

where $F(\eta) = F_1(\eta), \quad \eta = y(t + \beta)^{-1/2}$

(3.41)

### 3.4.3 Reduction to Ordinary Differential Equation

Substituting the values of partial derivatives in equation (3.22) and (3.21), one can derive as

$$
\frac{\partial u}{\partial t} = \frac{1}{2} (t + \beta)^{-1/2} \left[ F(\eta) - \eta F'(\eta) \right],
\frac{\partial u}{\partial y} = F'(\eta),
\frac{\partial^2 u}{\partial y^2} = (t + \beta)^{-1/2} F''(\eta)
$$

where primes denote ordinary derivative with respect to $\eta$.

Equation (3.22) reduces to
\[
\left( nb - \frac{b}{K} \right) (F')^{n-1} F'' + \left( 1 - \frac{1}{K} \right) F'' + \frac{1}{2} \eta F' - \left( \frac{1}{2} + N \right) F = 0 \tag{3.42}
\]

Where \( N = MH_0^2 \) will be referred as magnetic field parameter.

Further to transform, the boundary conditions in to constant form the free stream velocity must be of the form \( V(t) = (t + \beta)^{1/2} \). Hence, the auxiliary conditions reduce to,

\[
F(0) = 1, \quad F(\infty) = 0 \tag{3.43}
\]

Where primes denote ordinary derivative with respect to the similarity variable \( \eta \).

### 3.5 Results and Discussion

The differential equation (3.42) is highly non-linear differential equation and hence it is bit difficult to find its analytic solution. The numerical solution of equation (3.42) is obtained, subject to the boundary condition (3.43), for particular values of the flow parameters viz. \( b, K, n, N \) etc, by using MATLAB bvp4c.

#### 3.5.1 Numerical Procedure Using MATLAB Coding

The transformed highly non-linear ordinary differential equation (3.42) subject to the boundary conditions (3.43) is solved numerically by using bvp4c solver of MATLAB. This method is second order accurate and allows non-uniform grid size. The numerical algorithm to solve the non-linear boundary value problem is as explained below.

First, the boundary value problem of (3.42) in \( f \) is reduced to a first order system of two simultaneous ordinary differential equations:

\[
\frac{df_i}{d\eta} = F_i(\eta, f_1, f_2, \ldots, f_n); \quad (i = 1, 2 = n)
\]

Where,

\[
f_1 = F, \quad f_2 = F'
\]

And
Chapter 3: MHD Transient Flow of a Sisko Fluid in a Porous Medium

\[ F_1 = f_2, \quad F_2 = \left\{ \left( \frac{1}{2} + N \right) f_1 - \frac{1}{2} \eta f_2 \right\} + \left\{ \left( nb - \frac{b}{K} \right) f_2 \right\}^{n-1} + \left\{ 1 - \frac{1}{K} \right\} \]

Further, the boundary conditions become

\[ f_1(0) = 1, \quad f_1(\eta_{\infty}) = 0 \]

Here prime denotes differentiation with respect to \( \eta \). Next, after choosing \( \eta_{\infty} \), the ‘numerical infinity’ (see below), an initial guess for the missing initial condition is made and the uniform step length \( \Delta \eta \) for all the numerical solutions is chosen, for the closed interval \( [0, \eta_{\infty}] \). Then the system of initial value problems is solved by coding in MATLAB using bvp4c solver.

For the numerical solution we have considered \( \eta_{\infty} = 3 \) and the step length \( \Delta \eta = 0.25 \). The graphical representation is given in below diagrams which shows the behavior of the normalized velocity

\[ F(\eta) = \frac{u(y,t)}{(t + \beta)^{\frac{3}{2}}} \quad (3.44) \]

The various normalized velocity profiles for the different parameters with specific values are generated, as show in Figure 3.1 to Figure 3.5.

- In figure 3.1, controlling the parameters \( K, n \) and \( b \) it is observe that as the magnetic field increase, the normalized velocity approaches to its final value fast. In other words, in presence of magnetic field the velocity increase depends upon the strength of applied magnetic field.

- Figure 3.2 shows that increase in flow index \( n \), reduces the normalized velocity corresponding to same applied magnetic field. That is as known Non-Newtonian behavior of fluid increase, the velocity decrease.

- From the Figure 3.3, it is worth to observe that as porosity of the medium increase, the velocity decrease drastically.
• Figure 3.4 shows the special behavior of Non-Newtonian parameter $b$, in particular for the flow index $n = 3$, increase in $b$ will systematically increase the velocity, in presence of magnetic field.

• Figure 3.5 explains interesting comparisons of velocity profiles among the various Non-Newtonian fluids discovered by different flow indices.
### Table 3.1: Numerical solution for various magnetic parameters (Newtonian case)

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$F(\eta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K = 0.25$, $n = 1$, $b = 0.5$</td>
</tr>
<tr>
<td></td>
<td>$N = 0.1$</td>
</tr>
<tr>
<td>0.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.2500</td>
<td>2.6778</td>
</tr>
<tr>
<td>0.5000</td>
<td>4.1334</td>
</tr>
<tr>
<td>0.7500</td>
<td>5.3538</td>
</tr>
<tr>
<td>1.0000</td>
<td>6.3209</td>
</tr>
<tr>
<td>1.2500</td>
<td>7.0113</td>
</tr>
<tr>
<td>1.5000</td>
<td>7.3947</td>
</tr>
<tr>
<td>1.7500</td>
<td>7.4321</td>
</tr>
<tr>
<td>2.0000</td>
<td>7.0728</td>
</tr>
<tr>
<td>2.5000</td>
<td>4.8812</td>
</tr>
<tr>
<td>2.7500</td>
<td>2.8485</td>
</tr>
<tr>
<td>3.0000</td>
<td>0</td>
</tr>
</tbody>
</table>
### Table 3.2: Numerical solution for various magnetic parameters (Non-Newtonian case)

<table>
<thead>
<tr>
<th>η</th>
<th>$K = 0.25, \ n = 2, \ b = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normalized velocity $F(\eta)$</td>
</tr>
<tr>
<td></td>
<td>$N = 0.1$</td>
</tr>
<tr>
<td>0.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.2500</td>
<td>1.7635</td>
</tr>
<tr>
<td>0.5000</td>
<td>2.2257</td>
</tr>
<tr>
<td>0.7500</td>
<td>2.4210</td>
</tr>
<tr>
<td>1.0000</td>
<td>2.4037</td>
</tr>
<tr>
<td>1.2500</td>
<td>2.2400</td>
</tr>
<tr>
<td>1.5000</td>
<td>1.9887</td>
</tr>
<tr>
<td>1.7500</td>
<td>1.6906</td>
</tr>
<tr>
<td>2.0000</td>
<td>1.3689</td>
</tr>
<tr>
<td>2.2500</td>
<td>1.0350</td>
</tr>
<tr>
<td>2.5000</td>
<td>0.6942</td>
</tr>
<tr>
<td>2.7500</td>
<td>0.3489</td>
</tr>
<tr>
<td>3.0000</td>
<td>0</td>
</tr>
</tbody>
</table>
Chapter 3: MHD Transient Flow of a Sisko Fluid in a Porous Medium

Table 3.3: Numerical solution for various porosity parameters

<table>
<thead>
<tr>
<th>η</th>
<th>$K = 0.1$</th>
<th>$K = 0.2$</th>
<th>$K = 0.3$</th>
<th>$K = 0.4$</th>
<th>$K = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.2500</td>
<td>3.9048</td>
<td>2.2340</td>
<td>1.6563</td>
<td>1.3049</td>
<td>1.0780</td>
</tr>
<tr>
<td>0.5000</td>
<td>6.1505</td>
<td>3.1446</td>
<td>2.0959</td>
<td>1.4484</td>
<td>1.0631</td>
</tr>
<tr>
<td>0.7500</td>
<td>7.7165</td>
<td>3.7331</td>
<td>2.3342</td>
<td>1.4705</td>
<td>1.0000</td>
</tr>
<tr>
<td>1.0000</td>
<td>8.6120</td>
<td>4.0168</td>
<td>2.3974</td>
<td>1.4086</td>
<td>0.9130</td>
</tr>
<tr>
<td>1.2500</td>
<td>8.8793</td>
<td>4.0286</td>
<td>2.3182</td>
<td>1.2929</td>
<td>0.8141</td>
</tr>
<tr>
<td>1.5000</td>
<td>8.5894</td>
<td>3.8120</td>
<td>2.1310</td>
<td>1.1443</td>
<td>0.7086</td>
</tr>
<tr>
<td>1.7500</td>
<td>7.8329</td>
<td>3.4151</td>
<td>1.8664</td>
<td>0.9760</td>
<td>0.5986</td>
</tr>
<tr>
<td>2.0000</td>
<td>6.7050</td>
<td>2.8823</td>
<td>1.5484</td>
<td>0.7953</td>
<td>0.4852</td>
</tr>
<tr>
<td>2.2500</td>
<td>5.2939</td>
<td>2.2507</td>
<td>1.1938</td>
<td>0.6058</td>
<td>0.3686</td>
</tr>
<tr>
<td>2.5000</td>
<td>3.6720</td>
<td>1.5480</td>
<td>0.8135</td>
<td>0.4096</td>
<td>0.2488</td>
</tr>
<tr>
<td>2.7500</td>
<td>1.8945</td>
<td>0.7936</td>
<td>0.4143</td>
<td>0.2076</td>
<td>0.1260</td>
</tr>
<tr>
<td>3.0000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Chapter 3: MHD Transient Flow of a Sisko Fluid in a Porous Medium

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$F(\eta)$ at $K = 0.5$, $n = 3$, $N = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b = 0.5$</td>
</tr>
<tr>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.250</td>
<td>1.136</td>
</tr>
<tr>
<td>0.500</td>
<td>1.184</td>
</tr>
<tr>
<td>0.750</td>
<td>1.196</td>
</tr>
<tr>
<td>1.000</td>
<td>1.149</td>
</tr>
<tr>
<td>1.250</td>
<td>1.054</td>
</tr>
<tr>
<td>1.500</td>
<td>0.929</td>
</tr>
<tr>
<td>1.750</td>
<td>0.790</td>
</tr>
<tr>
<td>2.000</td>
<td>0.642</td>
</tr>
<tr>
<td>2.250</td>
<td>0.488</td>
</tr>
<tr>
<td>2.500</td>
<td>0.329</td>
</tr>
<tr>
<td>2.750</td>
<td>0.167</td>
</tr>
<tr>
<td>3.000</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.4: Numerical solution for different Non-Newtonian parameter
### Table 3.5: Numerical solution for different flow index of Non-Newtonian fluids

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$F(\eta)$</th>
<th>$K = 0.25$, $b = 1.5$, $N = 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n = 0.1$</td>
<td>$n = 0.3$</td>
</tr>
<tr>
<td>0.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.2500</td>
<td>1.8004</td>
<td>1.9554</td>
</tr>
<tr>
<td>0.5000</td>
<td>2.4102</td>
<td>2.7197</td>
</tr>
<tr>
<td>0.7500</td>
<td>2.8270</td>
<td>3.2909</td>
</tr>
<tr>
<td>1.0000</td>
<td>3.0447</td>
<td>3.6651</td>
</tr>
<tr>
<td>1.2500</td>
<td>3.0566</td>
<td>3.8321</td>
</tr>
<tr>
<td>1.5000</td>
<td>3.0303</td>
<td>3.7845</td>
</tr>
<tr>
<td>1.7500</td>
<td>2.9280</td>
<td>3.5792</td>
</tr>
<tr>
<td>2.0000</td>
<td>2.6758</td>
<td>3.2062</td>
</tr>
<tr>
<td>2.2500</td>
<td>2.2597</td>
<td>2.6626</td>
</tr>
<tr>
<td>2.5000</td>
<td>1.6758</td>
<td>1.9471</td>
</tr>
<tr>
<td>2.7500</td>
<td>0.9228</td>
<td>1.0596</td>
</tr>
<tr>
<td>3.0000</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Chapter 3: MHD Transient Flow of a Sisko Fluid in a Porous Medium

Figure 3.1: Effect of magnetic parameter on Newtonian fluid

Figure 3.2: Effect of magnetic parameter on Non-Newtonian fluid
Chapter 3: MHD Transient Flow of a Sisko Fluid in a Porous Medium

Figure 3.3: Effect of porosity parameter

Figure 3.4: Effect of Non-Newtonian parameter
Figure 3.5: Comparison of Non-Newtonian with different flow index
3.6 Conclusion

In this chapter, we have investigated the time dependent magnetohydrodynamics flow of a Sisko fluid filling the porous half-space. The governing equations have been solved using deductive group symmetry method. First assuming the general group of transformation-so called deductive group of transformation, we have derive the proper sub-group of transformations, which invariably transforms the governing highly non-linear partial differential equation together with auxiliary conditions, in to the ordinary differential equation. Then the reduced ordinary differential equation is solved numerically, using MATLAB solver and the normalized velocity profiles are generated for the different flow parameters. Finally, the effects of various parameters on Newtonian and Non-Newtonian fluid flow are studied. The remarks on behavior of velocity profiles are quite useful in the practical problems involving Newtonian and Non-Newtonian fluids with different flow index in engineering and science.