Chapter 8

Electric field enhancement in nano-tubes.

As discussed in the previous chapters, various novel targets are studied as alternative to planar targets in the quest for achieving high absorption of intense ultra-short laser pulses for energetic particle and radiation generation [69-85]. To explain interaction of laser with spherical gas clusters, a variety of models of the laser–cluster interaction have been put forward including inner shell excitation [75], ionization ignition [93], collective electron dynamics [104], Coulomb explosion [95], and nano-plasma model [96]. The last one explains the maximization of the absorption cross-section and the electric field in the vicinity of Mie resonance at \(3n_c\) in spherical clusters. This is also supported by a large number of experiments [96-100] which are consistent with this model. For cylindrical nano-structures, this maximization occurs when the electron density passes through \(2n_c\) [17-19, 177, 190, 202]. Nano-structures in other shapes like elongated and pointed nano-structures (nano-protrusions and nano-rods) have shown enhanced x-ray and energetic particle emission and this has been attributed to the enhanced electric field at the tip, also referred to as the “Lightning rod effect” [76, 78, 87]. It is desirable to design a nano-target with a geometry which will enable even higher field enhancement when irradiated by ultra-short laser pulses for efficient x-ray and energetic particle generation [203]. Unfortunately, the field enhancement in nano-structures discussed above is limited by their geometry and dielectric properties. Although the field is resonantly enhanced at Mie resonance in spherical clusters and nano-rods at \(3n_c\) and \(2n_c\) respectively during the ionization and expansion phase, the limitation is that this resonance is at a density much lower than the solid density and the resonance condition is also short lived (~ a few fs) [96, 102]. Even more importantly, as the high density laser heated
cluster plasma expands, the electric field inside the cluster is shielded i.e. the field inside the
cluster is actually less than the applied laser field strength, and the field enhancement occurs only
when the electron density is near to the resonance condition [96, 177, 190, 202, 204]. Further, a
lower resonance electron density requires the laser heated cluster to expand sufficiently long for
the electron density to decrease and approach the resonance condition. This time required is
typically ~100 fs to 1 ps [97, 101, 189, 202]. For ultra-short pulses of duration < 100 fs, the
clusters do not expand enough during the laser pulse to reach resonance at the peak of the pulse
[97, 101]. It is therefore desirable for maximum absorption and x-ray generation that the
resonance condition is met at the peak of the ultra-short laser pulse [96, 177, 190, 202, 204].
Therefore, for meeting this requirement, the laser pulse is generally stretched and there exists an
optimum pulse duration for a given size of a cluster [96, 177, 190, 202, 204]. However,
stretching of the pulse is not very desirable since it compromises the laser intensity as well as the
resultant x-ray emission duration. In the light of the above facts, it would be ideal to have a
structure that has resonance density close to the solid density, which will result in more
absorption, electron generation, and x-ray emission, as now more electrons would interact with
the enhanced field. Moreover, a high resonance density means the resonance condition can be
met even for pulses shorter than 100 fs, since now the nano-structure does not have to expand
much to reach the resonance condition. It is further desirable that the chosen nano-structure has
an extended resonance in time, so that the field enhancement continues during most of the entire
hydrodynamic evolution.

Although a target with the above mentioned features will be highly desirable, little work
has been done for designing such nano-structure targets. Nevertheless, recently, some other kinds
of nano-structures have been used for laser matter interaction studies, especially the ones which
have a hollow structure [79-85]. They show efficiently high x-ray generation and hot electron generation, but their resonance densities have not been reported [79-85]. To name a few, the hollow nano-targets that have been used for experiments with intense short pulses are: fullerene [79], nano-hole alumina [80], foam [81], nano-brush [82], carbon nano-tubes (CNTs) [83-85, 187] etc. The enhanced field and surface currents in hollow structures are believed to support hot electron generation and transport [83-85, 187] necessary for fast ignition scheme in inertial confinement fusion, and also for efficient x-ray generation. The electric field enhancement in the nano-structures make the effective laser intensity on the target very high, and consequently, it leads to x-ray yield enhancement that is proportional to the rescaled enhanced intensity [87]. In the class of hollow targets, CNTs are very attractive for ultra-short laser-plasma interaction [83-85, 187] as they are easily produced in bulk quantities with controllable inner and outer diameters [205]. To understand the reason for the x-ray enhancement from nano-tubes (as reported in references [83-85, 187]), it is necessary to investigate the effect of the hollow structure on the field enhancement and to determine the optimum geometric parameters of the nano-tubes for efficient absorption of ultra-short laser pulses (<1 ps).

Here we calculate the electric field enhancement inside a nano-tube plasma, and find out the plasma density at which the field enhancement occurs. We find that nano-tube has resonance density higher than a nano-rod and it can even be tuned to be close to the solid density. Therefore, a hollow structure helps in overcoming the limitation of a solid cluster or a nano-rod whose resonance densities are much smaller than the solid density. Moreover, it is observed that instead of a single resonance density (e.g. $3n_c$ in spherical clusters [96], or $2n_c$ in a solid nanocylinder [177, 190, 202]), a nano-tube shows field enhancement at two densities: $n_H = \frac{2n_c}{1 - \frac{a}{b}}$
and \( n_L = \frac{2n_e}{1 + \left(\frac{a}{b}\right)} \), where \( a \) and \( b \) are the instantaneous inner and outer radii. The high density resonance \( (n_H) \) occurs at a progressively higher density as one chooses nano-tubes of increasing degree of hollowness \( (a/b) \). Multiple resonances can occur in hollow nano-structures during the ionization and hydrodynamic expansion phases. Another interesting feature arises from the fact that in an expanding heated nano-tube, the plasma density as well as high resonance density \( (n_H) \) decrease with time. Simultaneous decrease in these two densities for nano-tubes of certain degree of hollowness may allow continued occurrence of resonance for a much longer time.

### 8.1 Theoretical considerations

The interaction of ultra-short laser pulses with spherical clusters/nano-rods is calculated considering it as a nano-plasma [96-100]. The cluster dipole moment and the internal electric field can be easily evaluated under the quasi-static approximation (The cluster size being much smaller than the incident laser wavelength) [96-100]. The Laplace equation is solved taking the Drude dielectric function [87, 96-100] and Mie theory in dipole limit to obtain the resonance condition and the internal electric field distribution.

We now proceed on similar lines to calculate the electric field inside a laser irradiated hydrodynamically evolving nano-tube. Consider a nano-tube with inner and outer radii \( a_o \) and \( b_o \) in an applied laser field of strength \( E_L \) \( (E_L = \frac{\lambda}{2}(Ee^{i\omega t} + c.c)) \). Let the laser field be oriented perpendicular to the axis of the nano-tube. A field applied on a nano-tube at any arbitrary direction can be resolved into two components, one parallel to the axis and the other perpendicular to the axis. The component of electric field applied parallel to the nano-tube axis is
not enhanced [177, 190, 202]. The enhancement in the perpendicular component depends on the geometric and dielectric parameters of the nano-tube.

Laser - nano-tube interaction can be studied in two phases, the first being the ionization phase and the second being the hydrodynamic expansion phase. In the ionization phase, at the onset of the interaction of the nano-tube with intense ultra-short laser pulse, there is plasma formation. The threshold of plasma formation is typically $10^{14} \text{ W/cm}^2$ for 100 fs pulse for planar solid and nano-structures [194]. This threshold is crossed near the foot of the intense pulse. Therefore the dielectric constant of the nano-tubes instantaneously modifies and becomes metal-like or plasma-like because of the generation of free electrons [200, 201]. The subsequent pulse further ionizes the nano-structure by optical field ionization and tunnel ionization, leading to creation of a solid density plasma of density $n_0$ [96, 97, 202]. In the ionization phase, the nano-tube’s geometry is almost preserved during interaction of the intense short pulse with the nano-tubes since the temperature rise is not enough to cause sufficient hydrodynamic expansion for the structure to get modified in such short time scales [96, 177, 190, 202, 204]. The nano-tube’s expansion is thus expected to be extremely slow during the ionization phase at the beginning of interaction and subsequently the temperature rises very quickly due to collisional absorption and maximum electron density is achieved near the peak of the pulse [96, 204]. In the hydrodynamic expansion phase, the energy absorption and the consequent sharp rise of the plasma temperature causes the expansion of the nano-plasma, and the electron density of the cluster starts to decrease monotonically [96, 204]. Since the plasma expansion takes place along the density gradient, the inner radius starts decreasing while the outer radius increases. Therefore, the expansion leads to modification of the inner and outer radii to some instantaneous value $a$ and $b$. It must be noted that during the ionization phase, the electron density increase monotonically (under the
assumption that target geometry and inner and outer radii remain fixed). On the other hand, when the nano-tube expands in the hydrodynamic expansion phase [102, 206], the electron density decreases monotonically (under the assumption that no further ionization or recombination takes place) [97, 202]. This modifies the dielectric constant \( \varepsilon = 1 - n_e/n_c \) of the nano-tubes plasma [97, 202] during both the phases of interaction. Here \( n_e \) is the instantaneous electron density and \( n_c \) is the critical density for the laser.

Like in the case of a nano-cluster or nano-rod, the electric field inside the hollow nano-tube is calculated using the Laplace equation \( \nabla^2 V = 0 \) solved under the above mentioned quasi-static approximation, where the outer radius considered is much smaller than the applied laser wavelength (i.e. \( b \ll \lambda \)). Solving this equation, one finds the dependence of the electric field in the nano-tube on the instantaneous value of \( a, b, \varepsilon \) and \( E_L \). Expanding in cylindrical coordinates and neglecting the \( Z \) variation (as the length of the cylinder is much larger compared to its outer diameter), the general solution is

\[
V = K + \ln r + \sum_{n=1}^{\infty} \left( A_n r^n + B_n r^{-n} \right) \cos n\theta + \left( C_n r^n + D_n r^{-n} \right) \sin n\theta \quad \text{.........(8.1)}
\]

Applying appropriate boundary conditions, the potential can be shown to be

\[
V_1 = A_1 r \cos \theta \quad \text{for (} r \leq a \quad \text{Region I)} \quad \text{.........(8.2)}
\]

\[
V_2 = (A_2 r + \frac{B_2}{r}) \cos \theta \quad \text{for (} a \leq r \leq b \quad \text{Region II)} \quad \text{.........(8.3)}
\]

\[
V_3 = -E_L r \cos \theta + \frac{B_3}{r} \cos \theta \quad \text{for (} r \geq b \quad \text{Region III)} \quad \text{.............(8.4)}
\]
Using the continuity of $D_n$, $E_r$ and potential $V$, one gets the four unknowns $A_1, A_2, B_2, B_3$ (all other $A_i$, $B_i$ being zero) in terms of $a$, $b$ and $\varepsilon$ as

\[
A_1 = \frac{-4E_r\varepsilon}{(\varepsilon+1)^2 - \left(\frac{a}{b}\right)^2 (\varepsilon-1)^2} \quad \text{........8.5(a)}
\]

\[
A_2 = \frac{-2E_r(\varepsilon+1)}{(\varepsilon+1)^2 - \left(\frac{a}{b}\right)^2 (\varepsilon-1)^2} \quad \text{........8.5(b)}
\]

\[
B_2 = \frac{-2E_r a^2 (\varepsilon-1)}{(\varepsilon+1)^2 - \left(\frac{a}{b}\right)^2 (\varepsilon-1)^2} \quad \text{........8.5(c)}
\]

\[
B_3 = \frac{E_r b^2(1-a^2)(\varepsilon^2-1)}{(\varepsilon+1)^2 - \left(\frac{a}{b}\right)^2 (\varepsilon-1)^2} \quad \text{........8.5(d)}
\]

The radial and azimuthal components of the electric field \( E_r = -\frac{\partial V}{\partial r} \) and \( E_\theta = -\frac{1}{r} \frac{\partial V}{\partial \theta} \) in the three regions can be derived from equation 8.2, 8.3 and 8.4. The magnitude of electric field ( \( \sqrt{E_r^2 + E_\theta^2} \)) in the three regions is thus given by

\[
E_I = A_1 \quad \text{....................8.6(a)}
\]

\[
E_{II} = \sqrt{A_2^2 + B_2^2 \frac{A_1 B_1}{r^2} + \frac{2A_1 B_1}{r^2} \cos 2\theta} \quad \text{....................8.6(b)}
\]

\[
E_{III} = \sqrt{E_r^2 + B_3^2 + \frac{2E_r B_3}{r^2} \cos 2\theta} \quad \text{....................8.6(c)}
\]

We now calculate the electric field enhancement in laser irradiated nano-tube plasma. In region I, the electric field is given by equation 8.6(a), and it is independent of $r$ and $\theta$. For region
II, one may define the root mean square (R.M.S.) electric field enhancement by taking the spatial average of the magnitude of field enhancement as

\[
\langle \frac{E_{II}}{E_L} \rangle = \frac{1}{\pi(b^2-a^2)} \sqrt{\int_0^{2\pi} \int_a^b \left( \frac{E_{II}}{E_L} \right)^2 r dr d\theta} \ldots \ldots 8.7(a)
\]

\[
= \frac{1}{E_L} \sqrt{A_2^2 + \frac{B_2^2}{a^2b^2}} \ldots \ldots 8.7(b)
\]

For the case of a nano-rod (a=0), the above equations provide the electric field inside the nano-rod to be \( \frac{2E_L}{\varepsilon+1} \), as expected [177, 190, 202]. By using the value of \( \varepsilon = 1 - n_e/n_c \) (where \( n_c \) is the electron density and \( n_c \) is the critical density), it is seen that the resonant field enhancement occurs at \( 2n_c \), in agreement with earlier reported result [177, 190, 202].

Next, we may note from equations 8.6(a) and 8.7(b), together with equations 8.5(a) – 8.5(c), that the electric field in region I and region II will peak for certain values of \( \frac{a}{b} \) given by

\[
\frac{a}{b} = \pm \left( \frac{\varepsilon + 1}{\varepsilon - 1} \right) \ldots \ldots (8.8)
\]

Therefore, the field enhancement in the case of nano-tube plasma occurs at two densities

\[
n_H = \frac{2n_c}{1 - \left( \frac{a}{b} \right)} \ldots \ldots (8.9)
\]

\[
n_L = \frac{2n_c}{1 + \left( \frac{a}{b} \right)} \ldots \ldots (8.10)
\]

It may be seen from equation 8.9 that the higher resonance density \( n_H \) can be increased by choosing a nano-tube of smaller wall thickness. This is in contrast to the case of solid clusters.
and nano-rods, where the field enhancement occurs only in the vicinity of $3n_c$ and $2n_c$ respectively.

### 8.2 Results and Discussion

From the preceding section, it is clear that the field enhancement depends on the ratio of the instantaneous inner and outer radii of the laser irradiated nano-tubes and the instantaneous dielectric constant of the plasma. First, one calculates the field enhancement factor during the ionization phase of laser nano-tube interaction where the electron density of the nano-tube plasma monotonically increases as the laser pulse intensity increases. The nano-tube gets ionized up to the solid density with no expansion (i.e. the initial inner and outer radii are at $a_0$ and $b_0$) [97, 202, 207]. This is a valid assumption since the optical field ionization and tunnel ionization are instantaneous processes.

Figure 8.1 shows the root mean square (R.M.S.) field enhancement during the ionization phase of laser irradiated nano-tubes of different degrees of hollowness (which is defined as the ratio of the initial inner and outer radii) subject to the condition that $b_0 \ll \lambda$, the assumption under which the Laplace equation was solved. For this calculation, one uses equation 8.7(b) and the value of $\varepsilon$ is varied since $n_e/n_c$ varies from 0 to 40 ($n_o/n_c$ is taken as 40 for Ti:sapphire laser (800 nm) irradiation of CNTs). Four values of degree of hollowness (0, 0.5, 0.75, and 0.95) were chosen.
It is seen from Fig. 8.1 that for a nano-rod, there is a single resonance at $2n_c$, also the electric field is highly shielded as the electron density increases and approaches the solid density. It can also be seen that nano-tubes show two resonances during ionization. To obtain a high electric field enhancement at a higher electron density, it is desirable to choose a nano-tube with greater degree of hollowness. This brings out an important point in the perspective of the spherical clusters and nano-rods where the resonance condition $(3n_c, 2n_c)$ is met near the foot of the laser pulse during the ionization phase. However, since the resonance is reached at low laser intensity (near the foot of the pulse), this resonance is of less significance [96-100]. It is desirable to meet the resonance condition at the peak of the laser pulse. If the resonance density is enhanced by choosing a hollow nano-structure, the resonance condition could be met when the applied field is also high [79-85]. Therefore, even in the ionization phase, one can make the resonance happen near the peak of the pulse so that it will have a significant role in the interaction with nano-tube.
FIG. 8.2: Initial field enhancement factor as a function of $a_0/b_0$ in two regions of the nano-tube: dotted curve $r<a_0$ Region I, solid curve $a_0<r<b_0$ Region II. Maximum initial field enhancement occurs for

$$\frac{a_0}{b_0} = \frac{n_0 - 2n_c}{n_0}.$$ 

Next, for the other phase of laser nano-tube interaction where one considers the hydrodynamic evolution, one can consider a zero time to mark the onset of expansion and variation of nano-tube inner and outer radii with time. One can define the zero time as the time when the nano-tube plasma is ionized up to the solid density $n_0$ assuming no expansion has occurred (i.e. the initial inner and outer radii are at $a_0$ and $b_0$) [97, 204, 207]. For calculating the initial value of $\varepsilon$, $n_c/n_c$ is taken as 40. This will help in identifying the class of nano-tubes with same degree of hollowness ($a_0/b_0$) for maximum initial field enhancement. Figure 8.2 shows the variation of initial field enhancement factor with the degree of hollowness ($a_0/b_0$) in region I and II. The value of $a_0/b_0$ lies between 0 and 1. For a nano-rod $a_0/b_0=0$. It is clearly seen that for a nano-rod the field is highly shielded. It is also seen that the field in region I is higher than that in region II. Further, for smaller values of $a_0/b_0$, there is a shielding of the applied electric field. However, as the degree of hollowness increases, field enhancement occurs. The maximum enhancement occurs for
\[
\frac{a_0}{b_0} = \frac{n_0 - 2n_e}{n_0} \quad \text{.....(8.11)}
\]

As may be seen from equation 8.9, this condition corresponds to the occurrence of the high density resonance at the solid density \(n_0\). Beyond this value of \(a_0/b_0\), the field enhancement starts reducing.

Now we examine the temporal evolution of the R.M.S. electric field during the expansion phase in the nano-tube plasma and also estimate the time scales of resonance, as done for clusters or nano-fibers [177, 204, 207]. This is done under a simplifying assumption of a constant temperature. As the plasma expands along the density gradient (perpendicular to the nano-tube axis), the instantaneous inner radius \(a\) decreases and is \(a_0 - c_s t\) and outer radius \(b\) increases and is \(b_0 + c_s t\), where \(c_s = \sqrt{\frac{ZkT}{M_i}}\) is the plasma expansion speed, where \(M_i\) is the ion mass. The value of \(c_s\) can be estimated from the work of Issac et al [192] who have done experiments on gas cluster using Ti:Sapphire laser with the duration ranging from 60 fs to 2 ps, and intensity up to \(10^{18}\) W/cm\(^2\). They have predicted a nearly constant electron temperature (between 1-2 keV) from experiments and simulations for the range of pulse duration in their experiments. Taking the typical value of plasma temperature of \(~1\) keV from their work, we estimate \(c_s\) to be \(~100\) nm/ps [177]. Of course, \(c_s\) will have some variation during the hydrodynamic expansion, but a constant value of \(c_s\) for a given average electron temperature helps in predicting the time scale at which the resonance would occur. For example, various experiments on spherical clusters and nano-fibers have shown that an assumption of constant \(c_s\) helps in predicting the resonance time scales quite accurately [97, 177]. Moreover, the simulations done by Ditmire et al [95] and Liu et al [204] show that the ion velocity \(c_s\) is extremely low initially during the ionization phase and after that it increases very rapidly when the electron density become close to solid density (this
happens approximately near the peak of the laser pulse), after this $c_s$ is almost constant. The electron density gradually decreases as the cluster expands with a variation discussed in the following text. On the basis of this picture, it is a good approximation to consider the zero time as that time when the nano-tube plasma is at solid density $n_0$ with a given inner and outer radii $a_0$ and $b_0$ and then it starts expanding eventually leading to the monotonic decrease of the electron density [177, 204, 207].

An important point during the hydrodynamic expansion of the nano-tube plasma is the "void closure", i.e. when $a = 0$. The time required for void closure ($t_v$) is $a_0/c_s$. If the void closure occurs before the condition for high density resonance is reached, one may term the nano-tube as a "thick" nano-tube. On the other hand, if the void closure takes place after the occurrence of high density resonance, the nano-tube may be referred to as a "thin" nano-tube. We first consider a thick nano-tube. Before the void closure, the plasma density varies with time as

$$n_e = \frac{n_0(b_0-a_0)}{b_0-a_0+2c_s t}$$

and after the void closure, it varies as

$$n_e = \frac{n_0(b_0^2-a_0^2)}{(a_0+b_0+c_s \tau)^2}$$

(where $\tau$ in the last expression is the time after void closure). Once the void closure occurs, the nano-tube plasma behaves like a nano-rod which will show resonance at $2n_c$. Next, in the case of a thin nano-tube, the time taken by the nano-tube plasma to reach the high density ($n_H$) resonance is

$$t_H = \frac{1}{c_s} \left[ (b_0-a_0) \frac{n_0}{2n_c} - b_0 \right] .$$

By comparing this time with the time of void closure, it is easily seen that the high density resonance will occur before void closure if

$$\frac{a_0}{b_0} > \frac{n_0 - 2n_c}{n_0 + 2n_c} .$$

Further, in a real situation, $t_H > 0$. This implies that

$$\frac{a_0}{b_0} < \frac{n_0 - 2n_c}{n_0} .$$

It can be shown that if this condition is
satisfied, then the low density \((n_L)\) resonance also occurs before the void closure. Therefore two resonances occur only when \(\frac{n_0 - 2n_c}{n_0 + 2n_c} < \frac{a_0}{b_0} < \frac{n_0 - 2n_c}{n_0} \). 

In the thin nano-tube category, we may identify two particular situations, viz. 
\[
\frac{a_0}{b_0} = \frac{n_0 - 2n_c}{n_0} as in equation 8.11, and the other being \(\frac{a_0}{b_0} > \frac{n_0 - 2n_c}{n_0} \). As seen from Fig. 8.2 and discussed earlier, the first condition \(\frac{a_0}{b_0} = \frac{n_0 - 2n_c}{n_0} \) corresponds to occurrence of high density resonance at the solid density. Such a nano-tube may be referred to as “resonant” nano-tube. Finally when \(\frac{a_0}{b_0} > \frac{n_0 - 2n_c}{n_0} \), the high density resonance does not occur at all, and only the low density resonance condition is fulfilled. We may refer to such a nano-tube as "ultra-thin". Thus, the nano-tubes can be categorized as

Thick nano-tube : \(0 < \frac{a_0}{b_0} < \frac{n_0 - 2n_c}{n_0 + 2n_c} \) \hspace{1cm} \ldots \ldots (8.12)

Thin nano-tube : \(\frac{n_0 - 2n_c}{n_0 + 2n_c} < \frac{a_0}{b_0} < \frac{n_0 - 2n_c}{n_0} \) \hspace{1cm} \ldots \ldots (8.13)

Ultra-thin nano-tube : \(\frac{a_0}{b_0} > \frac{n_0 - 2n_c}{n_0} \) \hspace{1cm} \ldots \ldots (8.14)

Resonant nano-tube : \(\frac{a_0}{b_0} = \frac{n_0 - 2n_c}{n_0} \) \hspace{1cm} \ldots \ldots (8.15)
FIG. 8.3: Time evolution of the electric field in various size nano-tubes: (a) Thin nano-tube, (b) Thick nano-tube, (c) Ultra-thin nano-tube, and (d) Resonant nano-tube (all for \( n_0/n_c = 40, c_s = 100 \text{ nm/ps} \)).

The points H, L and V indicate the time corresponding to occurrence of higher density resonance, lower density resonance, and void closure respectively.

Using equations 8.5(b), 8.5(c) and 8.7(b), one can plot the R.M.S. electric field enhancement evolution for nano-tubes with initial inner and outer radii \( a_0 \) and \( b_0 \) respectively, while they expand after being irradiated by an intense ultra-short laser pulse. The initial maximum plasma electron density is chosen as \( n_0/n_c = 40 \) and the expansion speed \( c_s = 100 \text{ nm/ps} \) [192]. We also take into account that the fact that hydrodynamic evolution causes the instantaneous inner radius \( a \) to decrease as \( a_0-c_st \) and outer radius \( b \) increases as \( b_0+c_st \) and this causes the decrease of electron density and the dielectric constant (\( \varepsilon \)) modifies accordingly.

Figure 8.3 shows the calculated variation of the R.M.S. electric field enhancement in region II for thick and thin carbon nano-tubes. Figure 8.3a depicts the case of a thick nano-tube \( (a_0 = 40 \text{ nm}, b_0 = 50 \text{ nm}) \), showing the occurrence of void closure at 400 fs, followed by \( 2n_c \) resonance at 840 fs. Figure 8.3b shows the electric field evolution for a thin nano-tube \( (a_0 = 47 \text{ nm}, b_0 = 50 \text{ nm}) \),...
nm). It is noted that while for the thick nano-tube there was an initial field shielding and the field enhancement occurred only near $2n_c$, the field inside the thin nano-tube is highly enhanced from the very beginning of heating and expansion and the enhancement takes place throughout the expansion of the nano-tube. The high (H) and low (L) density resonances are observed at 100 fs and 335 fs respectively, prior to the void closure (V) at 470 fs. Next, Fig. 8.3c shows the electric field evolution for ultra-thin nano-tube ($a_0 = 48$ nm, $b_0 = 50$ nm). It shows the occurrence of the low density resonance (L) at 250 fs, while the void closure (V) occurs at 480 fs. Like the thin nano-tube, even in the case of the ultra-thin nano-tube the field enhancement exists at all instants of the expansion. Finally, Fig. 8.3d shows the electric field evolution of a resonant nano-tube ($a_0 = 47.5$ nm, $b_0 = 50$). It is seen that the high density resonance (H) occurs over an extended time period from the beginning. The low density resonance (L) occurs at 295 fs and void closure at 475 fs. Thus, in all the cases of thin nano-tube, there is a greater field enhancement compared to the thick nano-tube.

An important aspect of the nano-tube - laser interaction arises from the fact that as the nano-tube (with appropriate choice of $a_0$ and $b_0$) expands, the higher density resonance will progressively occur at lower densities, so that the resonance condition can be sustained over a longer time. This is clearly seen from Fig. 8.4 which depicts the time evolution of the resonance densities of different nano-tubes (thick, thin, ultra-thin and resonant), plotted along with their density evolution for $n_0/n_c = 40$ and $c_s = 100$ nm/ps. The point of intersection of the decreasing nano-tube density with evolving resonance density gives the time of occurrence of resonance.
FIG.8.4: Time evolution of the nano-tube density (solid curve) along with that of the higher resonance density (dotted curve) and the lower resonance density (dashed curve) for (a) thick, (b) thin, (c) ultra-thin, and (d) resonant nano-tubes. The points H, L and V indicate the time corresponding to occurrence of higher density resonance, lower density resonance, and void closure respectively.

Figure 8.4a shows that the void closure (V) in the case of thick nano-tube \( (a_0 = 40 \text{ nm}, b_0 = 50 \text{ nm}) \) takes place before the occurrence of the resonance condition and the resonant density becomes \( 2n_c \), like in a solid nano-rod. In addition, the time duration over which the nano-tube plasma density is close to the resonance density is small. Figure 8.4b for the case of thin nano-tube \( (a_0 = 47 \text{ nm}, b_0 = 50 \text{ nm}) \) shows the occurrence of both higher and lower density resonances \( (H,L) \) before the void closure \( (V) \) occurs. In this case, the plasma density remains in the vicinity of the high resonance density for a longer time. Figure 8.4c shows that in the case of ultra-thin nano-tube \( (a_0 = 48 \text{ nm}, b_0 = 50 \text{ nm}) \), only the low density resonance \( (L) \) takes place before void closure \( (V) \) and the resonance condition is met over a small time. Figure 8.4d shows that in the case of “resonant” nano-tube \( (a_0 = 47.5 \text{ nm}, b_0 = 50 \text{ nm}) \) the high density resonance starts from the very onset of expansion, and the resonance continues for a very long time compared to the other categories of nano-tubes. These figures (Figs. 8.4a, b, c) also show the number of
resonances shown by thick, thin and ultra-thin tubes, as were seen earlier in Figs 8.3a-c. The overall analysis brings out the fact that the hollow structure has various advantages over clusters or nano-rods as resonance density of a nano-tube can be tuned close to the solid density and the resonance condition can be met for pulses shorter than 100 fs. In addition to this, there are resonances during the ionization and expansion phase, along with continued occurrence of high density resonance for a nano-tube with a particular degree of hollowness. Finally, since the electric field inside a structure governs the absorption, electron, ion and x-ray generation, therefore large field enhancement in laser irradiated nano-tubes make them very interesting target from application point of view.

To summarize, we have studied the evolution of the electric field enhancement inside the nano-tube plasma irradiated with intense short pulse laser. The hollowness of the nano-tubes determines the field enhancement and the electron density at which such structures exhibit resonance. It is found that a nano-tube exhibits two resonance at two electron densities during the ionization phase. During the hydrodynamic expansion phase also, a thin nano-tube exhibits resonant field enhancement at two densities (depending on its inner and outer radii). There exists a particular ratio of the inner to outer radii of the nano-tube where the field enhancement starts right at the solid density which may continue for a much longer time as the higher resonance density and the nano-tube density decrease simultaneously during the expansion of the heated nano-tube. While detailed theoretical calculations and computer simulations may be necessary to obtain exact quantitative information, the physical effects are quite well illustrated by taking simple analytical treatment and typical parameters of laser-nano-tube interaction. The observed features make nano-tubes of appropriately chosen inner and outer radii an attractive target for efficient absorption of intense ultra-short laser pulses. It will also be interesting to study the
interaction of nano-tubes with an intense few cycle laser pulse since the hydrodynamic motion is almost frozen in those time scales and the pulse interacts with a solid density plasma. In general, these calculations can also be extended to design efficient carbon nano-tubes based field emitters.