Chapter 2

Warped models

During the last decade the Randall-Sundrum (RS) model [93] and its variants have attracted a lot of attention, both theoretically and phenomenologically as this model solves the gauge hierarchy problem in a very elegant manner. Due to the AdS/CFT correspondence [94], such a theory defined in a slice of AdS space is conjectured to be dual to some strongly coupled 4D theory. In Sec. 2.1 we briefly review the construction of the RS model, including the derivation of the warped metric as a solution to the Einstein’s equations [93]. Then we show how this model solves the gauge hierarchy problem of the SM. After this, we present a short discussion on the bulk gauge and fermion fields coupled with an IR-brane localized Higgs field. In Sec. 2.2 we give the details of the warped models both without and with custodial protection of the $Zb_Lb_L$ coupling. We discuss the gauge sector and different quark representations of these models, and write various Lagrangian terms in the mass basis relevant to the phenomenology we discuss in the subsequent chapters.

2.1 Original RS model

Following Ref. [93], in this section we briefly review the construction of the RS model and present the derivation of the warped metric as a solution to the Einstein’s equations. We
consider a five dimensional spacetime with one extra spatial dimension $y$ compactified on an orbifold $S^1/\mathbb{Z}_2$, where $S^1$ denotes a circle with compactification radius $R$ and $\mathbb{Z}_2$ is a parity symmetry. In other words the fifth dimension $y$ is periodic with a period $2\pi R$ and $(x^\mu, y)$ is identified with $(x^\mu, -y)$, where $x^\mu$ denote the 4D Minkowskian coordinates. Thus, the $y$ coordinate is bounded in the interval $0 \leq y \leq \pi R$. The boundaries of this interval are called 3-branes. The branes at $y = 0$ and $y = \pi R$ are called the Ultraviolet (UV) or the Planck brane and the Infrared (IR) or the TeV brane respectively. As discussed in Ref. [95] and references therein, the $y$ direction is related to the renormalization scale of the 4D theory. The presence of the brane at $y = 0$ will make gravity dynamical in the 4D dual theory introducing the Planck scale $M_{Pl}$, a UV scale. Thus, the brane at $y = 0$ is called the “Planck brane” or the “UV brane”. The brane at $y = \pi R$ will break the conformal symmetry spontaneously in the IR and will introduce masses in the 4D theory. Thus, the brane at $y = \pi R$ is called the “IR brane”. Since the effective scale at $y = \pi R$ is the TeV scale, this brane is also known as the “TeV brane”. The region between the UV brane and the IR brane ($i.e.$ $0 < y < \pi R$) is called the bulk. The classical action for this setup can be split into three parts as follows

$$S = S_{\text{bulk}} + S_{\text{UV}} + S_{\text{IR}},$$

(2.1)
where $S_{\text{bulk}}$, $S_{\text{UV}}$ and $S_{\text{IR}}$ represent the actions for the bulk, the UV brane and the IR brane respectively, and they read as

$$S_{\text{bulk}} = \int d^4x \int_0^{\pi R} dy \sqrt{-G} \left( -\Lambda + 2M^3\mathcal{R} \right) \quad (2.2)$$

$$S_{\text{UV}} = \int d^4x \sqrt{-G} (\mathcal{L}_{\text{UV}} - V_{\text{UV}}) \delta(y) \quad (2.3)$$

$$S_{\text{IR}} = \int d^4x \sqrt{-G} (\mathcal{L}_{\text{IR}} - V_{\text{IR}}) \delta(y - \pi R) \quad , \quad (2.4)$$

where $G$ is the determinant of the 5D metric $G_{MN}(x^\mu, y)$ (where $M, N = 0, \ldots, 4$), $\Lambda$ is the 5D cosmological constant, $M$ is the 5D fundamental scale of gravity and $\mathcal{R}$ is the 5D Ricci scalar. In Eqs. (2.3) and (2.4), the 4D vacuum energy $V_{\text{UV}}$ and $V_{\text{IR}}$ act as gravitational sources even in the absence of particle excitations. Our strategy is to derive the background metric in absence of any particle excitation and then to add matter fields as perturbations on the background metric. Thus, we set $\mathcal{L}_{\text{UV}}, \mathcal{L}_{\text{IR}} = 0$ and write the 5D Einstein’s equations for the action $S$ as follows

$$\sqrt{-G} \left( \mathcal{R}_{MN} - \frac{1}{2} G_{MN} \mathcal{R} \right) = - \frac{1}{4M^3} \sqrt{-G} G_{MN} \left[ \Lambda + V_{IR} \delta(y - \pi R) + V_{UV} \delta(y) \right] \quad , \quad (2.5)$$

where $\mathcal{R}_{MN}$ is the 5D Ricci tensor. We assume that there exists a solution of Eq. (2.5) that respects 4D Poincare invariance in the $x^\mu$ directions. The general form of the 5D metric which satisfy this ansatz can be written as

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \quad , \quad (2.6)$$

where $\eta_{\mu\nu} = \text{diag}(1,-1,-1,-1)$ is the 4D Minkowskian metric. Our aim is to find out the unknown function $\sigma(y)$ appearing in Eq. (2.6). Using the metric in Eq. (2.6), the Einstein’s equations shown in Eq. (2.5) reduce to two differential equations as follows

$$\frac{d\sigma}{dy} = \sqrt{-\Lambda 24M^3} ; \quad \frac{d^2\sigma}{dy^2} = \frac{1}{12M^3R} \left[ V_{UV} \delta(y) + V_{IR} \delta(y - \pi R) \right] \quad . \quad (2.7)$$
The solution to the first order differential equation above consistent with the orbifold symmetry is

\[ \sigma = |y| \sqrt{-\frac{\Lambda}{24M^3}}. \] (2.8)

Since the metric is a periodic function in \( y \), using Eq. (2.8) we calculate \( \sigma'' \) as follows

\[ \frac{d^2 \sigma}{dy^2} = \frac{2}{R} \sqrt{-\frac{\Lambda}{24M^3}} [\delta(y) - \delta(y - \pi R)]. \] (2.9)

Comparing \( \sigma'' \) in Eq. (2.7) and Eq. (2.9), we find that a solution of Eq. (2.7) exists only if \( V_{UV}, V_{IR} \) and \( \Lambda \) are related in terms of a single scale \( k \) as

\[ V_{UV} = -V_{IR} = 24M^3k; \quad \Lambda = -24M^3k^2. \] (2.10)

Thus, the form of the 5D metric as a solution to the 5D Einstein’s equations for the RS warped geometry is given by

\[ ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2. \] (2.11)

We note that the above solution is valid only if \( \Lambda \leq 0 \). The case \( \Lambda = 0 \) gives the flat extra dimension, while for the \( \Lambda < 0 \) case, the 5D bulk is a slice of 5D Anti-de-Sitter space (AdS\(_5\)). Due to the non-vanishing negative 5D cosmological constant, the extra dimension has a finite curvature. Due to the presence of the \( e^{-2ky} \) factor in the metric, this space is called "warped". The \( x_\mu \) directions are flat and respects 4D Poincare invariance.

2.1.1 Solution to the hierarchy problem

Here we discuss how the RS geometry solves the gauge hierarchy problem. One can obtain a 4D effective theory by integrating over the extra dimension \( y \). Using the 5D metric in Eq. (2.11) in the 5D action \( S \), we obtain the 4D action corresponding to the
4D curvature term as

\[ S_{4D} \supset \int d^4 x \int_0^{\pi R} dy \ 2M^3 e^{-2k\pi R} \sqrt{-g} \widehat{R} , \]  

(2.12)

where \( \widehat{R} \) is the 4D Ricci scalar constructed from the 4D metric \( g_{\mu\nu} \) which has the form

\[ g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x) . \]  

(2.13)

The \( h_{\mu\nu}(x) \) describes local gravitational fluctuations on the background metric \( \eta_{\mu\nu} \). From Eq. (2.12) one can relate the 4D effective Planck scale of gravity \( M_{Pl} \) to the 5D gravity scale \( M \) as

\[ M^2_{Pl} = \frac{M^3}{k} (1 - e^{-2k\pi R}) \approx \frac{M^3}{k} \ \text{(since} \ e^{-2k\pi R} \ll 1) . \]  

(2.14)

We will see that in order to solve the gauge hierarchy we need \( k\pi R \sim 35 \) which makes \( e^{-2k\pi R} \ll 1 \) (cf. Eq. (2.16)). Now we move to a situation where \( \mathcal{L}_{IR} \neq 0 \) and consider a fundamental scalar field \( H \) on the IR brane with a vacuum expectation value (VEV) \( \langle H \rangle = v_0 \). The 4D action for this case is

\[ S_{4D} \supset \int d^4 x \sqrt{-g_{IR}} \left\{ g^\mu_{IR} \eta^\nu \partial_\mu H^\dagger \partial_\nu H - \lambda (H^\dagger H - v_0^2)^2 \right\} , \]  

(2.15)

where \( g^\mu_{IR} = e^{2k\pi R} \eta^\mu \) and \( g_{IR} = \text{det}(g^\mu_{IR}) = -e^{-8k\pi R} \). We absorb a factor \( e^{-k\pi R} \) in the definition of \( H \) to canonically normalize it and by replacing \( H \rightarrow e^{k\pi R} H \) we obtain

\[ S_{4D} \supset \int d^4 x \left\{ \eta^\mu \partial_\mu H^\dagger \partial_\nu H - \lambda (H^\dagger H - e^{-2k\pi R}v_0^2)^2 \right\} . \]  

(2.16)

In the above equation, we observe that the fundamental Higgs VEV is rescaled by a warp factor and the effective symmetry breaking scale \( v \) is given by \( v = e^{-k\pi R}v_0 \). According to the naturalness principle, we assume that all the fundamental parameters are of same order i.e. \( M, k, v_0 \sim \mathcal{O}(M_{Pl}) \). Thus, there is no large hierarchy present between the
fundamental parameters. But we can derive a scale $v \sim \mathcal{O}(\text{TeV})$ by choosing $k\pi R \sim 35$, the scale of EWSB from the Planck scale. Therefore, the RS model offers an intriguing solution to the gauge hierarchy problem by reducing the large hierarchy between the Planck scale and the scale of EWSB. This concludes the review of the original Randall-Sundrum model [93].

### 2.1.2 SM fields in the Bulk

In the original RS model only gravity is assumed to propagate into the bulk, while all the SM fields are assumed to be confined on the TeV brane. Localization here means that the fields are confined to a sub-space. Examples of such localization include solitonic solutions (see for example Ref. [96]) and D-Brane solutions (see for example Ref. [97]). For the gauge hierarchy problem, only the Higgs field has to be localized on the TeV brane. In addition to the gauge hierarchy problem, the fermion mass hierarchy problem of the SM can also be addressed by allowing SM fermions to propagate in the bulk [98–102]. In Fig. 2.1 we demonstrate an illustrative picture of warped extra dimension where fermions and gauge bosons propagate into the bulk, while Higgs is localized on the IR brane, and here we consider this scenario. Setting all interaction terms to zero, the free field action for gauge and fermion fields is given by

$$S = \int d^4x \int_0^{\pi R} dy \sqrt{-G} \left[ -\frac{1}{4} F_{MN} F^{MN} + \frac{1}{2} \bar{\psi} \left( i\Gamma_M (\partial_M + \omega_M) - ck \right) \psi \right] + \text{H.c.} , \quad (2.17)$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ is the field strength tensor of the 5D gauge field $A_M$. The 5D Dirac matrices and spin connections in curved spacetime is denoted by $\Gamma_M$ and $\omega_M$ respectively. The bulk mass of the 5D fermion $\psi$ is $m = ck$ where $c$ is the bulk mass parameter. We obtain the EOM for the gauge and the fermion fields using the variational principle $\delta S = 0$ which yields

$$\left[ -e^{2ky} \eta^{\mu\nu} \partial_\mu \partial_\nu + e^{s\pi ky} \partial_5 (e^{-s\pi ky} \partial_5) - M_\Phi^2 \right] \Phi(x^\mu, y) = 0 , \quad (2.18)$$
where $\Phi = \{ A_M, e^{-2ky}\psi_{L,R} \}$. Fermion field is scaled by a factor $e^{-2ky}$ as required for proper normalization and $L, R$ represent the Lorentz chiralities. In case of gauge fields, $s_A = 2$ and $M_A^2 = 0$ with the gauge choice $\partial_{\mu}A^\mu = 0$ and $A_5 = 0$. In case of fermions, $s_{\psi} = 1$ and $M_{\psi,n}^2 = c(c \pm 1)k^2$. In order to solve the EOM in Eq. (2.18), we decompose 5D gauge and fermion fields in a complete set $f^{(n)}_\Phi$ as follows

$$A_{\mu}(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} A^{(n)}_{\mu}(x^\mu)f^{(n)}_A(y)$$

$$\psi_{L,R}(x^\mu, y) = \frac{\psi^{(n)}_{L,R}(x^\mu)}{\sqrt{\pi R}} \sum_{n=0}^{\infty} f^{(n)}_{\psi_{L,R}}(y).$$

This decomposition is called Kaluza-Klein (KK) decomposition. The infinite sums appearing in the decompositions correspond to a tower of 4D KK states and each KK state is associated with a profile $f$ along the $y$ direction. Using the KK decomposition of $\Phi$ in Eq. (2.18) we find that $f$ satisfy the following equation

$$\left[ \partial_y^2 - s_\psi k \partial_y - (M^2 - e^{2ky}m^2_n) \right] f^{(n)}_\Phi(y) = 0,$$

where $m_n$ is the mass of the $n$-th KK mode satisfying the relation $\eta^{\mu\nu}\partial_\mu\partial_\nu\Phi^{(n)}(x^\mu) = m^2_n\Phi^{(n)}(x^\mu)$ relation. Eq. (2.21) is a second order differential equation which can be solved by specifying two boundary conditions (BCs) at the boundaries $y = 0$ and $y = \pi R$. Here we consider two types of BCs,

- Dirichlet (−) BC: The field $\Phi(x^\mu, y)$ or equivalently $f^{(n)}_\Phi(y)$ vanishes on the brane.

- Neumann (+) BC: The derivative of the field $\partial_y\Phi(x^\mu, y)$ vanishes on the brane.

By properly choosing the BCs for the field content of the theory, one can construct phenomenologically interesting models in agreement with the current experimental constraints. Now we discuss the solution of the EOM for the bulk gauge and fermion fields.
Gauge fields in the bulk

Solving the EOM for the gauge field using the KK decomposition given in Eq. (2.19) we obtain the bulk gauge boson profiles as \[ f_A^{(0)}(y) = 1; \quad f_A^{(n)}(y) = e^{ky/2} \frac{J_1 \left( \frac{m_n e^{ky}}{k} \right) + b_1(m_n) Y_1 \left( \frac{m_n e^{ky}}{k} \right)}{N_n} , \] where \( n = 1, 2, \ldots \) labels the \( n \)-th KK mode. The \( J_1(x) \) and \( Y_1(x) \) are the Bessel functions of order one of the first and the second kind respectively. We note that the zero mode profile \( f_A^{(0)}(y) \) for a massless gauge field is flat (i.e. not dependent on \( y \)) whereas the higher KK profiles \( f_A^{(n)}(y) \) are exponentially peaked towards the TeV brane. The flat zero mode, \( f_A^{(0)}(y) = 1 \) exists only for \((+, +)\) BCs. Here the signs in the bracket indicate the BCs for each field on the UV and IR brane respectively. These profiles satisfy the following orthonormality conditions,

\[
\frac{1}{\pi R} \int_0^{\pi R} dy f^{(m)}(y) f^{(n)}(y) = \delta_{mn} ,
\]

from which one can determine the normalization \( N_n \). The KK mass \( m_n \) and the coefficient \( b_1(m_n) \) depend on the choice of the BCs on the branes. Here we consider gauge fields with \((+, +)\) and \((-+, +)\) BCs.

- For \((+, +)\) BCs, i.e. \( \partial_y f_A^{(n)}(y) \big|_{y=0,\pi R} = 0 \):

\[
b_1(m_n) = -\frac{J_1 \left( \frac{m_n}{k} \right) + \left( \frac{m_n}{k} \right) J'_1 \left( \frac{m_n}{k} \right)}{Y_1 \left( \frac{m_n}{k} \right) + \left( \frac{m_n}{k} \right) Y'_1 \left( \frac{m_n}{k} \right)} = b_1(m_n e^{k\pi R}) ,
\]

which can be solved numerically for \( m_n \) and \( b_1(m_n) \). For instance, solving Eq. (2.24) numerically for the first KK mode with \((+, +)\) BCs we find \( m_1^{(+,+)} \approx 2.45 k e^{-k\pi R} \).

- For \((-+, +)\) BCs, i.e. \( f_A^{(n)}(y) \big|_0 = 0 \) and \( \partial_y f_A^{(n)}(y) \big|_{\pi R} = 0 \):

\[
b_1(m_n) = \frac{J_1 \left( \frac{m_n}{k} \right)}{Y_1 \left( \frac{m_n}{k} \right)} = -\frac{J_1 \left( \frac{m_n e^{k\pi R}}{k} \right) + \left( \frac{m_n}{k} \right) e^{k\pi R} J'_1 \left( \frac{m_n}{k} e^{k\pi R} \right)}{Y_1 \left( \frac{m_n e^{k\pi R}}{k} \right) + \left( \frac{m_n}{k} e^{k\pi R} \right) Y'_1 \left( \frac{m_n}{k} e^{k\pi R} \right)} ,
\]
Solving the above equation numerically we find that the first KK gauge boson mass with \((-+,+)\) BCs is \(m_1^{(-,+)} \approx 2.40ke^{-k\pi R}\).

We note that \(m_1^{(-,+)} < m_1^{(+,+)}\) and we define \(M_{KK} = m_1^{(+,+)}\) i.e. the mass of the lowest gauge KK excitation.

**Fermion fields in the bulk**

![Graphs showing mass vs. c-parameter for different values of \(M_{KK}\)]

Figure 2.2: Masses of the first KK fermion with \((-+,+)\) (left) and \((+,+)\) (right) BCs as functions of \(c\)-parameter for \(M_{KK} = 3\) and 5 TeV.

Solving the EOM for the fermion field using the KK decomposition given in Eq. (2.20) we obtain the bulk profiles for left-handed fermion as [99]

\[
\begin{align*}
    f^{(0)}_{\Psi_L} (y) &= \sqrt{\frac{(1 - 2c)k\pi R}{e^{(1-2c)k\pi R} - 1}} e^{-cky}, \\
    f^{(n)}_{\Psi_L} (y) &= \frac{e^{ky/2}}{N_n} \left[ J_\alpha \left( \frac{m_n}{k} e^{ky} \right) + b_\alpha(m_n) Y_\alpha \left( \frac{m_n}{k} e^{ky} \right) \right],
\end{align*}
\]  

where \(n = 1, 2, \ldots\) labels the \(n\)-th KK mode and \(\alpha = |c + 1/2|\). The special functions \(J_\alpha\) and \(Y_\alpha\) are the Bessel functions of order \(\alpha\) of the first and the second kind respectively. Due to orbifold BCs the fermionic zero modes are chiral and they are identified with the SM fermions, while all the higher fermionic KK states are vectorlike in nature with respect to the gauge group. We note that a massless zero mode \(f^{(0)}_{\Psi_L} (y)\) exists only for \((+,+)\) BCs. The profiles for the right-handed modes can be obtained by replacing \(c\) by
\(-c\) in the above formulae. We also note that the left-handed zero mode \(f_{0}^{(0)}(y)\) is flat for \(c = 1/2\), peaked towards the UV brane for \(c > 1/2\) and peaked towards the IR brane for \(c < 1/2\). The fermionic profiles satisfy the following orthonormality conditions,
\[
\frac{1}{\pi R} \int_0^{\pi R} dy \ e^{ky} f^{(m)}(y) f^{(n)}(y) = \delta_{mn},
\]  
(2.28)
from which one can determine the normalization, \(N_n\). The coefficient \(b_\alpha(m_n)\) and KK mass \(m_n\) are determined through the BCs on the branes. The vector-like mass \(m_n\) is set by the compactification scale \(M_{KK}\), and is given as the solution of Eqs. (2.29) and (2.30) below.

- For fermions obeying \((-,+)\) BCs, i.e. \(f^{(n)}(y)|_{y=0} = 0\) and \((\partial_y + ck)f^{(n)}(y)|_{y=\pi R} = 0\), we obtain
\[
b_\alpha(m_n) = - \frac{J_\alpha \left(\frac{m_n}{k}\right)}{Y_\alpha \left(\frac{m_n}{k}\right)} = - \left(\frac{c + \frac{1}{2}}{2}\right) J_\alpha \left(\frac{m_n}{k}\right) + J'_\alpha \left(\frac{m_n}{k}\right) \left(\frac{m_n}{k}\right) + \left(\frac{m_n}{k}\right) Y'_\alpha \left(\frac{m_n}{k}\right) \left(\frac{m_n}{k}\right) = b_\alpha(m_n e^{\pi kR})
\]  
(2.29)
This condition can be solved numerically for \(m_n\) and \(b_\alpha(m_n)\). The first fermion KK mass \(m_1\) with \((-,+)\) BC as functions of the bulk mass parameter \(c\) for \(M_{KK} = 3\) and 5 TeV is shown in Fig. 2.2(a).

- For fermions obeying \((+,-)\) BCs, i.e. \((\partial_y + ck)f^{(n)}(y)|_{y=0,\pi R} = 0\), we obtain
\[
b_\alpha(m_n) = - \frac{J_\alpha \left(\frac{m_n}{k}\right)}{Y_\alpha \left(\frac{m_n}{k}\right)} = - \left(\frac{c + \frac{1}{2}}{2}\right) J_\alpha \left(\frac{m_n}{k}\right) + J'_\alpha \left(\frac{m_n}{k}\right) \left(\frac{m_n}{k}\right) = b_\alpha(m_n e^{\pi kR})
\]  
(2.30)
This condition can be solved numerically for \(m_n\) and \(b_\alpha(m_n)\). The first fermion KK mass \(m_1\) with \((+,-)\) BCs as functions of the bulk mass parameter \(c\) for \(M_{KK} = 3\) and 5 TeV is shown in Fig. 2.2(b).

In Fig. 2.2(a) we see that the \(m_1\) for \((-,+)\) BCs can be significantly smaller in some \(c\)-parameter range and the LHC signatures of \((-,+)\) fermions might be very promising. Therefore, in this thesis our main aim is to study the LHC signatures of \((-,+)\) fermions.
How the warped geometry solves the fermion mass hierarchy of the SM can be understood from the exponential localization of fermion zero modes along the extra dimension (see Eq. 2.26). Different fermions have different localization along the 5D bulk depending on their bulk mass parameters $c$. The overlap of the fermion shape functions with the Higgs on the IR brane depends exponentially on $c$-parameters. By choosing all the $c$-parameters to be $\mathcal{O}(1)$ numbers but slightly different from each other, the 4D effective hierarchical Yukawa couplings can be generated without introducing hierarchy in $c$-parameters. In this manner the warped geometry provides a nice explanation of the fermion mass hierarchy problem.

2.2 Custodially Protected RS Model

In the previous section we reviewed the warped-space extra dimensional model that has been proposed by Randall-Sundrum (RS) as a solution to the gauge hierarchy problem of the SM [93]. The RS model is a theory defined on a slice of AdS$_5$ space. Due to the AdS/CFT correspondence [94] certain strongly coupled 4D theories can be interpreted as weakly coupled 5D theories in the AdS$_5$ background. Therefore, it is possible to calculate some observables perturbatively in the framework of the RS model. The fermion mass hierarchy of the SM can also be addressed by allowing SM fields to propagate in the bulk satisfying electroweak precision test constraints with accessible $M_{KK}$ scale at the LHC [98, 99] and without badly spoiling flavor changing neutral current constraints [103, 104]. In particular the most stringent constraints come from the measurements of the Peskin-Takeuchi parameters [105] and the $Zb_Lb_L$ coupling. The Peskin-Takeuchi parameters are a set of three measurable quantities, called $S$, $T$, and $U$, which are very sensitive to the new physics contributions to the electroweak radiative corrections. They are parametrized
as

\[ S = \frac{4s_w^2c_w^2}{\alpha(M_Z)} \left[ \Pi'_{ZZ}(0) - \frac{c_w^2}{s_wc_w} \Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0) \right] \]  
\[ T = \frac{1}{\alpha(M_Z)} \left[ \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} \right] \]  
\[ U = \frac{4s_w^2}{\alpha(M_Z)} \left[ \Pi'_{WW}(0) - \frac{c_w^2}{s_wc_w} \Pi'_{ZZ}(0) - 2s_wc_w \Pi'_{Z\gamma}(0) - s_w^2 \Pi'_{\gamma\gamma}(0) \right] \]

where \( \alpha(M_Z) \) is the fine structure constant measured at the scale \( M_Z \). Here \( \Pi_{VV} \) denotes the vacuum polarization functions of the gauge boson \( V \) measured at the scale \( q^2 = 0 \) and the \( \Pi'_{VV} \) is the derivative of \( \Pi_{VV} \) with respect to \( q^2 \). The \( s_w \) and \( c_w \) are the sine and cosine of the weak mixing angle respectively. The Peskin-Takeuchi parameters are defined in such a way that they are all equal to zero at a reference point in the Standard Model, with a particular value chosen for the Higgs boson mass. Usually \( U \) is small in typical BSM theories. Assuming \( U = 0 \) and \( M_h = 125 \) GeV, a combined analysis of electroweak precision measurements leads to the constraint, \( S = 0.04 \pm 0.09 \) [88]. The \( T \) parameter is a measure of the violation of the custodial symmetry in the electroweak sector and very sensitive to the new physics effects (\( S \) parameter is also sensitive). The LEP data put very stringent bound on the \( T \) parameter, \( T = 0.07 \pm 0.08 \) [88]. Another EWPT observable which is very precisely measured is the \( Z\bar{b}_Lb_L \) coupling and in the SM it reads

\[ \kappa_{Zb_Lb_L} = g_Z \left[ -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right] . \]

Experimentally the bound on the shift of the \( Z\bar{b}_Lb_L \) coupling from the SM value, \( \Delta \kappa_{Zb_Lb_L} \) with 95\% C.L. is given by [88]

\[ -2 \times 10^{-3} \lesssim \Delta \kappa_{Zb_Lb_L} \lesssim 6 \times 10^{-3} . \]  

In a simple extension of the RS model with SM fields in the bulk and the bulk gauge group being the SM gauge group \( SU(2)_L \otimes U(1)_Y \), the mass of the lowest KK excitation
of the gauge boson, $M_{KK}$ is constrained by electroweak precision tests (in particular
the $T$ parameter) to be above 8 TeV [106]. Therefore, this simple extension will likely
remain beyond the reach of the LHC. However, as shown in Ref. [106] this situation
can be significantly improved by extending the bulk gauge group to $G = SU(2)_L \otimes
SU(2)_R \otimes U(1)_X$. The custodial symmetry in the Higgs sector offers an $SU(2)_R$ symmetry
in the bulk [106] and protects the $T$-parameter from receiving large tree level corrections.
In this scenario the limit relaxes to $M_{KK} \gtrsim 2 - 3$ TeV which could be discovered at
the LHC. However, this scenario is still strongly constrained due to a large shift to
the $Zb_L b_L$ coupling. As shown in Ref. [107] the correction to the $Zb_L b_L$ coupling can
be kept under control by embedding the third generation quarks ($t_L$ and $b_L$) into the
bidoublet representation (i.e. $(2, 2)/3$) of $G$ together with an extra discrete $\mathbb{Z}_2$ (which
implies $SU(2)_L \leftrightarrow SU(2)_R$) symmetry of the theory. The $\mathbb{Z}_2$ symmetry (we call it \(P_{LR}\)
symmetry) between two $SU(2)$ groups in $G$ implies that the Lagrangian is invariant under
the exchange of $SU(2)_L$ and $SU(2)_R$. As a consequence, in a left-right symmetric theory
the $SU(2)_L$ and $SU(2)_R$ gauge couplings have to be equal and fermions are embedded in
the left-right symmetric representations of the gauge group.

Next we give the particle content of the warped model with bulk gauge group $G$ and
work out various Lagrangian terms in the mass basis. For the quark content of the theory
we present various quark representations in models both without and with the custodial
protection of the $Zb_L b_L$ coupling.

2.2.1 Gauge sector

The bulk gauge group of the custodially protected RS model is larger than the SM
gauge group and therefore, the particle content in this model is larger than the SM
particle content. Here, we list all the gauge bosons associated with the bulk gauge group
$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_X$, and the corresponding gauge couplings.

- $SU(3)_c$ gauge bosons are $G^A_{\mu}$ ($A = 1, \cdots, 8$) and the gauge coupling is $g_S$. 

• $SU(2)_L$ gauge bosons are $W^1_{L\mu}$, $W^2_{L\mu}$, $W^3_{L\mu}$ and the gauge coupling is $g_L$.

• $SU(2)_R$ gauge bosons are $W^1_{R\mu}$, $W^2_{R\mu}$, $W^3_{R\mu}$ and the gauge coupling is $g_R$.

• $U(1)_X$ gauge bosons is $X_\mu$ and the gauge coupling is $g_X$.

To obtain the correct low energy spectrum, the bulk gauge group of the custodially protected RS model can be broken by an appropriate choice of BCs on the UV brane to the SM gauge group, and the SM gauge group is finally broken to $U(1)_{EM}$ by a nonzero Higgs VEV as in the SM [106]. Since $SU(3)_c$ is not broken, we do not always show $SU(3)_c$ explicitly. In short, the breaking pattern can be shown as

$$ SU(2)_L \otimes SU(2)_R \otimes U(1)_X \xrightarrow{\text{UV brane}} SU(2)_L \otimes U(1)_Y \xrightarrow{(H)} U(1)_{EM} \quad (2.36) $$

The symmetry breaking is achieved by the following assignment of BCs

$$ W^a_{L\mu}(+, +), \quad B_\mu(+, +), \quad W^b_{R\mu}(-, +), \quad Z_{X\mu}(-, +), \quad (2.37) $$

where $a = 1, 2, 3$ and $b = 1, 2$. The field $Z_X$ and $B$ are the linear combinations of $W^3_R$ and $X$ as follows

$$ Z_{X\mu} = \cos \phi W^3_{R\mu} - \sin \phi X_\mu, \quad B_\mu = \sin \phi W^3_{R\mu} + \cos \phi X_\mu \quad (2.38) $$

where $\tan \phi = g_X/g_R$. At this point, $W^a_L$ and $B$ have massless zero modes before EWSB in their KK decompositions. We define $W^{\pm}_{L,R}$, $Z$ and $A$ as follows

$$ W^\pm_{L\mu} = \frac{1}{\sqrt{2}} \left( W^1_{L\mu} \mp W^2_{L\mu} \right), \quad W^\pm_{R\mu} = \frac{1}{\sqrt{2}} \left( W^1_{R\mu} \mp W^2_{R\mu} \right) \quad (2.39) $$

$$ Z_\mu = \cos \psi W^3_{L\mu} - \sin \psi B_\mu, \quad A_\mu = \sin \psi W^3_{L\mu} + \cos \psi B_\mu \quad (2.40) $$

where $\tan \psi = g_X/\sqrt{g^2_R + g^2_X}$. It is important to note that the angle $\psi$ is analogues to the weak mixing angle $\theta_W$ in the SM. Because of mixing between the gauge boson zero
modes and heavy KK modes, $\psi$ and $\theta_W$ are slightly different from each other.

### 2.2.2 Model without $Z\bar{b}_L b_L$ protection

To discuss fermion content of the theory, we present various quark representations which are phenomenologically interesting. We begin our analysis following Ref. [106] with the simplest quark representations (although the $Z\bar{b}_L b_L$ coupling is not protected in this case) where the third generation quarks transform under $G$ as

$$Q_L \equiv (2,1)_{\frac{1}{6}} = \begin{pmatrix} t^{(++)}_L \\ b^{(++)}_L \end{pmatrix}; \quad Q_{t_R} \equiv (1,2)_{\frac{1}{6}} = \begin{pmatrix} t^{(++)}_R \\ b^{(-+)}_L \end{pmatrix}; \quad Q_{b_R} \equiv (1,2)_{\frac{1}{6}} = \begin{pmatrix} t^{(-+)}_R \\ b^{(++)}_R \end{pmatrix}. \tag{2.41}$$

Here we consider only the third generation quarks because the couplings of the third generation quarks with the Higgs are significantly bigger than the first two generations. Since they are localized closer to the Higgs profile (i.e. closer to the IR brane) as compared to the first two generations. Thus, the mixing of third generation quarks with higher KK modes through the off-diagonal mass terms generated after EWSB can be important [108]. We use the notation for the field representations as $(\mathbf{l}, \mathbf{r})_X$ where $\mathbf{l}$ and $\mathbf{r}$ denote $SU(2)_L$ and $SU(2)_R$ representations respectively, and $X$ denotes the $U(1)_X$ charge. The signs in the bracket associated with each field indicate the BCs for each field on the UV and IR brane respectively. The “+” denotes a Neumann BC and “−” stands for a Dirichlet BC. The fields with $(+, +)$ BCs on the extra dimensional interval $[0, \pi R]$ have zero modes and these zero modes are identified with the SM fields, while the new fields $t'$ and $b'$ (the “custodians”) have no zero modes by applying $(-, +)$ BCs. All the zero-modes (i.e. SM fields) are chiral, while all the higher KK excitations are vectorlike with respect to the SM gauge group.
The Higgs field which is responsible for the EWSB transforms as bidoublet under $\mathcal{G}$,

$$
\Sigma \equiv (2,2)_0 = \begin{pmatrix}
\phi_0^0 & \phi^+
\end{pmatrix},
$$

where $\phi_0$ denotes the physical Higgs boson whose VEV eventually leads to EWSB, $\phi^\pm$ and $\phi_0^*$ denote the Goldstone bosons. In the unitary gauge, these Goldstone bosons can be gauged away. These degrees of freedom appear as the longitudinal polarizations of the massive gauge bosons. The electroweak symmetry is broken by a nonzero VEV $\langle \Sigma \rangle = \text{diag}(v,v)/\sqrt{2}$ (where $v$ is the Higgs boson VEV, $v \approx 246 \text{ GeV}$). Throughout this thesis we work in the unitary gauge in which the Goldstone bosons are the longitudinal polarizations of the gauge bosons.

To reproduce the large top mass requires the localization of either the $Q_L$ or the $Q_{t_R}$ near the IR brane. If we take the $Q_L$ to be too close to the IR brane, the $b_L$-couplings (since $b_L$ is a part of $Q_L$) will receive large corrections [109]. This leaves the possibility of $Q_{t_R}$ is to be localized near the IR brane. Thus, the $b'$ which belongs to the $Q_{t_R}$ is most likely the lightest KK excitation and the $b \leftrightarrow b'$ mixing is large due to the large off-diagonal term in the mixing matrix. Therefore, the $b'$ promises to have the best observability at the LHC, and we will only study its phenomenology for the model without $Z\bar{b}_Lb_L$ protection.

**Lagrangian**

We want to write down the 4D effective couplings of quarks shown in Eq. (2.41) with the SM gauge bosons and Higgs. One can write down an equivalent 4D theory starting from a 5D theory by using KK reduction in which one performs a KK expansion of the fields and then integrate over the extra dimension. The EWSB makes some zero modes massive like in the SM, and mixes various KK modes. After diagonalization of the various mass matrices the lightest eigenmodes of each mass matrix are identified with the SM states.

The kinetic energy (K.E.) terms for the quark multiplets defined in Eq. (2.41) are
given by
\[
\mathcal{L}_{KE} \supset \bar{Q}_L i \gamma^\mu D_\mu Q_L + \bar{Q}_{t R} i \gamma^\mu D_\mu Q_{t R} + \bar{Q}_{b R} i \gamma^\mu D_\mu Q_{b R} ,
\]  

(2.43)

where $D_\mu$ is the covariant derivative for the SM gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ written in the mass basis of the gauge bosons after EWSB as follows

\[
D_\mu = \partial_\mu - ig_S T^a G_\mu^a - ie Q A_\mu - i \frac{g_W}{\sqrt{2}} \left( T^+ W_\mu^+ + T^- W_\mu^- \right) - ig_Z \left( T^3 - s^2_W Q \right) Z_\mu .
\]  

(2.44)

The K.E. term of the Lagrangian $\mathcal{L}_{KE}$ expressed in the mass basis of gauge boson is

\[
\mathcal{L}_{KE} \supset \sum_q \left( e Q_q \bar{q} \gamma^\mu q A_\mu + g_5 \bar{q} \gamma^\mu T^a q G_\mu^a \right) + \frac{g_W}{\sqrt{2}} \left[ \bar{t}_L \gamma^\mu t_L \right. + \left. + \bar{t}_R \gamma^\mu t_R + \left( -\frac{1}{2} + \frac{1}{3} s^2_W \right) \bar{b}_L \gamma^\mu b_L \right. \\
+ \left. \left( \frac{1}{3} s^2_W \right) \bar{b}_R \gamma^\mu b_R + \left( -\frac{1}{2} + \frac{1}{3} s^2_W \right) \bar{y} \gamma^\mu y + \left( \frac{1}{2} - \frac{2}{3} s^2_W \right) \bar{l} \gamma^\mu l \right] Z_\mu .
\]  

(2.45)

After KK reduction, each term in the Lagrangian is associated with an overlap integral which is not shown explicitly above and can be written in a general form

\[
\mathcal{I}_{q q_v V} = \frac{1}{\pi R} \int_0^{\pi R} dy \ e^{k_y} f_{q_1}(y) f_{q_2}(y) f_V(y) ,
\]  

(2.46)

where $q, q_{1,2} = \{ t_{L,R}, b_{L,R}, l', b' \}$ and $V$ is the vector bosons, either massless $V_0 = \{ A, G \}$ or massive $V_M = \{ W^\pm, Z \}$. Photon and gluons will remain massless after EWSB since $U(1)_{EM}$ and $SU(3)_c$ are unbroken. Therefore, the zero mode profiles of $V_0$, $f_{V_0}^{(0)}(y)$ will remain flat (i.e. $f_{V_0}^{(0)}(y) = 1$) after EWSB along the extra dimension. Thus, the overlap integrals $\mathcal{I}_{q q_v V_0}$ become unity using the orthonormality condition of the normalized fermion wavefunctions. On the other hand, $\mathcal{I}_{q q_v V_M}$ differ from unity since the zero modes $V_M^{(0)}$ of the EW gauge bosons mix with their higher KK modes due to EWSB. But the mixing changes the overlap integrals $\mathcal{I}_{q q_v V_M}$ from unity only by a few percent. In our analysis we neglect this small mixing effect and take all the $\mathcal{I}_{q q_v V_M} = 1$ for simplicity. Later we give more quantitative comparison of mixing effects in quark sector and in gauge sector.
The couplings $q_1q_2V_M$ can be modified due to the mixing in the quark sector or mixing in the EW gauge boson sector. For LHC phenomenology, it is sufficient to consider only the dominant mixing effects i.e. mixing between zero mode and first KK excitations. In this thesis, we keep mixings between zero-mode and first KK modes in the quark sector as these can be bigger owing to the smaller mass of the custodians with ($-, +$) BCs. Whereas, we ignore mixing effects in the gauge sector as these effects are only a few percent compared to the mixing effects in the quark sector.

To compare the mixing effects in the quark sector with the EW gauge boson sector more quantitatively, we, for example, consider the $b' \rightarrow tW$ decay. The $b/tW$ vertex can be modified due to $b \leftrightarrow b'$ mixing as well as mixing in the $W$ sector. The contribution to the $b' \rightarrow tW$ decay rate due to $b \leftrightarrow b'$ mixing is proportional to the $(M_{bb'}/M_b)^2$ (in the limit of large $M_b$), while due to $W_L^{(0)} \leftrightarrow W_L^{(1)}$ mixing it is proportional to $\left( \sqrt{k\pi R} (g_R/g_L) M_{W_L}^2 / M_{W^*_R}^2 \right)^2$ [110]. An additional $\sqrt{k\pi R}$ appears in the gauge sector mixing, due to an IR-brane-peaked Higgs. The gauge KK boson mass $M_{W_R^*}$ is constrained to be about 2 TeV by EWPT (see Ref. [111] and references therein). Thus, the contribution due to gauge KK mixing is about 1.3% of the quark KK mixing contribution for $M_{b'} = M_{W_R^*} = 2$ TeV (we take $k\pi R \sim 35$ as discussed after Eq. (2.16) and assume $g_L = g_R$), and even smaller for lighter $b'$ masses. Therefore, the mixing effects in the gauge sector have little impact on the phenomenology we discuss in this thesis and we do not consider any gauge KK mixing anymore.

The top and the bottom quarks Yukawa couplings are obtained from the invariant combination $(\mathbf{2}, \mathbf{1})_{1/6} (\mathbf{2}, \mathbf{2})_{0} (\mathbf{1}, \mathbf{2})_{1/6}$. The 5D Yukawa interactions are given by [62]

\[
\mathcal{L}_Y \supset -\tilde{\lambda}_t \tilde{Q}_L \Sigma t_R - \tilde{\lambda}_b \tilde{Q}_L \Sigma b_R + \text{H.c.}
\]

\[
\mathcal{L}_Y \supset -\tilde{\lambda}_t \left( \bar{t}_L t_R \phi_0^+ + \bar{t}_L b'_R \phi^+ - \bar{b}_L t_R \phi^- + \bar{b}_L b'_R \phi^0 \right)
- \tilde{\lambda}_b \left( \bar{t}_L t'_R \phi_0^+ + \bar{t}_L b_R \phi^+ - \bar{b}_L t'_R \phi^- + \bar{b}_L b_R \phi^0 \right) + \text{H.c. ,}
\]

(2.47)

where $\tilde{\lambda}_{t,b}$ are dimensionless 5D Yukawa coupling constants which we take to be $\mathcal{O}(1)$. 

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One can write down an equivalent 4D theory by performing a KK expansion of the fields and then integrating over the extra dimension. After EWSB, the off-diagonal terms in the bottom mass matrix resulting from Eq. (2.47) lead to the mixing of the fields \((b^{(0)}, b^{(n)}, b^{(n)}_L, b^{(n)}_R)\) where \(n \geq 1\) denotes the \(n\)-th KK states. To simplify our analysis, we consider only the dominant mixing (i.e. \(b^{(0)} \leftrightarrow b^{(1)}\) mixing) and ignore mixing to all heavier KK states. We call \(b^{(0)}\) and \(b^{(1)}\) as \(\bar{b}\) and \(\bar{b}'\) respectively and write the bottom mass matrix in the \((\bar{b}, \bar{b}')\) basis as follows:

\[
\mathcal{L} \supset - \left( \begin{array}{c} \bar{b}_L \\ \bar{b}'_L \\
\end{array} \right)^* \left( \begin{array}{cc} M_b & \bar{M}_{b'} \\
0 & M_{b'} \\
\end{array} \right) \left( \begin{array}{c} \bar{b}_R \\ \bar{b}'_R \\
\end{array} \right) + \text{H.c. },
\] (2.48)

where \(M_b = \tilde{\lambda}_b \frac{e^{kR}}{\sqrt{2} \kappa R} f_{Q_L}^{(0)}(\pi R) f_{Q_{b_R}}^{(1)}(\pi R)\), the \(M_{b'}\) is the vectorlike mass of the \(b'\), and the off-diagonal mass term \(M_{b'b'} = \tilde{\lambda}_b \frac{e^{kR}}{\sqrt{2} \kappa R} f_{Q_L}^{(0)}(\pi R) f_{Q_{b_R}}^{(1)}(\pi R)\) is induced after EWSB, and \(f_{\psi}\)’s are the fermion wavefunctions which depend on the fermion bulk mass parameters \(c_{\psi}\). The vectorlike mass \(M_{b'}\) is set by the compactification scale (see sec. 2.1.2 for bulk fermions). In Fig. 2.2, we have shown vectorlike fermion masses (first KK excitation) for two different BCs.

The mass matrix in Eq. (2.48) is diagonalized by a bi-orthogonal rotation and we denote the sine (cosine) of the mixing angles by \(s_{L,R} (c_{L,R})\).

\[
\left( \begin{array}{c} b_L \\ b'_L \\
\end{array} \right) = \left( \begin{array}{cc} \cos \theta_L & -\sin \theta_L \\
\sin \theta_L & \cos \theta_L \\
\end{array} \right) \left( \begin{array}{c} b_{1L} \\ b_{2L} \\
\end{array} \right) ; \quad \left( \begin{array}{c} b_R \\ b'_R \\
\end{array} \right) = \left( \begin{array}{cc} \cos \theta_R & -\sin \theta_R \\
\sin \theta_R & \cos \theta_R \\
\end{array} \right) \left( \begin{array}{c} b_{1R} \\ b_{2R} \\
\end{array} \right),
\] (2.49)

where \(\{b_1, b_2\}\) are the mass eigenstates. The mixing angles are given by

\[
tan(2\theta_L) = -\frac{2M_{b'}M_{b'b'}}{(M_{b'}^2 - M_b^2 - M_{b'b'}^2)} ; \quad tan(2\theta_R) = -\frac{2M_bM_{b'b'}}{(M_{b'}^2 - M_b^2 + M_{b'b'}^2)} .
\] (2.50)

The mass eigenstates are given by

\[
M_{b_{1,2}}^2 = \frac{1}{2} M_{b'}^2 \left[ 1 + x_b^2 + x_{b'b'}^2 \right] \mp \sqrt{\left( 1 + x_b^2 + x_{b'b'}^2 \right)^2 - 4x_b^2},
\] (2.51)
where $x_b = M_b/M_\nu$ and $x_{bb'} = M_{bb'}/M_\nu$. In the limit of large $M_\nu$, i.e., $x_b, x_{bb'} \ll 1$, the mixing angles behave as $\sin \theta_L \sim x_{bb'}$, $\sin \theta_R \sim x_b x_{bb'}$ and the mass eigenvalues become

\[ M_{b_1} = M_b \left[ 1 + \mathcal{O} \left( x_b^4, x_{bb'}^4 \right) \right] ; \quad M_{b_2} = M_\nu \left[ 1 + \frac{1}{2} x_{bb'}^2 + \mathcal{O} \left( x_b^4, x_{bb'}^4 \right) \right]. \tag{2.52} \]

The Lagrangian in the mass basis consists of the following interactions [63],

- **Interactions with photon ($A$) and gluon ($G$):**

\[ \mathcal{L}_{A+G} \supset -\frac{e}{3} \left[ \bar{b}_1 \gamma^\mu b_1 + \bar{b}_2 \gamma^\mu b_2 \right] A_\mu + g_S \left[ \bar{b}_1 \gamma^\mu T^\alpha b_1 + \bar{b}_2 \gamma^\mu T^\alpha b_2 \right] G_\mu^\alpha \tag{2.53} \]

- **Interactions with $W$-boson (charged current):**

\[ \mathcal{L}_W \supset \frac{g_W}{\sqrt{2}} \left[ c_L \bar{b}_L \gamma^\mu b_L - s_L \bar{b}_L \gamma^\mu b_2 L \right] W_\mu^+ + \text{H.c.} \tag{2.54} \]

- **Interactions with $Z$-boson (neutral current):**

\[ \mathcal{L}_Z \supset g_Z \left[ \left( \frac{1}{2} c_L^2 + \frac{1}{3} s_L^2 \right) \bar{b}_1 \gamma^\mu b_1 L + \left( \frac{1}{3} c_W^2 \right) \bar{b}_{1R} \gamma^\mu b_{1R} \\
+ \left( \frac{1}{2} s_L^2 + \frac{1}{3} s_W^2 \right) \bar{b}_2 \gamma^\mu b_2 L + \left( \frac{1}{3} c_W^2 \right) \bar{b}_{2R} \gamma^\mu b_{2R} \\
+ \left\{ \left( \frac{1}{2} c_L s_L \right) \bar{b}_1 \gamma^\mu b_2 L + \text{H.c.} \right\} \right] Z_\mu. \tag{2.55} \]

- **Interactions with Higgs boson:**

\[ \mathcal{L}_h \supset -\frac{1}{v} \left[ (M_b \ c_{L\nu} - M_{bb'} \ c_{LsR}) \bar{b}_1 b_{1R} + (M_b \ s_{LsR} - M_{bb'} \ s_{L\nu}) \bar{b}_2 b_{2R} \\
+ (M_b \ c_{LsR} - M_{bb'} \ c_{L\nu}) \bar{b}_1 b_{2R} + (M_b \ s_{L\nu} - M_{bb'} \ s_{LsR}) \bar{b}_2 b_{1R} \right] h + \text{H.c.} \tag{2.56} \]
As mentioned earlier, the $Zb_L b_L$ coupling is very precisely measured. The shift in the $Zb_L b_L$ coupling can be defined as

$$\Delta \kappa_{Zb_L b_L} = \kappa_{BSM} - \kappa_{SM} = \frac{g_Z}{2} (1 - \epsilon_L^2) = \frac{g_Z}{2} \kappa_L^2.$$  \hfill (2.57)

The experimental constraints shown in Eq. (2.35) require that this shift be less than about 1%, roughly implying $s_L \lesssim 0.1$, i.e. equivalently $M_{b'} \gtrsim 10 M_{bb'} \approx 3$ TeV. We have discussed the model without $Zb_L b_L$ protection for simplicity, but in the following subsections we discuss models with $Zb_L b_L$ protection which will relax this constraints.

We notice off-diagonal couplings $b_2 b_1 Z$ and $b_2 b_1 h$ are present in the mass basis. The off-diagonal $Z$ coupling is due to the fact that the $b'$ has different $T^3_L$ quantum number compared to $b_L$, and therefore going to the mass basis leads to an off-diagonal coupling. The off-diagonal Higgs coupling is because of the presence of a $b'$ vectorlike mass that is independent of Higgs VEV, due to which diagonalizing the mass matrix does not diagonalize the Higgs interactions.

### 2.2.3 Models with $Zb_L b_L$ protection

In this section we consider a class of models where $Zb_L b_L$ coupling is protected using the custodial symmetry as detailed in [107]. The $Zb_L b_L$ coupling can receive corrections since the $SU(2)_L$ charge ($T^3_L$) of $b_L$ can be modified after EWSB. The diagonal subgroup $SU(2)_V$ of $SU(2)_L \otimes SU(2)_R$ remains unbroken even after EWSB ensuring that $SU(2)_V$ charge ($T^3_V$) does not get any correction, i.e. $\delta T^3_V = \delta T^3_L + \delta T^3_R = 0$. The $P_{LR}$ symmetry ensures that $T^3_L = T^3_R$ or equivalently $\delta T^3_L = \delta T^3_R$. This immediately yields $\delta T^3_L = 0$. One way to achieve $T^3_L = T^3_R$ for $b_L$ is to embed the third generation left handed quarks ($t_L$ and $b_L$) into the bidoublet representation (i.e. $(2, 2)_{2/3}$) of the bulk gauge group $G = SU(2)_L \otimes SU(2)_R \otimes U(1)_X$ and the theory should be made invariant under a discrete $\mathbb{Z}_2$ $(SU(2)_L \leftrightarrow SU(2)_R)$ symmetry. The component fields of the bidoublet representation
are

\[ Q_L \equiv (2, 2)_2 = \begin{pmatrix} t_L^{(++)} & \chi^{(-+)} \\ b_L^{(++)} & t'^{(-+)} \end{pmatrix}. \]  

(2.58)

In the bidoublet representation above, the \( SU(2)_L \) acts vertically and \( SU(2)_R \) acts horizontally. Note that to complete the bidoublet representation, two new quarks namely \( \chi \) (charge 5/3) and \( t' \) (charge 2/3) have been introduced. The K.E. term for \( Q_L \) is

\[ \mathcal{L}_{KE} \supset \text{Tr} \left[ \bar{Q}_L \gamma^\mu D_\mu Q_L \right], \]  

(2.59)

where \( D_\mu \) is the covariant derivative defined in Eq. (2.44). The Higgs field also transforms as a bidoublet representation of the gauge group \( \mathcal{G} \) as shown in Eq. (2.42).

It is possible to write down an invariant top quark Yukawa coupling with either the \( t_R = (1, 1)_{2/3} \) or with \( t_R \subset (1, 3)_{2/3} \oplus (3, 1)_{2/3} \) [107]. We will elaborate on both these possibilities in the following subsections. The invariant bottom quark Yukawa coupling can be written in many ways by embedding \( b_R \) in various multiplets of \( \mathcal{G} \) as detailed in [107]. The \( c \)-parameter required for obtaining the correct bottom mass implies that all the \((-+, +)\) partners of \( b_R \) are heavier than 3 TeV. Thus, the mixing effects of these heavier quarks with the lighter modes are much smaller and phenomenologically uninteresting. Therefore, we ignore all \( b_R \) partners in our analysis and show couplings of \( b_R \) wherever they are relevant.

**Model with** \( t_R \subset (1, 1)_{2/3} \)

In this subsection we explore the possibility where \( t_R \) is a singlet under both \( SU(2)_L \) and \( SU(2)_R \), and this can be represented as

\[ Q_{t_R} \equiv (1, 1)_{2} = t^{(++)}_R. \]  

(2.60)
The K.E. term for $Q_{t_R}$ can be written as (K.E. term for $Q_L$ is given in Eq. (2.59))

$$\mathcal{L}_{KE} \supset \hat{Q}_{t_R} i\gamma^\mu D_\mu Q_{t_R}.$$  \hfill (2.61)

Using the invariant operator $(\overline{2}, 2)_{2/3}(\overline{2}, 2)_0(1, 1)_{2/3}$ one can write down the 5D top-quark Yukawa coupling as follows

$$\mathcal{L}_Y \supset \hat{\lambda}_t \text{Tr} \left[ Q_L \Sigma \right] Q_{t_R} + \text{H.c.}$$  \hfill (2.62)

$$\mathcal{L}_Y \supset \hat{\lambda}_t \left( \bar{t}_L t_R \phi^*_0 - \bar{b}_L t_R \phi^* + \bar{t}_R t_R \phi^* + \bar{t}_R t_R \phi_0 \right) + \text{H.c.} ,$$  \hfill (2.63)

where $\hat{\lambda}_t \equiv k\lambda_t$ is the dimensionless 5D Yukawa coupling. The K.E. terms in Eq. (2.59) and (2.61) can be expressed in the mass basis of gauge bosons as

$$\mathcal{L}_{KE} \supset \sum_q \left( eQ_q \bar{q} \gamma^\mu q A_\mu + gS \bar{q} \gamma^\mu T^a q G^a \right) + \frac{g_W}{\sqrt{2}} \left[ (\bar{t}_L \gamma^\mu b_L + \bar{\chi} \gamma^\mu t') W^\mu + \text{H.c.} \right]$$

$$+ g_Z \left[ \left( \frac{1}{2} - \frac{2}{3} \frac{s_W^2}{\sqrt{2}} \right) \bar{t}_L \gamma^\mu t_L + \left( -\frac{2}{3} \frac{s_W^2}{\sqrt{2}} \right) \bar{t}_R \gamma^\mu t_R + \left( -\frac{1}{2} + \frac{1}{3} \frac{s_W^2}{\sqrt{2}} \right) \bar{b}_L \gamma^\mu b_L \right]$$

$$+ \left( \frac{1}{3} \frac{s_W^2}{\sqrt{2}} \right) \bar{b}_R \gamma^\mu b_R + \left( \frac{1}{2} - \frac{5}{3} \frac{s_W^2}{\sqrt{2}} \right) \bar{\chi} \gamma^\mu \chi + \left( -\frac{1}{2} - \frac{2}{3} \frac{s_W^2}{\sqrt{2}} \right) \bar{t}^\gamma \gamma t' \right] Z_\mu .$$  \hfill (2.64)

In the quark sector, the top-mass matrix including zero-mode and the lightest KK mode mixing but neglecting the smaller mixings to heavier KK states is

$$\mathcal{L}_t \supset \left( \begin{array}{c} \bar{t}_L \\ \bar{t}_t \end{array} \right) \left( \begin{array}{cc} M_t & 0 \\ M_{t'} & M_v \end{array} \right) \left( \begin{array}{c} t_L \\ t' \end{array} \right) + \text{H.c.}$$  \hfill (2.65)

where $M_t = \hat{\lambda}_t \sqrt{2} \frac{N_{kR}}{k} f_Q^{(0)}(\pi R) f_Q^{(1)}(\pi R)$, the $M_v$ is the vectorlike mass of $t'$, and $M_{t'} = \hat{\lambda}_t \sqrt{2} \frac{N_{kR}}{k} f_Q^{(0)}(\pi R) f_Q^{(1)}(\pi R)$ is the off-diagonal mass term induced after EWSB. We have not shown mass matrix for the bottom sector as in this model the new heavy charge $-1/3$ vectorlike quarks could only arise as the partners of the $b_R$ and we ignore them since they are very heavy. The above mass matrix is diagonalized by a bi-orthogonal rotation as
follows

\[
\begin{pmatrix}
 t_L \\
 t'_L
\end{pmatrix} = \begin{pmatrix} c_L & -s_L \\ s_L & c_L \end{pmatrix} \begin{pmatrix} t_1_L \\
 t_2_L
\end{pmatrix}, \quad \begin{pmatrix} t_R \\
 t'_R
\end{pmatrix} = \begin{pmatrix} c_R & -s_R \\ s_R & c_R \end{pmatrix} \begin{pmatrix} t_1_R \\
 t_2_R
\end{pmatrix},
\]
(2.66)

where \( \{ t_1, t_2 \} \) are the mass eigenstates (ignoring mixings to higher KK states), with the mixing angles given by

\[
\tan(2\theta_L) = -\frac{2M_tM_{t'}}{M_t^2 - M_{t'}^2 + M_{t''}^2}; \quad \tan(2\theta_R) = -\frac{2M_{t''}M_{t'}}{M_{t''}^2 - M_t^2 - M_{t'}^2}.
\]
(2.67)

The mass eigenvalues \( m_{1,2} \) are given by

\[
M_{t_{1,2}} = \frac{M_{t'}}{2} \left[ (1 + x_t^2 + x_{t''}^2) \mp \sqrt{(1 + x_t^2 + x_{t''}^2)^2 - 4x_t^2} \right],
\]
(2.68)

where \( x_t = M_t/M_t' \) and \( x_{t''} = M_{t''}/M_t' \). In the limit of large \( M_{t'} \), i.e., \( x_t, x_{t''} \ll 1 \), the mixing angles behave as \( \sin\theta_R \sim x_{t''}, \sin\theta_L \sim x_t x_{t''} \) and the mass eigenvalues become

\[
M_{t_1} = M_t \left[ 1 + \mathcal{O}(x_t^4, x_{t''}^4) \right]; \quad M_{t_2} = M_t' \left[ 1 + \frac{1}{2} x_{t''}^2 + \mathcal{O}(x_t^4, x_{t''}^4) \right].
\]
(2.69)

In the mass basis the final interactions we obtain are as below

- **Interactions with photon (A) and gluon (G):**

\[
\mathcal{L}_{A+G} \supset e \left[ \left( \frac{5}{3} \right) \bar{\chi}\gamma^\mu \chi + \left( \frac{2}{3} \right) \bar{t}_1 \gamma^\mu t_1 + \left( \frac{2}{3} \right) \bar{t}_2 \gamma^\mu t_2 + \left( -\frac{1}{3} \right) \bar{b} \gamma^\mu b \right] A_\mu \\
+ g_S \left[ \bar{\chi} \gamma^\mu T^a \chi + \bar{t}_1 \gamma^\mu T^a t_1 + \bar{t}_2 \gamma^\mu T^a t_2 + \bar{b} \gamma^\mu T^a b \right] G_{\mu}^a.
\]
(2.70)

- **Interactions with W-boson (charged current):**

\[
\mathcal{L}_W \supset \frac{g_W}{\sqrt{2}} \left( c_L \bar{t}_1 L \gamma^\mu b_L - s_L \bar{t}_2 L \gamma^\mu b_L + s_L \bar{\chi} L \gamma^\mu t_{1L} + c_L \bar{\chi} L \gamma^\mu t_{2L} \\
+ s_R \bar{\chi} R t_{1R} + c_R \bar{\chi} R t_{2R} \right) W_\mu^+ + \text{H.c.}
\]
(2.71)
• Interactions with $Z$-boson (neutral current):

\[
\mathcal{L}_Z \ni g_Z \left\{ \left[ \frac{1}{2} \cos 2\theta_L - \frac{2}{3} s_W^2 \right] \bar{t}_{1L} \gamma^\mu t_{1L} + \left[ -\frac{1}{2} \cos 2\theta_L - \frac{2}{3} s_W^2 \right] \bar{t}_{2L} \gamma^\mu t_{2L} \\
+ \left[ -\frac{1}{2} s_R^2 - \frac{2}{3} s_W^2 \right] \bar{t}_{1R} \gamma^\mu t_{1R} + \left[ -\frac{1}{2} s_R^2 - \frac{2}{3} s_W^2 \right] \bar{t}_{2R} \gamma^\mu t_{2R} \\
+ \left[ \left( -\frac{1}{2} \sin 2\theta_L \right) \bar{t}_{2L} \gamma^\mu t_{1L} + \left( -\frac{1}{2} s_R c_R \right) \bar{t}_{2R} \gamma^\mu t_{1R} + \text{H.c.} \right] \\
+ \left[ -\frac{1}{2} - s_W^2 \left( -\frac{1}{3} \right) \right] \bar{t}_L \gamma^\mu t_L + \left[ -\frac{1}{2} - s_W^2 \left( \frac{5}{3} \right) \right] \bar{\chi} \gamma^\mu \chi \right\} Z_\mu .
\] (2.72)

• Interactions with Higgs boson:

\[
\mathcal{L}_h \ni -\frac{1}{v} \left\{ (M_t \ c_L c_R + M_{t'} \ s_L c_R) \bar{t}_{1L} t_{1R} + (M_t \ s_L s_R - M_{t'} \ c_L s_R) \bar{t}_{2L} t_{2R} \\
+ (-M_t \ s_L s_R - M_{t'} \ s_L s_R) \bar{t}_{1L} t_{2R} + (-M_t \ s_L c_R + M_{t'} \ c_L c_R) \bar{t}_{2L} t_{1R} \right\} h + \text{H.c.} .
\] (2.73)

We notice off-diagonal couplings in the neutral current and Higgs sectors. This characteristic signature of vectorlike quarks has been discussed just before sec. 2.2.3.

Model with $t_R \subset (1, 3)_{2/3} \oplus (3, 1)_{2/3}$

In this subsection we pursue another option in which the $t_R$ is embedded into a $(1, 3)_{2/3}$ representation of $\mathcal{G}$. As explained in Ref. [107], due to the required $P_{LR}$ invariance to protect the $Z\bar{b}_L b_L$ coupling, a $(3, 1)_{2/3}$ must also be added. Thus, the multiplet containing the $t_R$ is

\[
Q_{tr} \equiv Q_{tr}' \oplus Q_{tr}'' = \begin{pmatrix} 1 & \chi^{(++)} \\ \frac{1}{\sqrt{2}} \bar{t}_{R'}^{(++)} & -\frac{1}{\sqrt{2}} \bar{t}_{R'}^{(++)} \end{pmatrix} \oplus \begin{pmatrix} 1 & \chi^{(-+)} \\ \frac{1}{\sqrt{2}} b_{R'}^{(-+)} & -\frac{1}{\sqrt{2}} b_{R'}^{(-+)} \end{pmatrix},
\] (2.74)

where $Q_{tr}' \equiv (1, 3)_{2/3}$ and $Q_{tr}'' \equiv (3, 1)_{2/3}$. The top Yukawa couplings are obtained from [112]

\[
\mathcal{L}_Y \ni -\sqrt{2} \bar{\lambda}_t' \text{Tr} \left[ Q_{tr}' \Sigma \right] - \sqrt{2} \bar{\lambda}_t'' \text{Tr} \left[ Q_{tr}'' \bar{\Sigma} \right] + \text{H.c.}
\] (2.75)

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where $\tilde{\lambda}'_t, \tilde{\lambda}''_t$ are 5D Yukawa couplings and $P_{LR}$ invariance of the theory requires $\tilde{\lambda}'_t = \tilde{\lambda}''_t$, which we just denote as $\tilde{\lambda}_t$ henceforth. The $P_{LR}$ invariance also implies that the $c$-parameters for $Q'_{tR}$ and $Q''_{tR}$ are equal i.e. $c_{Q'_{tR}} = c_{Q''_{tR}}$. The factor of $\sqrt{2}$ is introduced for proper normalization of the K.E. terms. The EM charge of the $b', b''$ is $-1/3$, the $t''$ is $2/3$ and the $\chi', \chi''$ is $5/3$.

One can write bottom Yukawa couplings that respects the custodial symmetry of the theory. Many possibilities for $b_R$ representations are discussed in Ref. [107]. For example with $Q_L = (2, 2)_{2/3}$, the $b_R$ can be embedded into the representation $Q'_{bR} = (1, 3)_{2/3}$ and the bottom Yukawa coupling is obtained from, $\mathcal{L}_Y \supset -\lambda_b \text{Tr} \left[ Q_L \Sigma Q'_{bR} \right] + \text{H.c.}$. However, this choice breaks the $P_{LR}$ symmetry but the resulting shifts in the $Zb_Rb_R$ coupling are acceptable since the $c_{b_R}$ choice required to get the correct bottom mass makes the new states in the $Q'_{b_R}$ multiplet all very heavy ($> 3$ TeV). Therefore, in our analysis we have ignored the mixing effects and the signatures of these heavy $b_R$ partners.

After EWSB due to $\langle \phi_0 \rangle = v/\sqrt{2}$, with the restrictions due to $P_{LR}$ symmetry mentioned earlier the mass matrices are [112]

- Mass matrix for charge $-1/3$ states ($b$ sector):

$$\mathcal{L}_b \supset - \begin{pmatrix} b_L & \bar{b}'_L & \bar{b}_L \end{pmatrix} \begin{pmatrix} M_b & \sqrt{2} M_{b'\nu} & \sqrt{2} M_{b''\nu} \\ 0 & M_{\nu'} & 0 \\ 0 & 0 & M_{\nu''} \end{pmatrix} \begin{pmatrix} b_R \\ b'_R \\ b''_R \end{pmatrix} + \text{H.c.} \quad (2.76)$$

where due to $P_{LR}$ symmetry we have $M_{\nu'} = M_{\nu''}$ and $M_{b'\nu} = M_{b''\nu}$.

- Mass matrix for charge $2/3$ states ($t$ sector):

$$\mathcal{L}_t \supset - \begin{pmatrix} t_L & \bar{t}'_L & \bar{t}_L \end{pmatrix} \begin{pmatrix} M_t & 0 & M_{t'\nu} \\ -M_{t''} & M_{\nu'} & -M_{t''\nu} \\ 0 & -M_{t'\nu'} & M_{\nu''} \end{pmatrix} \begin{pmatrix} t_R \\ t'_R \\ t''_R \end{pmatrix} + \text{H.c.} \quad (2.77)$$
• Mass matrix for charge 5/3 states (χ sector):

$$
\mathcal{L}_\chi \supset - \begin{pmatrix} \bar{\chi}_L & \bar{\chi}'_L & \bar{\chi}''_L \end{pmatrix} \begin{pmatrix} M_\chi & \sqrt{2}M_{\chi\chi'} & \sqrt{2}M_{\chi\chi''} \\ \sqrt{2}M_{\chi\chi'} & M_{\chi'} & 0 \\ \sqrt{2}M_{\chi\chi''} & 0 & M_{\chi''} \end{pmatrix} \begin{pmatrix} \chi_R \\ \chi'_R \\ \chi''_R \end{pmatrix} + \text{H.c.} \quad (2.78)
$$

where due to $P_{LR}$ symmetry we have $M_{\chi'} = M_{\chi''}$ and $M_{\chi\chi'} = M_{\chi\chi''}$.

In all the three mass matrices, the $M_q$ (except $M_b$ and $M_t$) denotes the vectorlike masses, and the EWSB generated off-diagonal masses $M_{pq}$ which are given by

$$
M_{pq} = \hat{\lambda}_{p,q} \frac{\psi^{k\pi R}}{\sqrt{2} k\pi R} f^{(n)}_{Q_{pL}}(\pi R) f^{(m)}_{Q_{qR}}(\pi R) \quad (2.79)
$$

The chiral masses also arise after EWSB and they are

$$
M_{b,t} = \hat{\lambda}_{b,t} \frac{\psi^{k\pi R}}{\sqrt{2} k\pi R} f^{(0)}_{Q_{bL}}(\pi R) f^{(0)}_{Q_{tR}}(\pi R) \quad (2.80)
$$

In the above expressions $\hat{\lambda}_{b,t} \equiv k\lambda_{b,t}$ is the dimensionless 5D Yukawa couplings.

Next, our aim is to work out couplings in the mass basis. For this, let us define the flavor eigenstates $\psi^\alpha \equiv (\psi ~ \psi' ~ \psi'')^T$ and the mass eigenstates as $\psi^i \equiv (\psi_1 ~ \psi_2 ~ \psi_3)^T$ for each of the $\psi = \{b, t, \chi\}$ sectors (where $\alpha, i = \{1, 2, 3\}$). We perform a bi-orthogonal rotation (we take the masses to be real for simplicity) $\psi^\alpha_L = R^{\alpha i}_{\psi L} \psi^i_L$ and $\psi^\alpha_R = R^{\alpha i}_{\psi R} \psi^i_R$ to diagonalize each of the mass matrices in Eqs. (2.76)-(2.78).

The gluonic and photonic interactions are standard and we do not show them explicitly. We have checked numerically that mixing effects in the gauge sector can give only a few percent correction to the couplings we are interested in. Therefore, we ignore differences in the overlap integrals and take all $I = 1$ while deriving Lagrangian terms. In unitary gauge the interactions we obtain in the mass basis are as below:
• Interactions with $W$ boson (charged current):

$$\mathcal{L}_W \supset \frac{g_W}{\sqrt{2}} \left[ R^{1*}_{iL} R^{1j}_{bL} \bar{l}_{L} \gamma^\mu b_L^j + R^{1*}_{iL} R^{2j}_{iL} \bar{l}_{L} \gamma^\mu t_L^j + R^{1*}_{iL} R^{2j}_{iR} \bar{l}_{R} \gamma^\mu t_R^j \right.$$

$$\left. + \sqrt{2} \left( R^{3*}_{iL} R^{1j}_{bL} \bar{l}_{L} \gamma^\mu b_L^j - R^{3*}_{iL} R^{2j}_{iL} \bar{l}_{L} \gamma^\mu t_L^j + R^{3*}_{iL} R^{2j}_{iR} \bar{l}_{R} \gamma^\mu t_R^j \right) \right] W^+_{L\mu} + \text{H.c.} \ . \quad (2.81)$$

• Interactions with $Z$ boson (neutral current):

$$\mathcal{L}_Z \supset g_Z \left[ R^{\alpha*\alpha}_{\psi L,R} \left( q_{\psi L,R}^{3L} - Q_{\psi}^2 S_W \right) R^{\alpha j}_{\psi L,R} \bar{\psi}_{L,R} \gamma^\mu \psi_{L,R} Z^\mu \right] , \quad (2.82)$$

where $Q_{\psi} = \{-1/3, 2/3, 5/3\}$ are EM charges and the $q_{\psi}^{3L}$ are the $SU(2)_L$ charges of $\psi = \{b, t, \chi\}$ as given below

$$q_{\psi}^{3L} = \{-1/2, 0, -1\} \ , \ q_{\psi}^{3L} = \{1/2, -1/2, 0\} \ , \ q_{\psi}^{3L} = \{1/2, 0, 1\}$$

$$q_{\psi}^{3L} = \{0, 0, -1\} \ , \ q_{\psi}^{3L} = \{0, -1/2, 0\} \ , \ q_{\psi}^{3L} = \{1/2, 0, 1\} \quad (2.83)$$

• Interactions with Higgs boson:

$$\mathcal{L}_h \supset -\frac{1}{v} \left[ M_b \ R^{11*}_{bL} R^{11}_{bR} b_{L} b_{R} + M_t \ R^{11*}_{tL} R^{11}_{tR} t_{L} t_{R} + M^{\psi}_{\psi} \ R^{\alpha*\alpha}_{\psi L} R^{\beta j}_{\psi R} \bar{\psi}_{L} \psi_{R} \right] h + \text{H.c.} \ . \quad (2.84)$$

where $M^{\psi}_{\alpha\beta} (\alpha \neq \beta = \{1, 2, 3\})$ are the off-diagonal mass terms of $\psi$-sector induced after EWSB.

Here too we find numerically the presence of non-zero off-diagonal couplings in the neutral current and Higgs sectors. This characteristic signature of vectorlike quarks has been discussed just before sec. 2.2.3.

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**b-mass matrix diagonalization**

We have shown in the earlier sections some simple analytical derivation of mixing angles and Lagrangian terms for those cases where mass matrices were $2 \times 2$ dimensions. But it is not always possible to give simple analytical results for mass matrices with dimensions $3 \times 3$ or more. That is why, for the case of $t_R \subset (1, 3)_{2/3} \oplus (3, 1)_{2/3}$ model we present the general structure of the interaction terms and use numerical diagonalization for the LHC phenomenology. However, for the $b$-sector we derive some simple analytical results in some limiting cases.

In case of $b$ mass matrix in Eq. (2.76), due to $P_{LR}$ symmetry we have $M_{\nu} = M_{\nu'}$ (= $M$ say) and $M_{bb'} = M_{bb'}$ (= $m$ say). Taking $M_b = 0$ in the $b$ mass matrix since $M_b \ll M$ and defining $r = m/M$, we find two orthogonal rotation matrices in the following form [112]

\[
R_L = \frac{1}{\sqrt{1 + 2r^2}} \begin{pmatrix} -1 & 0 & \sqrt{2r} \\ r & -\frac{\sqrt{1 + 2r^2}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ r & \frac{\sqrt{1 + 2r^2}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}; \quad R_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}
\]  

(2.85)

with the mass eigenvalues $0, M, M\sqrt{1 + 2r^2}$. The $b_1$ is identified as the SM $b$-quark, and the zero eigenvalue will be lifted when non-zero $M_b$ is included. In unitary gauge the interaction terms in the mass basis are (we will not show charged current interactions since they involve diagonalization of $t$ and $\chi$ sectors)

- **Interactions with $Z$ boson (neutral current):**

\[
\mathcal{L}_Z \supset g_Z \left\{ \left( -\frac{1}{2} - s_W^2 Q_b \right) \bar{b}_{1L} \gamma^\mu b_{1L} + \left( -s_W^2 Q_b \right) \bar{b}_{1R} \gamma^\mu b_{1R} + \left( -\frac{1}{2} - s_W^2 Q_b \right) \bar{b}_{2L} \gamma^\mu b_{2L} + \bar{b}_{2R} \gamma^\mu b_{2R} + \bar{b}_{3L} \gamma^\mu b_{3L} + \bar{b}_{3R} \gamma^\mu b_{3R} \right) + \left[ \left( \frac{-r}{\sqrt{2} + 4r^2} \right) \bar{b}_{1L} \gamma^\mu b_{2L} + \left( \frac{1}{\sqrt{4 + 8r^2}} \right) \bar{b}_{2L} \gamma^\mu b_{3L} + \bar{b}_{2R} \left( -\frac{1}{2} \right) b_{3R} + H.c. \right] \right\} Z_\mu
\]

(2.86)

where $Q_b = -1/3$. We have taken all $\mathcal{I}_{\psi \psi V} = 1$ as earlier, ignoring corrections to this due to EWSB $(0) - (1)$ gauge boson mixing which are at most a few percent.
Note that the $b_1b_1Z$ interactions come out standard due to the custodial protection.

- Interactions with Higgs boson:

$$\mathcal{L}_h \supset \frac{m}{v} \left[- \left( \frac{2\sqrt{2}r}{\sqrt{1+2r^2}} \right) \bar{b}_{3L}b_{3R} + \left( \frac{2}{\sqrt{1+2r^2}} \right) \bar{b}_{1L}b_{3R} \right] h + \text{H.c.} . \quad (2.87)$$

The Higgs interactions are got by replacing $v \rightarrow v(1 + h/v)$.

Interestingly we observe that in Eqs. (2.86) and (2.87), some possible interaction terms (like $b_1b_3Z$, $b_1b_2h$ etc.) are not present. This is because, due to the $P_{LR}$ symmetry of the theory the $b$-mass matrix has a special structure and some couplings will become zero after mixing. In the next chapter we will show various parameters and couplings for the different warped models we have discussed here.