ABSTRACT

Let $G_0$ be a simply connected non-compact real simple Lie group with maximal compact subgroup $K_0$. Let $T_0 \subset K_0$ be a maximal torus. Assume that $\text{rank}(G_0) = \text{rank}(K_0)$ so that $G_0$ has discrete series representations. We denote by $\mathfrak{g}, \mathfrak{t},$ and $\mathfrak{k}$, the complexifications of the Lie algebras $g_0, t_0$ and $k_0$ of $G_0, K_0$ and $T_0$ respectively. Denote by $\Delta$ the root system of $\mathfrak{g}$ with respect to $\mathfrak{t}$. There exists a positive root system known as the Borel-de Siebenthal positive system such that there is exactly one non-compact simple root, denoted $\nu$. We assume that $G_0/K_0$ is not Hermitian. In this case one has a partition $\Delta = \bigcup_{-2 \leq i \leq 2} \Delta_i$ where $\alpha \in \Delta$ belongs to $\Delta_i$ precisely when the coefficient of $\nu$ in $\alpha$ when expressed as a sum of simple roots is equal to $i$. Let $G$ be the simply connected complexification of $G_0$. Denote by $L_0$ and $\check{L}_0$, the centralizer in $K_0$ of a certain circle subgroup $S_0$ of $T_0$ and its image in $G$ (under the homomorphism $p : G_0 \rightarrow G$ defined by the inclusion $g_0 \hookrightarrow g$) respectively so that the root system of $(L_0, T_0)$ is $\Delta_0$. Any $\check{L}_0$-representation is regarded as an $L_0$-representation via $p$.

Let $\gamma$ be the highest weight of an irreducible representation of $\check{L}_0$ such that $\gamma + \rho_\mathfrak{g}$ is negative on $\Delta_1 \cup \Delta_2$. Here $\rho_\mathfrak{g}$ denotes half the sum of positive roots of $\mathfrak{g}$. Then $\gamma + \rho_\mathfrak{g}$ is the Harish-Chandra parameter of a discrete series representation $\pi_{\gamma+\rho_\mathfrak{g}}$ of $G_0$ called a Borel-de Siebenthal discrete series representation. The $K_0$-finite part of $\pi_{\gamma+\rho_\mathfrak{g}}$ is admissible for a simple factor $K_1 \subset K_0$. It turns out that $S_0 \subset K_1$ and $K_1/L_1 = K_0/L_0$ is a Hermitian symmetric space where $L_1 = L_0 \cap K_1$. One has a Hermitian symmetric pair of non-compact type $(K_0^*, \check{L}_0)$ dual to the pair $(K_0, L_0)$. The element $\gamma$ also determines a holomorphic discrete series representation $\pi_{\gamma+\rho_\mathfrak{k}}$ of $K_0^*$.

In this thesis we address the following question: Does there exist common $L_0$-types between the Borel-de Siebenthal discrete series representation $\pi_{\gamma+\rho_\mathfrak{g}}$ and the holomorphic discrete series representation $\pi_{\gamma+\rho_\mathfrak{k}}$? We settle this question completely in the quaternionic case, namely, when $t_1 \cong \mathfrak{su}(2)$. In the general case, affirmative answer is obtained under the following two hypotheses—(i) there exists a (non-constant) relative invariant for the prehomogeneous space $(L_0^C, u_1)$, where $u_1$ is the representation of $L_0$ on the normal space at the identity coset for the (holomorphic) imbedding $K_0/L_0 \hookrightarrow G_0/L_0$, and, (ii) the longest element $w_0^k$ of the Weyl group of $K_0$ normalizes $L_0$. The proof uses, among others, a decomposition theorem of Schmid and Littelmann’s path model.