Chapter 3

Investigation of the Thermal Conductivity of Niobium in the Temperature Range 1.8-5 K

A cavity made of large grain or single crystal niobium operating below 2 K may have a better thermal stability due to the reduction of phonon scattering by grain boundaries (causing the so-called “phonon-peak”). Many measurements have shown that the scattering of phonons and electrons with the fluxoids decreases the observed thermal conductivity [47,48]. At low temperature (T<<Tc), for instance, in niobium between the temperature 1.8-3K, $k_{ph}$ monotonically increases due to the decrease of scattering of phonons with the electrons. This is due to the electron decoupling resulting from the condensation into Cooper pairs. The thermal conductivity in the mixed state has been measured in previous studies [47-51]. They showed that with the increasing magnetic field in the mixed state more and more fluxoids enters the superconductor and as a result both electron and phonon mean free path (mfp) decrease.

3.1 Present Work

Present work consisted of thermal conductivity measurements of large grain niobium samples in the Meissner state and in the mixed state in the temperature range 1.8 – 5 K and for magnetic fields up to the surface critical field, $H_{c3}$. Also the effect of initial trapped vortices (field-cooled states) on the Meissner and mixed state conductivity of type II superconductor was studied. The results show that when the sample is cycled through Tc, with no external field applied, the sample re-gain the Meissner state and the thermal conductivity has the same value as with no trapped flux. The zero field thermal conductivity data has been fitted with the semi empirical
parametrization of F. Koechlin and B Bonin [52]. The results for the specimen with the trapped vortices are interpreted with phonon-vortex scattering, using the qualitative model of Vinen et.al [48]. The thermal conductivity as function B in the mixed state is analyzed with the Houghton-Maki theory [58].

### 3.2 System Design

A system to measure the magnetization curve and the thermal conductivity of the cylindrical sample rod of diameter 6 mm and 120 mm long was designed and built. The schematic of the system and the picture of the sample rod is shown in Fig.3.1[61]. A heater made with constantan wire glued on a Cu block with Epoxy is clamped near the base of the sample. Two calibrated Cernox resistors are soldered with indium on two small Cu blocks which are clamped to the rod at a distance of about 40 mm. A pickup coil (< 200 turns, 0.29 mm diameter Cu wire) is inserted in the middle of the sample, between the two Cernox resistors. The sample is clamped on a Cu block which is inserted in a copper tube and sealed with indium wire. A stainless steel 2 ¾” Conflat flange was brazed on the other end of the tube. The tube is bolted to a “T” section where a flange with feed-through connectors is bolted on the side.
The assembly is bolted to the vacuum line on a vertical test stand (the pressure in the Cu tube is \(< 10^{-5} \) mbar at 4.3 K). Some heat shields were inserted in the vacuum line to minimize radiation losses. A superconducting magnet up to 1 T (0.1% field homogeneity over the sample length) made by Cryomagnetics surrounds the Cu tube carrying the sample.
3.3 Measurement Methods

The thermal conductivity as a function of the average temperature of the sample is calculated using Fourier’s law where the power supplied to the heater, $P$, the temperature difference, $\Delta T$, the distance $d$ between the two Cernox, and the cross-sectional area of the sample $A$ are measured:

$$k = \frac{P \cdot d}{\Delta T \cdot A}$$ (3.1)

The heater power and the sample temperature are controlled with a LakeShore 332 Temperature Controller.

The magnetization of the sample as a function of the applied field is obtained by linearly ramping the current in the superconducting magnet (the field-to-current ratio is 12.9 mT/A) at a rate of about 0.1 A/s while measuring the voltage from the pick-up coil with a Keithley 2182 nanovoltmeter. The magnetization is calculated from the following formula [59].

$$M(B_a) = \frac{-1}{1-N_D} \int_0^n \frac{V(B_a')-V_n}{V_s-V_n} dB_a$$ (3.2)

where $V_n$ and $V_s$ are the voltages in the normal and superconducting state respectively and $N_D$ is the demagnetization factor, estimated to be about 0.007 for our samples. A Power Ten power supply (0-10 V, 0-100 A) controlled by an American Magnetics 412 Programmer, remotely controlled by a PC, provides the current to the superconducting solenoid. For calibration purposes, we also measured the critical field $B_c$ as a function of temperature for an Indium rod.
(99.99% purity) made by melting the Indium in a stainless steel mold. The data, showed in Fig. 3.2, were fitted with the classical formula

$$B_c(T) = B_c(0) \left( 1 - \frac{T^2}{T_c^2} \right)$$

(3.3)

**Fig. 3.2:** Critical magnetic field as a function of temperature measured on an Indium rod, 99.99% purity. The solid line is a least-square fit with Eqn. (3.3), and resulted in $T_c = 3.35 \pm 0.03$ K and $B_c(0) = 270 \pm 2$ mT, in good agreement with published data\(^\text{16}\).

### 3.4 Zero field Temperature Dependence of the Thermal Conductivity

The thermal conductivity $k$ is a sum of contributions from electrons and phonons, $k = k_{\text{el}} + k_{\text{lat}}$.

The electron heat conduction in the superconducting state is reduced because the electrons which have condensed into Cooper pairs do not contribute to any disorder or entropy transport any more. The remaining fraction of electrons which contribute to heat transport decreases exponentially with decreasing temperature. According to BRT theory \([52,53]\),
\[
\frac{K_{es}}{K_{en}} = R(y) \quad \quad R(y) \leq 1
\]

where,
\[
R(y) = (f(0))^{-1} \left[ f(-y) + y \ln(1 + \exp(-y)) + y^2 / \left(2(1 + \exp(y))\right)\right]
\]

and
\[
y = \Delta(T) / K_nT = (\Delta(T) / K_nT_c)(T_c / T).
\]
The approximation \( y \approx \alpha T_c / T \) is valid if \( T / T_c \leq 0.6 \).

Finally \( f(-y) \) is defined as,
\[
f(-y) = \int_0^\infty \frac{zdz}{1+\exp(z+y)} \quad \text{with} \quad f(0) = \pi^2 / 12.
\]

**Fig.3.3** The ratio of \( K_{es} / K_{en} = R(y) \) as a function of reduced temperature, \( T / T_c = \alpha / y \) (within the experimental temperature range).

Here, \( T_c \) is the superconductor critical temperature, \( \Delta(T) \), the superconductor energy gap and \( \alpha \approx 1.76 \) in the BCS theory, but may take values in the range \( 1.75 \leq \alpha \leq 1.95 \) because of strong coupling effects. The ratio \( K_{es} / K_{en} = R(y) \) is plotted as a function of \( T / T_c \), taking \( \alpha = 1.76 \) in Fig.3.3 within the experimental temperature range 1.8-5K used for our experiments.
The lattice thermal conductivity is limited by the different scattering mechanism of phonons with point defects, dislocations (line defects), grain boundaries, sample walls, and electrons. The general expression for lattice conductivity is 

\[ k_{\text{latt}} = \frac{1}{3} C_v \cdot v^2 \cdot \frac{1}{\tau} \]

where \( 1/\tau \) is the total scattering rate of phonons from different scattering mechanisms, \( C_v \) is the specific heat per unit volume and \( v \) is the average velocity of the carriers of thermal energy. The resultant lattice thermal conductivity taking into account the phonon scattering by the electrons and by the crystal boundaries is given by,

\[ K_{\text{latt},\text{e}} \equiv \left[ \frac{1}{\exp(y)DT^2} + \frac{1}{Bl_{ph}T^3} \right]^{-1} \]  

(3.5)

Where, \( D \) and \( B \) are two constants and \( l_{ph} \) is the phonon mean free path and for our large grain niobium sample it is the smallest sample dimension as the grain size of the sample was bigger than the diameter of the sample rod. The total heat conductivity of the superconducting metal is obtained by adding the electron term \( K_{e}(T) \) and the lattice term \( K_{\text{latt},\text{e}}(T) \). This of course is valid for temperatures \( T \) lower than the critical temperature, because \( y = \Delta(T)/K_B T \) is defined only in this domain,

\[ K_e(T) \equiv R(y) \left[ \frac{\rho_{295 K}}{L \cdot RRR \cdot T} + A \cdot T^2 \right]^{-1} + \left[ \frac{1}{\exp(y)DT^2} + \frac{1}{BIT^3} \right]^{-1} \]  

(3.6)

Where \( L \) is the Lorentz constant, \( A \) is the coefficient of momentum exchange with the lattice vibrations, \( D \) is the coefficient of momentum exchange with the normal electrons and \( B \) is a constant which depends on the material and mechanism of scattering. In order to obtain \( K_e(T) \) using this model, it is necessary to give experimental values to three variables: temperature \( T \),
residual resistivity ratio RRR and the phonon mean free path $l_{ph}$. On the other hand the theoretical parameters $A, L, \alpha, B, D$ are obtained by fitting these five parameters to the experimental results. The RRR value is generally determined by the well known relationship established by Padamsee [21], $RRR = 4 k_{s,4.2K}$. But to determine the RRR value in our studies we have measured the thermal conductivity at 4.2K in normal ($k_{en}$) and superconducting state ($k_{cs}$) and then used the following procedure to have the correct value of the measured RRR. The standard formula of RRR is replaced by $RRR = \delta k_{s,4.2K}$, where $\delta$ is defined by $\delta = \rho_{295K} / R(y)LT$. To calculate the RRR value we take $\rho_{295K} = 1.44 \times 10^{-7} \Omega \cdot m$, $L = 2.45 \times 10^{-8} W K^{-2}$, $T = 4.2K$ and $R(y)$ is experimentally determined using Eqn.3.4. It is found that the value of the parameter $\delta$ is 4.7 in case of BCP cleaned samples whereas for the heat treated samples it varied from 4.2-4.5.

Experimental thermal conductivity measurement data for four different samples named as A0, B0, C0 and D0 and A1, B1, C1 and D1 are shown in Fig. 3.4 and Fig. 3.5. The corresponding RRR values are shown in the insets of Fig.3.4 and Fig.3.5. The RRR values for the samples are calculated using the empirical formula $RRR = 4x K_{d,2}$, where $K_{d,2}$ is the thermal conductivity at 4.2K. The samples A0,B0,C0 and D0 are degreased ultrasonically after the EDM wire cut and then about 180µm are etched away from all the samples surface by buffered chemical polishing (BCP- 1:1:1, HF+HNO$_3$+H$_3$PO$_4$). After the first set of measurement all the samples are degassed in a vacuum furnace at 600°C for 10 hour in a vacuum better than $10^{-6}$ Torr. Then a light BCP (1(HF):1(HNO$_3$):2(H$_3$PO$_4$)) is carried out on all the heat treated samples to remove about 20µm. This set of samples is named as A1, B1, C1 and D1 respectively.
Fig. 3.4 Experimental thermal conductivity data for 180 µm buffered chemical polished Niobium samples from four different ingots. Solid lines are the fitting curves.

Fig. 3.5 Experimental thermal conductivity data for 600°C heat treated Niobium samples from four different ingots. Solid lines are the fitting curves.
Table 3.1 Theoretical fitting parameters of the thermal conductivity data.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>A</th>
<th>A</th>
<th>1/D</th>
<th>B</th>
</tr>
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<tbody>
<tr>
<td>Samples</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A0</td>
<td>1.87</td>
<td>5.03E-05</td>
<td>482</td>
<td>1.02E+03</td>
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<tr>
<td>B0</td>
<td>1.78</td>
<td>3.05E-05</td>
<td>523</td>
<td>3.78E+02</td>
</tr>
<tr>
<td>C0</td>
<td>1.81</td>
<td>4.07E-05</td>
<td>348</td>
<td>2.02E+03</td>
</tr>
<tr>
<td>D0</td>
<td>1.86</td>
<td>2.97E-05</td>
<td>423</td>
<td>1.24E+03</td>
</tr>
<tr>
<td>A1</td>
<td>1.86</td>
<td>1.00E-05</td>
<td>402</td>
<td>3.01E+03</td>
</tr>
<tr>
<td>B1</td>
<td>1.77</td>
<td>7.00E-07</td>
<td>299</td>
<td>1.06E+03</td>
</tr>
<tr>
<td>C1</td>
<td>1.80</td>
<td>1.00E-06</td>
<td>237</td>
<td>6.32E+03</td>
</tr>
<tr>
<td>D1</td>
<td>1.86</td>
<td>1.64E-05</td>
<td>289</td>
<td>2.33E+03</td>
</tr>
</tbody>
</table>

The experimental curves were fitted with the above model taking \( L = 2.45 \times 10^{-8} W K^{-2} \) and the theoretical parameters \( A, \alpha, B, D \) obtained from the fitting curves are listed in Table 3.1.

The resultant error in \( k \) calculation is \( \frac{(\Delta k/k)^2}{1/2} \approx 6\% \). The experimental fit parameters are in good agreement with theoretical parameters reported in Ref. [52]. Table 3.1 shows that the BCS gap parameter \( \alpha \) doesn’t change in all four samples before and after the 600°C degassing. The 600°C, 10 hour degassing is given to remove interstitial hydrogen and mechanical stress relaxation. Thus 600°C degassing has no effect on the BCS electron-phonon coupling constant, \( \lambda \sim N(\varepsilon_F).V \), where \( N(\varepsilon_F) \) is density of states for normal electrons at Fermi energy and \( V \) is matrix element of scattering interaction. It suggests that some other scattering mechanism such as
electron-defect is responsible for the change in thermal conductivity after 600°C heat treatment. The parameter $B \propto 1/a_0$ increases after the degassing, where $a_0$ is the lattice constant. The increase in B implies that the lattice constant $a_0$ decreases and hence the interstitial hydrogen concentration reduces which plays a role in the lattice parameter expansion when it is trapped at the tetrahedral positions of the BCC niobium lattice.

The temperature dependence of thermal conductivity of sample C1 in superconducting state $K_s$ and in normal conducting state $K_n$ is plotted in Fig. 3.6. The solid line which fits the normal state data points is obtained from the first term on the right hand side of Eqn. 3.6 excluding the $R(y)$ term. The parameter $A$ is the best fit value as shown in Table 3.1 for the sample C1 and $L = 2.45 \times 10^{-8} W K^{-2}$, $\rho_{205} = 1.45 \times 10^{-7}$ and $RRR = 138$ are used as material constants for the fit in

![Fig. 3.6: Thermal conductivity data in normal and superconducting state for the sample C1. Solid lines are the fitting curves with Eqn.3.6.](image)
superconducting state. So the fit parameters are in well agreement for both the normal state and superconducting state data. As a result this model can be applied to calculate the electronic and phonon conductivity in normal and superconducting state.

3.5 Field Dependence of Thermal Conductivity

The core of the trapped flux line is represented by a region of radius $\xi$, the GL coherence length, within which the modulus of the order parameter, $\Delta(r)$, is significantly reduced. Within this core the magnetic field and the superfluid velocity are large enough to cause depairing, so that we expect to find bound excitations that are localized within the core. From the studies of Caroli et al [54], for clean materials with large GL parameter $k$, it was confirmed that except for an energy gap $\sim \Delta_\infty/E_F$, where $\Delta_\infty$ is the BCS energy gap in the Meissner state and $E_F$ is the Fermi energy, the density of states is similar to that in a normal metal cylinder of radius $\xi$. The effect of magnetic field in the materials of smaller $k_{GL}(\lambda_L/\xi)$ has been studied by Hansen [55] and by Bergk and Tewordt [56]. They found that the small gap may disappear, but the density of states remains of the same order of magnitude.

The properties of these excitations (for small and large $k_{GL}$) are quite different from those of normal electrons: coherence factors are generally different, and group velocities along the flux lines are expected to be much smaller than those of the normal electrons. The low group velocity means that the contribution of the bound excitations to the thermal conductivity (measured parallel to the flux lines) should be very small. In addition to these bound excitations there will be unbound excitations. The unbound excitations behave as BCS quasiparticles at distances larger than the penetration depth from the vortex core. Near the flux lines they will be modified as they interact with the magnetic field, the superfluid velocity, and the modulation of $\Delta(r)$. This
interaction will cause scattering among quasiparticles which in turn will reduce the electronic thermal conductivity in the mixed state due to the trapped flux lines in the material at favorable locations. In the Meissner state, phonons interact with the electronic excitations, and this interaction plays an important role in thermal conduction in the Meissner state of a superconductor at fairly low temperatures. When the magnetic flux is trapped within the material, phonons will again interact with the bound excitations in the core. The strength of this interaction can be calculated from the Ref. of Caroli et al [54], taking into account much higher frequencies for the thermally excited phonons.

Before presenting the experimental results a theoretical review of the field dependence of the phonon and electron conductivity is discussed below.

i) At low inductions (H<<Hc2) and at low temperature (T<<Tc) the phonons are scattered by a random array of vortices and these behave as if they were cylinders of normal metal [57,58]. The qualitative expression of the phonon conductivity as a function of B is given by,

$$\frac{K_{ph}(0)}{K_{ph}(B)} = 1 + \sigma \frac{B}{K_{ph}(0)} \frac{H_{c2}}{K_{ph}}$$

(3.7)

where $\sigma$ is the average scattering diameter of a vortex line for the thermal phonons ($\sigma \sim 0.5$). Similarly the electronic thermal conductivity as a function of B is given by,

$$\frac{K_{e}(0)}{K_{e}(B)} = 1 + \frac{l_{ea}B}{\Phi_0}$$

(3.8)
where $l_e$ is the mean free path of electrons in zero field, $\Phi_0$ is flux quanta and $a$ is the effective scattering diameter of a vortex line for the free excitations.

ii) At large inductions i.e. close to $H_{C2}$ and at low temperature, the field dependent thermal conductivity has been analyzed by Houghton-Maki [58]. They have determined the thermal conductivity when (a) the temperature gradient is parallel to the applied magnetic field and (b) the temperature gradient is perpendicular to the applied magnetic field. The result of Houghton-Maki for the temperature gradient parallel to the applied magnetic field is given by,

$$\Delta K = -6\mu \left[ (1 - \mu^2) J_1 \left( \frac{1}{4} \pi - \mu \right) \right]$$

where, $J_1 = \int_0^\pi \frac{\cos \theta}{\cos \theta + \mu} d\theta = \frac{\pi}{2} + \frac{2\mu}{\sqrt{1-\mu^2}} \text{Tanh}^{-1} \left( \frac{\mu-1}{\sqrt{1-\mu^2}} \right)$ and $\mu$ is the transport coefficient given by, $\mu = 2 \sqrt{\frac{\pi}{\hbar^2 k_c v_F^2}} \Delta l_e$ in which $k_c$ is the reciprocal lattice vector of the vortex lattice and $v_F$ is the Fermi velocity, $\Delta$ is the order parameter and $l_e$ is the electron mean free path and $\Delta K$ is the difference between the normal and superconducting state electronic thermal conductivities.

The experimental thermal conductivity measurement of the sample-A1 and C1 with and without the trapped vortices is shown in Fig.3.7. The bulk magnetization measurement allows us to get the values of remnant magnetization for those samples. The bulk magnetization measurement for A1 and C1 is shown in Fig.3.8. The magnetization curves show that the trapped magnetic flux in A1 and C1 is 58 and 80 mT, respectively. Fig.3.7 shows that the effect on the electronic part of
the thermal conductivity by the scattering of electrons with the vortex cores is negligibly small. Whereas the phonons are strongly scattered causing an

![Figure 3.7](image_url)

**Fig. 3.7** Plot of $k_s$ versus $T$, ■ C1-Zero field, *C1-Trapped flux 81 mT, △ A1-Zero field, ○ A1-Trapped flux 60 mT, solid lines are the fitting curves of Eq. (6).

almost zero contribution to the net thermal conductivity. The field dependence of phonon and electronic conductivity of Eqn.3.7 and 3.8 are used to interpret the thermal conductivity results with trapped vortices for the sample A1 and C1. $K''_{ph}$ is being calculated from the Eqn. 3.5 in case of normal metals using the value of the parameters $D$ and $B$ from the Table 3.1.

The phonon conductivity in the normal state for A1 and C1 are 0.01 and 0.017 W/m-K respectively. The value of electron mfp $l_e$ is calculated using the expression $1/\rho = 2e^2S_F l_e/(3(2\pi\hbar)^3$, where $S_F$ is the area of the Fermi surface in momentum space in the first Brillouin zone and $\rho$ is the electrical resistivity. A value of
**Fig. 3.8** Magnetization curves at \( T = 2 \text{K} \) with and without remnant magnetization for \( \cdots \text{A1}, \quad \text{C1} \).

\( S_F = 2.23 \times 10^{-47} \text{ kg}^2 \text{m}^2/\text{sec}^2 \), the value for \( \rho \) are \( 2.37 \times 10^{-9} \) and \( 1.12 \times 10^{-9} \) \( \Omega \cdot \text{m} \) are used for A1 and C1 to calculate the electron mfp. The thermal conductivity model defined in Sec. III gives a reasonable fit of the experimental data points as shown in Fig.3.9, when corrected using Eqn.3.7 and 3.8 to take into account the presence of trapped vortices. The error in k calculation is 

\[
\left\langle \frac{\Delta k}{k} \right\rangle^{1/2} \approx 6\%.
\]

Fig.3.9 shows that the deviation of the theoretical curve from the experimental data increases with the increase of trapped flux which signifies that the other mechanisms, such as inter vortex tunneling and collective mode excitations of the flux line lattice play a role at higher fields. As the flux lines act as normal metal cores randomly distributed over the sample cross section, they can be treated as point like scattering centers to the thermally excited phonons and BCS quasi-particles. The thermal conductivity results of sample A1 and C1 with trapped
vortices are fitted with Eqn.3.9 taking into account that the vortex cores are randomly distributed point like scattering centers. The fitting parameters in presence and absence of trapped vortices are summarized in Table 3.2.

Fig.3.9 Effect of trapped vortices on the superconducting state thermal conductivity in sample A1 and C1. Solid lines represent the qualitative theoretical model by Vinen et.al. at low inductions and at low temperature.

Table 3.2 Theoretical fitting parameters in presence and absence of trapped flux lines

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Sample A1</th>
<th>Sample C1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Zero Flux</td>
<td>Trapped Flux (58mT)</td>
</tr>
<tr>
<td>α</td>
<td>1.86</td>
<td>1.78</td>
</tr>
<tr>
<td>L</td>
<td>2.45x10⁻⁸</td>
<td>2.45x10⁻⁸</td>
</tr>
<tr>
<td>A</td>
<td>1.00x10⁻⁵</td>
<td>1.77x10⁻⁴</td>
</tr>
<tr>
<td>1/D</td>
<td>402</td>
<td>1216</td>
</tr>
<tr>
<td>B</td>
<td>3.01x10³</td>
<td>3.35x10²</td>
</tr>
</tbody>
</table>
From Table 3.2 we can see that the gap parameter $\alpha$ and the coefficient of momentum exchange of electrons with the lattice vibration, $A$, decreases while the coefficient of momentum exchange of phonons with the electrons, $D$, increases in the presence of trapped vortices. The parameter $A \propto N^{2/3}$ and the parameter $D \propto N^{-2}$, where $N$ is the effective number of conduction electrons per atom. As the vortices are trapped inside the superconductor the effective number of conduction electrons reduces because of the bound excitations within the vortex core. The reduction in the gap energy $\alpha$ is due to the low energy excitations close to the vortex core. The energy gap of these excitations is very small and is given by $\varepsilon_0 \sim \Delta_0^2/E_F$. So the effective energy gap will be $\Delta_{\text{eff}} \sim (\Delta_0 - \varepsilon_0)$. Taking $E_F = 0.00018m_0c^2 = 91.8$ for niobium and $\Delta_0=1.86(A1)$ and $1.78(C1)$ from the experimental fit data of the zero trapped vortex sample, we get $\varepsilon_0 \sim 0.04 (A1)$ and $0.034(C1)$ leading to an effective energy gap of $\Delta_{\text{eff}} \sim 1.82 (A1)$ and $1.74(C1)$. This simple explanation gives an error of 2.4% for the fit parameter $\alpha$. The variation of thermal conductivity with the applied magnetic field for the sample B1 with and without trapped vortices at 2K is shown in Fig.3.10.
Fig. 3.10 Field dependence of the thermal conductivity with and without trapped vortices measured with the field parallel to the heat flow direction at 2K.

We can see that the value of $H_{C1}$ and $H_{C2}$ are independent on the sample condition from the point of view of trapped vortices. Both the curves are showing the same behavior in the mixed state. In the mixed state the phonon contribution is negligibly small; it is the electronic contribution which increases with the increasing magnetic field as more and more quasiparticles start contributing to the thermal conductivity. Eventually as the applied field reaches $H_{C2}$, the bulk of the superconductor is in the normal state while the surface remains in the superconducting state to an extent of the order of coherence length $\xi_0$. The figure shows a constant thermal conductivity between $H_{C2}$ and a value of applied magnetic field beyond the third critical field $H_{C3}$. So we couldn’t find any effect of the surface sheath on the measured thermal conductivity, as expected. Above $H_{C2}$ the specimen shows the thermal conductivity behavior of the normal metal. But there is marked difference of the measured thermal conductivity data in the region $0 \leq H \leq H_{C1}$ for
the sample with and without trapped vortices. No magnetic flux enters the samples up to the first 
flux penetration at 180 mT. Below $H_{c1}$, the difference between the thermal conductivity values of 
the two curves in Fig.3.10 is due to the strong scattering of phonons by the vortex cores as 
explained earlier.

At large inductions and at low temperature the field dependent thermal conductivity is 
represented by the Houghton-Maki theory described by the Eqn.3.9. A plot of $\mu$ versus $\Delta k/k_e$ is 
shown in Fig.3.11.

![Diagram showing $\Delta k/k_e$ as a function of $\mu$.](image)

**Fig.3.11** $\Delta k/k_e$ as a function of $\mu$ in sample B1. (Δ) Sample B1 without any trapped flux in zero 
field, (■) sample B1 with an initial remnant magnetization of about 58 mT, solid line is the 
theoretical curve of Houghton-Maki.

Fig.3.11 shows that when there are trapped vortices in zero field, the experimental data points for 
$0.2 \leq \mu \leq 0.6$ lie below the theoretical curve as well as the baseline measurement data in zero 
field without the trapped vortices. This might be due to the additional contributions from the 
bound excitations in the form of tunneling. Although the exact cause is not yet clear but there is 
definitely an additional contribution to the thermal conductivity in case of initial remnant
magnetization. A future experiment with different remnant magnetization at zero field and the corresponding thermal conductivity measurement will produce a systematic deviation from the Houghton-Maki theory and the evidence for this new contribution to the thermal conductivity can be established.

Finally, the measurements of the thermal conductivity of sample C1 after zero-field cooling, after the applied magnetic field was cycled from zero up to $H_{c2}$ and then back to zero, and after warming-up the sample above $T_c$ followed by zero-field cooling is shown in Fig.3.12. The remnant magnetization after cycling the applied magnetic field is about 81mT.

![Graph](image-url)

**Fig.3.12** Plot of thermal conductivity in superconducting state in zero remnant field, with remnant field and heating up the sample through $T_c$ to exclude the flux lines to reproduce the zero field curve

As shown in Fig.3.12, the phonons are strongly scattered by the vortex cores. By raising the temperature of the sample above $T_c$ (9.25K), the remnant magnetic field is homogeneously
distributed throughout the normal conducting sample. By lowering the temperature below $T_c$, the magnetic flux is expelled from the superconductor and a new thermal conductivity measurement reproduces the data obtained after the first measurement, in absence of any applied magnetic field.

3.6 Conclusion

The thermal conductivity as a function of temperature measured on large grain niobium samples in the Meissner state is well described by the model of Ref. [52] within the experimental error of $\pm 6\%$. The measurements clearly show the presence of a phonon peak at around 2K. One important observation is that the phonon peak is eliminated by the presence of trapped vortices due to the strong scattering of phonons with vortex cores. When the vortices are trapped inside the sample, the fit parameters indicate a reduction of the gap energy $\alpha$ due to the low energy excitations having very small energy gap $\sim \Delta_0^2/E_F$ close to the vortex core. Also the effective number of conduction electrons decreases due to the bound excitations in the vortex cores. The dependence of the thermal conductivity with the applied magnetic field for the samples with and without trapped vortices show the same $H_{C1}$ and $H_{C2}$ values as from the magnetization measurement. Finally when the temperature of the samples is cycled above $T_c$, the thermal conductivity measured for the sample in absence of an applied magnetic field is restored. The temperature dependence of the thermal conductivity at low temperature and low magnetic field agrees qualitatively with the model of Vinen et al. In the vicinity of $H_{C2}$ the thermal conductivity agrees quite well with Houghton-Maki theory for the virgin sample i.e without any trapped vortices. But if there is initial flux trapped within the sample, the measured thermal conductivity deviates from the Houghton-Maki theory and observed an increase in thermal conductivity in the
range of $0.2 \leq \mu \leq 0.6$. Future experiments with different initial trapped vortices and subsequent measurement of the thermal conductivity in the range of $0.2 \leq \mu \leq 0.6$ might help to interpret the deviation from the theory.