Abstract

C. F. Gauss has started the theory of hypergeometric series in 1812. Thirty
three years later, E. Heine has developed systematically the theory of basic
hypergeometric series. Since then a number of well-known mathematicians
have contributed to the development of the theory of hypergeometric and basic
hypergeometric series. Among them are J. Thomae, L. J. Rogers, A. C. Dixon, J.
Dougall, L. Saalschütz, F. J. W. Whipple, F. H. Jackson, Srinivasa Ramanujan,

Motivated by their works, the research work was carried out and the findings
are presented in this thesis. The thesis consists of six chapters.

In chapter 1, we present a detailed literature survey along with necessary
definitions, notations and some identities which will be employed in the
subsequent chapters.

In his lost notebook, Ramanujan listed many beautiful mathematical
formulae, majority of which fall under the purview of \(q\)-series. These include
mock theta functions, partial theta functions, \(q\)-continued fractions, modular
equations. One of the identities involving partial theta functions is called a
reciprocity theorem of Ramanujan. This theorem has received the attention
of many mathematicians during the past two decades hence several proofs and
generalizations are now available in the literature. In chapter 2, of the thesis
we give a unified approach to the proofs of Ramanujan’s reciprocity theorem
and to its three and four variable generalizations. Also, we present interesting
applications of the reciprocity theorems.

A special case of the reciprocity theorem of Ramanujan is the well-known
Jacobi’s triple product identity. In 1915, MacMahon has given finite versions
of the Jacobi’s Triple product identity, since then the finite versions of several classical identities such as quintuple product identity and Ramanujan’s $1\psi_1$ summation formula were found. Motivated by these works, in chapter 3 of the thesis we present finite versions of all the three reciprocity theorems along with their applications.

Andrews’ in his paper on Ramanujan’s lost notebook—I partial $\theta$—functions, has established a generalization of Ramanujan’s $1\psi_1$ summation formula. In 2005, Liu has established an equivalent form of Andrews’ identity. In chapter 4 of the thesis we present a new and a bilateral form of the Andrews’ identity along with a number of applications.

In 1936, Bailey carried out systematic investigations of summation and transformation formula for bilateral basic hypergeometric series and found the summation for the very-well-poised $6\psi_6$-series. In 1950, Bailey has derived three transformation formulae for bilateral basic $2\psi_2$-series. In chapter 5 of the thesis we present two proofs of Bailey’s $2\psi_2$ transformation formula. We also present finite forms of the Bailey’s $2\psi_2$ transformation formula and of its iterate. We further use $2\psi_2$ transformation formula to prove the two and three variable reciprocity theorems and present some more applications of these identities.

Finally, in chapter 6, we present a new transformation formula for bilateral basic $6\psi_6$ series and deduce therefrom the Bailey’s $6\psi_6$ summation formula. We also deduce a number of identities which are interesting from number theoretic point of view and also some theta function identities.