Chapter 8

Conclusion

In this thesis, we studied the microscopic and macroscopic descriptions of black holes in string theory. In the microscopic side, we studied the counting of a class of states called twisted BPS states in a supersymmetric theory. In the macroscopic side, we studied the attractor mechanism in gauged supergravity. We first summarise the contents of the thesis, and then highlight the main results and open questions.

In Chapter 2, we studied the microscopic state counting in string theory [3]. We computed the generating functions for a class of 1/2 BPS states called twisted BPS states in CHL orbifold theories, when the twists do not commute with the orbifold group. The generating functions turn out be ratios of the theta functions for the orbifold group and the twist generating group. The orbifold partition function counts the states which are invariant under the orbifolding group. The twists count states invariant under the twist generating group within the states invariant under the orbifolding group. So, it is natural to expect that the number of twisted states in orbifold theory should be lesser than untwisted states. We verified this expectation by computing the asymptotic expansion of the degeneracy. One direction where this work may be extended is to do an analogous computation for 1/4 BPS states. It is also possible that the twists may provide a controlled way to break supersymmetry and help extend the counting problem to situations with reduced super-
In the macroscopic side, we studied the attractor mechanism in gauged supergravity. We covered the necessary background materials in attractor mechanism in supergravity in chapter 3. In chapter 4, we studied $AdS$ black holes and homogeneous extremal black brane horizons known as Bianchi attractors [7,53]. These metrics are of the general form,

$$ds^2 = L^2 \left[ -\hat{r}^{2u_0} d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} + \hat{r}^{2(u_i+u_j)} \eta_{ij} \omega^i \otimes \omega^j \right].$$

(8.0.1)

and display homogeneous symmetries. In addition, they also exhibit scale invariance in all directions. We were interested in embedding these metrics as attractor solutions in gauged supergravity.

We covered the necessary background material on $\mathcal{N} = 2, d = 5$ gauged supergravity in chapter 5. We saw that gauging the symmetries of the scalar manifold gives rise to interesting structure such as a potential for the scalar fields. We also saw that gauging the $R$ symmetry of the theory leads to terms in the potential which can support $AdS$ vacuum. We highlighted these features through a simple example of a gauged supergravity theory with one vector multiplet.

In chapter 6, we discussed the generalised attractors in $\mathcal{N} = 2, d = 5$ gauged supergravity characterised by constant anholonomy coefficients [17]. The generalised attractor points are obtained by solving the field equations when all the bosonic fields of the theory become constants in tangent space. The field equations at the attractor points are algebraic, and the moduli are determined in terms of the charges as expected for attractor solutions. We constructed the attractor potential from the scalar field equations and showed that it can be written independently from squares of the bosonic terms in the fermion supersymmetry transformations. We constructed some explicit examples of homogeneous extremal black brane horizons studied in chapter 4, as generalised attractor solutions in gauged supergravity.
The generalised attractor procedure relied on extremization of an attractor potential and not on supersymmetry. From the discussions in chapter 3, we see that this method is general and can include non-supersymmetric attractors. In chapter 7 we investigated the stability of the Bianchi attractors in gauged supergravity. We considered the scalar perturbations about the critical value and performed a fluctuation analysis in the field equations. Since the stress energy tensor depended on the scalar fluctuations even at first order, we demanded that the fluctuation be regular. We then analysed the scalar field equations and solved for the fluctuations. We obtained the conditions under which the fluctuations are well defined and demanded regularity at the horizon to determine the conditions for stability.

The main results of the study of attractor mechanism in gauged supergravity are,

- We have extended the study of generalised attractors in $\mathcal{N} = 2, d = 4$ gauged supergravity [14] to $\mathcal{N} = 2, d = 5$ gauged supergravity in [17].
  - The field equations become algebraic at the attractor points. The moduli are determined as functions of the charges by extremising an attractor potential.
  - The attractor potential can be constructed independently from fermionic shifts in gauged supergravity.
  - The homogeneous extremal near horizon geometries known as the Bianchi attractors [7, 53] are generalised attractor solutions in gauged supergravity.
  - We constructed explicit examples of Bianchi I, Bianchi II and Bianchi VI type solutions as generalised attractors in a simple gauged supergravity theory with one vector multiplet.

- We have studied the stability of the Bianchi attractor solutions in gauged supergravity under scalar fluctuations about the critical value [21].
  - The stress energy tensor in gauged supergravity (with a generic gauging of the symmetries of the scalar manifold) depends on scalar fluctuations even at first
order perturbation.

- Fluctuations which do not have regular behaviour near the horizon will backreact strongly leading to significant deviation from the attractor geometry. This indicates an instability.

- The scalar fluctuations are well defined when the critical point in consideration is a maxima of the attractor potential.

- Regularity of the fluctuations near the horizon require the near horizon geometry to factorise as $Li f_{\omega_0} \times M$,

$$
\begin{align*}
    ds^2 &= L^2 \left( -L^2 \omega d\hat{t}^2 + \frac{d\hat{r}^2}{L^2} \right) + \frac{L^2}{\hat{r}^2} \left( \eta_{ij} \omega^i \otimes \omega^j \right), \\
    &= L^2 \left( -r^{2m_0} d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} \right) + L^2 \left( \eta_{ij} \omega^i \otimes \omega^j \right),
\end{align*}
$$

where $M = M_I, M_{II} \ldots M_{IX}$ are the homogeneous subspaces invariant under the Bianchi type symmetries.

We now discuss some of the implications of our results. Our study of generalised attractors indicates that there are several possible end points for an attractor flow in five dimensional gauged supergravity. Even in the simple example of the gauged supergravity with one vector multiplet, the Bianchi I, II and VI solutions all exist for the same value of the critical point. It will be interesting to study this further and see if there is a preferred end point. We have answered this partially by the stability analysis which indicates that the Bianchi type metrics which factorise as (8.0.2) represent stable end points. This factorisation is reminiscent of the fact that near horizon geometry of extremal black holes in four dimensions factorise as $AdS_2 \times S^2$. It would also be very interesting to see if the results of the stability analysis are model independent.

We have studied the generalised attractors by extremising an attractor potential. This method is more generic and describes non-supersymmetric attractor points as well. The construction of the attractor potential from fermionic shifts in the supersymmetry transformation indicates that supersymmetry may have a very important role to play in this
construction. We hope to explore this in future.

In the gauged supergravity model we considered, the critical points of the attractor potential coincide with the critical point of the scalar potential in gauged supergravity. This is very similar to the situation in ungauged supergravity where the critical points of the effective potential coincide with the critical points of the central charge. It would be interesting to see if the potential of gauged supergravity is related to the central charge of the theory. This may be related to the issue regarding the fermionic shifts and we hope to explore this in future.

Another implication from the study of generalised attractors is the similarity in the description of generalised attractors in the $\mathcal{N} = 2, d = 4$ theory [14] and the $\mathcal{N} = 2, d = 5$ theory. In ungauged supergravity a large class of BPS solutions in four and five dimensions are related to each other [168, 169]. It would be interesting to see if this connection extends to gauged supergravity.

One of the most interesting problems in the study of attractor mechanism in gauged supergravity is the construction of the flow equations. These equations require a full analytical or numerical black hole solution interpolating between an asymptotic $AdS$ geometry and a near horizon geometry. To construct the flow equations for generalised attractors, it is necessary to construct black brane solutions interpolating between the Bianchi type geometries and the $AdS$ geometry. This will help prove the attractor mechanism for black branes in gauged supergravity. We hope to address this in future.

There are classes of near horizon geometries which are more general than the ones considered in the thesis. Typical examples are metrics which are conformal to the Lifshitz metric, which belongs to the Bianchi I class. These family of metrics exhibit different scaling symmetries, hyperscale violation and occur as gravity duals in studies of doped matter in AdS/CMT [170–173]. Such solutions have been studied systematically in ungauged supergravity [174]. It has been shown that hyperscale violating metrics can arise upon dimensional reduction of some null deformations of the AdS factors that appear in the near
horizon geometry of various extremal brane configurations in string theory [175]. More recently, hyperscale violating metrics conformal to Lifshitz have also been constructed in gauged supergravity [176]. In [53], examples of Bianchi attractors that exhibit hyperscale violation have been constructed in Einstein-Maxwell-Dilation theories. It would be interesting to develop a technique similar to the generalised attractor approach to systematically obtain Bianchi attractors which exhibit hyperscale violation in gauged supergravity. Our previous attempts to obtain such metrics as generalised attractors point that the assumptions of constant anholonomy and constant fields in tangent space need to be relaxed suitably. Furthermore, the supersymmetry of such hyperscale violating metrics need to be studied and perhaps this will shed some light on modification of the generalised attractor assumptions.

Another important issue is string embedding. In general, the embedding of Bianchi attractors in gauged supergravity does not imply string embedding. For instance, It appears that the gauged supergravity models that we have considered [18, 19] are not embeddible in string theory ¹. Perhaps, one way to approach this problem is to look for low dimensional gauged supergravity models with known string embeddings, construct the Bianchi attractors in these theories and then attempt a ten dimensional lift. Perhaps, it is also possible to approach this question top-down from flux compactifications [9]. Gauged supergravities can arise as low energy effective theories from flux compactifications. The various flux parameters associated in a given compactification can be grouped in a tensorial form on which the duality action is manifest. This tensorial form is called an embedding tensor which encodes all the various possible gaugings of the supergravity [10]. This may help narrow down possible models where one could then look for generalised attractor solutions.

The stability condition (7.4.6), is similar to the condition for an isotropic universe in the AdS-Kasner metric. It implies that the near horizon geometries must be free from any

¹We would like to thank Prof Marco Zagermann for informing us about this issue and for several helpful discussions.
anisotropies with respect to the radial direction. This manifests as a loss of scale invariance in the spatial directions. However, the homogeneous symmetries in the spatial directions and scale invariance along the radial, time directions are preserved. The stability analysis predicts that the generalised attractors which are stable under scalar fluctuations about the attractor value, factor into a direct product form, have homogeneous symmetries and are isotropic with respect to a radial flow. In this respect, the stability conditions certainly narrows down the possible IR candidate geometries. However, one should exercise caution as some model dependent information has gone into the final stages of this calculation.