Chapter 3

Attractor mechanism in supergravity

3.1 Introduction

In chapter 2, we studied the counting of microscopic states of bound states of D-Branes in a string theory. We saw that the BPS nature of the bound state configurations played an important role in the calculation of the microscopic degeneracy. In this chapter, we will focus on the macroscopic side. Here the BPS nature of the black hole simplifies the analysis of the Killing spinor equations which arise from the vanishing of fermionic supersymmetry transformations. Exact black hole solutions can often be found by solving the Killing spinor equations rather than the second order Einstein field equations. Once again, it is the BPS nature of the black hole that allows the comparison of the statistical entropy calculated at the weak coupling limit of the theory with the Bekenstein-Hawking entropy of the black hole in the strong coupling limit.

The attractor mechanism explains the macroscopic entropy of extremal black holes in supergravity [4–6]. The moduli fields for a given extremal black hole, flow radially to a fixed value at the horizon regardless of their asymptotic values. The corresponding black hole solution is called an attractor and the mechanism has been named as the attractor mechanism. Solving the attractor equations relates the fixed values of the moduli in terms
of the quantised charges of the black hole. As a result the entropy of the black hole is
determined completely in terms of its charges. The attractor mechanism works not mainly
because of supersymmetry but due to extremality of the black hole [94–96] and hence it
can also be extended to the case of non-supersymmetric black holes [95, 97–100]. In the
non-supersymmetric case, one can no longer use the Killing spinor equations to study
the attractor. For single centered, extremal non-supersymmetric black holes the attractor
mechanism is understood in terms of an effective black hole potential. The attractor point
corresponds to an extremum of this black hole potential. Some review articles covering
the subject are [101–103].

The near horizon geometry of an extremal black hole in spacetime corresponds to the
attractor point in the moduli space. The attractor geometry for black holes preserving su-
persymmetry is always stable. For the non-supersymmetric case, the attractors are stable
when the critical point is an absolute minima of the effective black hole potential. In the
asymptotically flat case this is strictly true [95, 97]. Thus, for the stable attractors, the
matrix of second derivatives of the effective potential should have positive eigenvalues.

The organisation of this chapter is as follows. In §3.2 we review some essential material
in $\mathcal{N} = 2$ supergravity related to special geometry. We then discuss the supersymmetry
conditions that give rise to the attractor behaviour and black hole entropy in §3.3. In the
next section §3.4, we demand regularity of the horizon, and consequent analysis reduces
the scalar field equations to extremization of an effective potential. This leads to the dis-
ussion on non supersymmetric attractors and their stability in §3.5. We then summarise
in §3.6.
3.2 Preliminaries

The \( N = 2, d = 4 \) supergravity coupled to vector and hyper multiplets has the following field contents. The gravity multiplet consists of,

\[
\{ e^a_\mu, \psi^A, A^0_\mu \},
\]  

(3.2.1)

where \( e^a_\mu \) is the vielbein with \( a = 0, 1, 2, 3 \), \( \psi^A \) are the gravitinos with \( A = 1, 2 \) and \( A^0_\mu \) is the graviphoton. The chirality conditions are given by \( \gamma_5 \psi^A = 1 = -\gamma_5 \psi^A \). The vector multiplet consists of,

\[
\{ A^i_\mu, \lambda^{iA}, \zeta^i \},
\]  

(3.2.2)

where \( A^i_\mu \) are the gauge bosons with \( i = 1, 2, \ldots n_V \), the gauginos are denoted by \( \lambda^{iA} \) and the complex scalars are written as \( \zeta^i \), where \( i = 1, 2, \ldots n_V \). The graviphoton and the gauge bosons \( A^i_\mu \) coming from the \( n_V \) vector multiplets are together denoted by \( A^\Lambda_\mu \), with \( \Lambda = 0, 1, \ldots n_V \). The scalars in the vector multiplet, parametrise a special Kähler manifold.

A Kähler manifold has mutually compatible complex structure, Riemannian structure and a symplectic structure [104]. The metric on a Kähler manifold is Ricci flat, hermitian and is derived from a Kähler potential,

\[
g_{ij} = \partial_i \partial_j K.
\]  

(3.2.3)

A Kähler manifold is special Kähler when there exists local holomorphic sections \( (X^\Lambda, F_\Lambda) \) which can be used to express the the Kähler potential as,

\[
K = -\ln( i(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda)) .
\]  

(3.2.4)

Since the Kähler manifold is also symplectic one can introduce the symplectic sections
The hypermultiplet consists of, 

\[ \{\zeta^\alpha, q^u\} , \]  

(3.2.9) 

where \( q^u \) are the scalars in the hypermultiplet with \( u = 1,2,\ldots,4n_H \) and \( \zeta^\alpha \) are the hyperinos with \( \alpha = 1,2,\ldots,2n_H \). The quaternions \( q^u \) parametrise a quaternionic manifold of dimension \( 4n_H \). The quaternionic manifold is also an example of a Kähler manifold with mutually compatible Riemannian, complex and a symplectic structures \([105, 106]\). The
metric on the quaternion Kähler manifold is defined by,

\[ ds^2 = h_{uv} dq^u \otimes dq^v . \]  

(3.2.10)

The manifold is called quaternionic as the three complex structures \( J^x \) that exist on the manifold satisfy a quaternionic identity,

\[ (J^x)^w (J^y)^v = -\delta^{xy} (Id)^v_u + \epsilon^{xyz} (J^z)^v_u , \]  

(3.2.11)

where \( x = 1, 2, 3 \). The metric \( h_{uv} \) is hermitian with respect to \( J^x \),

\[ (J^x)^u (J^x)^v h_{uv} = h_{vw} , \]  

(3.2.12)

as expected for a Kähler manifold.

With the preliminaries in hand, the bosonic part of the Lagrangian is given by,

\[ \mathcal{L} = \sqrt{-g} [R + g_{ij}\partial^\mu z^i \partial_\mu z^j + h_{uv} q^u \partial_\mu q^v + i(\tilde{N}_\Lambda \Sigma \mathcal{F}^{-\Lambda} \mathcal{F}^{-\Sigma\mu\nu} - N_\Lambda \Sigma \mathcal{F}^{+\Lambda} \mathcal{F}^{+\Sigma\mu\nu})] , \]  

(3.2.13)

where \( g_{ij} \) is the metric on the special Kähler manifold, \( h_{uv} \) is the metric on the quaternion manifold. The self dual and anti-self dual form field strengths of the gauge fields are defined as,

\[ \mathcal{F}^{\pm}_{\mu\nu} = \frac{1}{2} (\mathcal{F}_{\mu\nu} \mp i \epsilon_{\mu\nu\rho\sigma} \mathcal{F}^{\rho\sigma}) , \]  

(3.2.14)

where \( \epsilon_{0123} = 1. \)

### 3.3 Supersymmetry, attractors and black hole entropy

In this section, we illustrate the emergence of the attractor mechanism from supersymmetry considerations. The supersymmetry transformations of the fermions in the theory are
given by,

\[ \delta \psi_A = D_\mu \epsilon_A + \epsilon_{AB} T_{\mu\nu} \gamma^\nu \epsilon^B, \]
\[ \delta \lambda^I = i \gamma^\mu \partial_\mu \zeta^I \epsilon^A + \frac{1}{2} T_{\mu\nu} \gamma^\mu \epsilon_B \epsilon^{AB}, \]
\[ \delta \zeta_\alpha = i \mathbf{U}^B_\mu \partial_\mu q^\mu \epsilon^A \epsilon_{AB} C_{ab}. \]  

(3.3.1)

where \( D_\mu \epsilon_A = \partial_\mu \epsilon_A + \frac{1}{4} \omega^{ab}_\mu \gamma_{ab}, \mathbf{U}^B_\mu \) are the quaternionic vielbein and \( T_{\mu\nu} \) and \( T_{\mu\nu}^{-1} \) are symplectic invariant combinations of the field strength defined by,

\[ T_{\mu\nu} = M_\Lambda \mathcal{F}_{\mu\nu}^{-\Lambda} - L_\Lambda \bar{N}_{\Lambda \Sigma} \mathcal{F}_{\mu\nu}^{-\Sigma}, \]
\[ \mathcal{F}_{\mu\nu}^{-1} = g^{ij} (D_j \bar{M}_\Lambda \mathcal{F}_{\mu\nu}^{-\Lambda} - D_j \bar{L}_\Lambda \bar{N}_{\Lambda \Sigma} \mathcal{F}_{\mu\nu}^{-\Sigma}). \]  

(3.3.2)

We are interested in static, spherically symmetric, charged, supersymmetric black hole solutions of the type,

\[ ds^2 = -e^{2U} dt^2 + e^{-2U} (dr^2 + r^2 d\Omega_2^2), \quad U \equiv U(r), \]

(3.3.3)

which asymptote to Minkowski space. In supersymmetric theories, the ADM mass is given by the central charge of the supersymmetry algebra. In general, the central charge is a function of the moduli (\( \zeta^I \)) and the physical charges of the black hole. Extremization of the central charge relates the moduli to the charges and the black hole entropy is then given by the value of the central charge at the extremum values [6]. The principle of extremization of the central charge follows from the requirement that the near horizon geometry and the asymptotic geometry represent maximally supersymmetric solutions of (3.3.4).

It is easy to see that the Minkowski space satisfies the conditions,

\[ \delta \psi_{A\mu} = 0, \quad \delta \lambda^I = 0, \quad \delta \zeta_\alpha = 0, \]  

(3.3.4)
for arbitrary $\epsilon_A$, when there are no vector fields and when all scalars in the theory take arbitrary constant values,

$$T_{\mu\nu} = 0, \quad F^{-i}_{\mu\nu} = 0, \quad \psi^i = \psi^i_0, \quad q^\mu = q^\mu_0. \quad (3.3.5)$$

The flat space solution thus preserves the full $\mathcal{N} = 2$ supersymmetry of the theory. The other solution is the the near horizon geometry of the black hole solution when,

$$e^{-2\psi} \to \frac{M^2}{r^2} \quad \text{as} \quad r \to 0. \quad (3.3.6)$$

where $M^2 = \frac{A}{4\pi}$ is the ADM mass of the black hole. The near horizon metric takes the form $AdS_2 \times S^2$,

$$ds^2 = -\frac{r^2}{M^2}dt^2 + \frac{M^2}{r^2}dr^2 + M^2d\Omega^2_2, \quad (3.3.7)$$

also known as the Bertotti-Robinson universe. This is a solution [6] of the supersymmetry equations (3.3.4) with,

$$F^{-i}_{\mu\nu} = 0, \quad \partial_\mu \psi^i = 0, \quad \partial_\mu q^\mu = 0. \quad (3.3.8)$$

This solves the gaugino and hyperino conditions. The Killing spinor integrability condition from the gravitino variation (3.3.1) gives terms proportional to $\gamma^\mu, \gamma^{\mu\nu}$. The coefficients of each of these terms must identically vanish as these matrices form a complete basis. This gives,

$$\frac{1}{4}R_{\mu\nu}^{\lambda\sigma} - 2T_{\mu}^{\lambda\nu}T_{\nu}^{\lambda\sigma} = 0, \quad (3.3.9)$$

$$\mathcal{D}_\nu T_{\mu\lambda}^- = 0,$$

which are the Einstein equations and the condition for a covariantly constant graviphoton field strength respectively. These conditions are necessary for the solution to exist and to

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1We have used Planck units $G = 1, \hbar = 1, c = 1, \kappa_B = 1, k = 1.$
preserve the $\mathcal{N} = 2$ supersymmetry.

The central charge of the supersymmetry algebra is given by [107],

$$Z = -\frac{1}{2} \int_{S^2} T^- = L^\Lambda q_\Lambda - M_\Lambda p^\Lambda,$$

$$Z_i = D_i Z = -\frac{1}{2} \int_{S^2} F^{+j} g_{ij}^\prime.$$

The gaugino condition together with $\mathcal{F}^\prime_{\mu^\prime} = 0$ and $\frac{d}{dr} z^i(r) = 0$ implies,

$$D_i Z = 0,$$

which is solved by,

$$p^\Lambda = i(\bar{Z} L^\Lambda - Z \bar{L}^\Lambda), \quad q_\Lambda = i(\bar{Z} M_\Lambda - Z \bar{M}_\Lambda),$$

where we have used (3.2.5) and the fact that $N_{\Lambda \Sigma}$ is Kähler covariant. Then (3.3.12) determine the sections $X^\Lambda$ completely in terms of the charges up to Kähler gauge transformations, which are fixed by choosing the gauge $X^0 = 1$ in (3.2.6). Defining,

$$|Z_c(q, p)| = |Z|_{D_i Z = 0},$$

we see from (3.3.12) that the central charge is purely a function of the charges carried by the black hole. Since we are looking at BPS solutions, $|M| = |Z_c(q, p)|$ and hence the black hole entropy in Planck units is given by,

$$S_{BH} = \frac{A}{4} = \pi M^2 = \pi |Z_c(q, p)|^2.$$

This result is also arrived at by studying the flow equations for the scalar fields in the background of the full black hole solution. For the magnetically charged black hole, this was arrived at by requiring the gaugino supersymmetry transformations to vanish resulting
in a first order equation which relates the moduli in terms of the ratio of the magnetic charges \[4\]. \(^2\) This result was further generalised to include electrical and dyonic black holes in \[5\].

### 3.4 Regularity

In this section, we discuss the approach of \[94\], where the radial equations are obtained from an effective one dimensional action. Regularity of the metric and moduli fields on the horizon gives rise to the \(AdS_2 \times S^2\) near horizon geometry and an extremization condition on the effective black hole potential.

A general static, spherically symmetric, non-extremal black hole solution in a Einstein-Maxwell-Dilaton theory is specified by the ansatz \[94\],

\[
ds^2 = -e^{2U} dt^2 + e^{-2U} \left( \frac{c^4 d\rho^2}{\sinh^4 c\rho} + \frac{c^2}{\sinh^2 c\rho} d\Omega_2^2 \right),
\]

\[ (3.4.1) \]

where \(c\) is the extremality parameter defined by \(c^2 = 2ST\), with \(S\) being the entropy and \(T\) being the temperature of the black hole. The extremal limit corresponds to \(c \to 0\) and we get,

\[
ds^2 = -e^{2U} dt^2 + e^{-2U} \left( \frac{d\rho^2}{\rho^4} + \frac{1}{\rho^2} d\Omega_2^2 \right).
\]

\[ (3.4.2) \]

In order to have a regular area for the horizon we require the condition,

\[
e^{-2U} \to \frac{A}{4\pi \rho^2},
\]

\[ (3.4.3) \]

as \(\rho \to -\infty\). In this limit, the metric becomes the direct product form, \(AdS_2 \times S^2\) after the change of variables to \(r = -\frac{1}{\rho}\),

\[
ds^2 = -\frac{4\pi}{A} r^2 dt^2 + \frac{A}{4\pi} \left( \frac{dr^2}{r^2} + d\Omega_2^2 \right).
\]

\[ (3.4.4) \]

\(^2\)The solution for the moduli fields usually occur as ratios of the charges as is evident from \(3.3.12\).
Thus we get the Bertotti-Robinson metric which we assumed earlier to be the near horizon geometry by demanding regularity near the horizon.

Similarly we will determine a condition on the black hole potential by requiring regularity of the moduli near the horizon. The black hole potential is defined as,

$$V(p, q, z, \bar{z}) = |Z|^2 + |D_iZ|^2,$$  \hspace{1cm} (3.4.5)

and the effective one dimensional Lagrangian reads as,

$$\mathcal{L}_{\text{eff}}(z(\rho), \bar{z}(\rho), U(\rho)) = \left(\frac{dU}{d\rho}\right)^2 + g_{ij} \frac{dz^i}{d\rho} \frac{dz^j}{d\rho} + e^{2U} V(p, q, z, \bar{z}),$$  \hspace{1cm} (3.4.6)

together with the constraint equation given by,

$$\left(\frac{dU}{d\rho}\right)^2 + g_{ij} \frac{dz^i}{d\rho} \frac{dz^j}{d\rho} - e^{2U} V(p, q, z, \bar{z}) = 0.$$  \hspace{1cm} (3.4.7)

The second order field equations are,

$$e^{-2U} \frac{d^2U}{d\rho^2} = V(p, q, z, \bar{z}),$$  \hspace{1cm} (3.4.8a)

$$e^{-2U} \frac{d}{d\rho} \left( g_{ij} \frac{dz^i}{d\rho} \right) = 2 \frac{d}{d\rho} (V(p, q, z, \bar{z})), \hspace{1cm} (3.4.8b)$$

where we have used the constraint equation (3.4.7) for simplification. The field equations (3.4.8) obtained from the effective one dimensional Lagrangian along with the constraints are equivalent to the Einstein equations. The scalar field equation for $z^i$ can be further simplified using,

$$\partial_k g_{ij} = \partial_i \partial_j K = \partial_i g_{kj},$$  \hspace{1cm} (3.4.9)

and using the constraint equation (3.4.7) to get,

$$\frac{d^2 z^i}{d\rho^2} = e^{2U} \partial^i V.$$  \hspace{1cm} (3.4.10)
We have already seen that near the horizon regularity requires (3.4.3). The above equation becomes,
\[
\frac{d^2 z_i}{d\rho^2} = \frac{4\pi}{A\rho^2} \partial^i V ,
\] (3.4.11)
and is solved by,
\[
z_i' = \frac{4\pi}{A} \partial^i V \ln \frac{1}{\rho} + z_c' .
\] (3.4.12)
In this coordinate system, the horizon of the black hole is located at \( \rho = -\infty \). We see that the scalars have a regular behaviour near the horizon only if \( \partial^i V = 0 \). This also implies that the scalars become constants at the horizon. Thus, demanding regularity near the horizon reduces the scalar field equations to an extremization condition on the black hole potential:
\[
\frac{\partial V(z, \bar{z}, q, p)}{\partial z^l} = 0 .
\] (3.4.13)
Solving the above equation relates the moduli in terms of the charges \( z_i' = z_i'(q, p) \). The equation for \( U \) evaluated at the horizon gives the entropy as,
\[
\frac{A}{4\pi} = V(q, p, z_i'(q, p), \bar{z}_i'(q, p)) ,
\] (3.4.14)
which is the value of the black hole potential evaluated at the critical points. The extremization of the black hole potential is compatible with the condition (3.3.11) obtained in the supersymmetric case. To see this we use the identities [107],
\[
\bar{D}_i Z = 0 , \quad D_i \bar{D}_j \bar{Z} = g^{ij} \bar{Z} , \quad D_i D_j Z = i c_{ijk} g^{kk'} \bar{D}_{kj} \bar{Z} ,
\] (3.4.15)
where \( c_{ijk} \) is symmetric in all indices and satisfies \( \bar{D}_l c_{ijk} = 0 \). Using the above it can be shown that,
\[
\partial_i V = \partial_i (|Z|^2 + |D_i Z|^2) = 2Z D_i Z + i c_{ijk} g^{ip} g^{kk'} \bar{D}_i \bar{Z} \bar{D}_k Z .
\] (3.4.16)
Thus \( D_i Z = 0 = \bar{D}_i \bar{Z} \) implies the condition \( \partial_i V = 0 \). Note that we have not used any
supersymmetry in this discussion. Also note that \( \partial_i V = 0 \) does not always imply \( D_i Z = 0 \), which is valid only for supersymmetric attractors. This suggests that the procedure of extremization of an effective potential is generic to capture attractors in non-supersymmetric theories as well. We explore some aspects of non-supersymmetric attractors and their stability conditions in the next section.

3.5 Non-supersymmetric attractors and stability

In the previous section, we studied some features of non-supersymmetric attractors in \( \mathcal{N} = 2 \) supergravity theory. However, the attractor mechanism is much more general and all it requires is an extremal black hole with Minkowski asymptotics in any theory of gravity with generic matter content. In this section, we take cue from the previous discussion on the effective potential approach and review the non-supersymmetric attractors and their stability conditions [95]. We consider generic four dimensional Einstein-Maxwell-dilatonic theories with abelian gauge fields given by the Lagrangian,

\[
\mathcal{L} = R - 2\partial_\mu \phi \partial_\nu \phi - a_{IJ}(\phi) F^I_{\mu\nu} F^J_{\mu\nu},
\]

where \( I \) refers to the number of \( U(1) \) gauge fields. The function \( a_{IJ} \) is similar to the period matrix \( N_{\Lambda \Sigma} \) and we consider the dilatonic couplings to be \( a_{IJ} = e^{\beta_i \phi} \delta_{IJ} \). We consider the magnetically charged black holes of Reissner-Nordstrom type for this discussion. The black hole ansatz is of the form,

\[
ds^2 = -a(r)^2 dt^2 + \frac{dr^2}{a(r)^2} + b(r)^2 d\Omega_2^2.
\]

with the magnetic field strength,

\[
F^l_{\theta \phi} = p^l \sin \theta,
\]
where $p^I$ are the magnetic charges. The independent components of the Einstein field equations read,

$$
\partial^2 r (a^2 b^2) - 2 = 0,
$$
$$
\partial^2 r b + b(\partial_r \phi)^2 = 0,
$$
(3.5.4)

together with a constraint,

$$
a^2 b^2 (\partial_r \phi)^2 + V_{\text{eff}}(\phi) = b^2 (1 - a^2 (\partial_r b)^2 - \frac{1}{2} (\partial_r a)^2 (\partial_r b)^2),
$$
(3.5.5)

where $V_{\text{eff}}(\phi, p) = a^I j(\phi) p^I p^J$ is the effective potential. The scalar field equations are given by,

$$
2b^2 \partial_r (a^2 b^2 \partial_r \phi) - \frac{\partial V_{\text{eff}}}{\partial \phi} = 0.
$$
(3.5.6)

As expected from our discussion in the previous section, all the field equations can be derived from an effective one dimensional Lagrangian,

$$
L_{\text{eff}} = \partial_r b \partial_r (a^2 b) - a^2 b^2 (\partial_r \phi)^2 - \frac{V_{\text{eff}}(\phi)}{b^2},
$$
(3.5.7)

together imposing the constraint (3.5.5). For the double extreme Reissner-Nordstrom black hole $a(r) = (1 - \frac{r}{r_h})$ and $b(r) = r$ and we can see that the field equations near the horizon give,

$$
\left. \frac{\partial V_{\text{eff}}}{\partial \phi} \right|_{\phi_c} = 0, \quad V_{\text{eff}}(\phi_c, p) = r_h^2.
$$
(3.5.8)

The entropy of the black hole is then given by,

$$
S = \frac{A}{4\pi} = r_h^2 = V_{\text{eff}}(\phi_c, p),
$$
(3.5.9)

which agrees with the discussions in previous sections.

To discuss the stability condition, we consider for simplicity two gauge fields such that
the effective potential becomes,

\[ V_{\text{eff}} = e^{\beta_1 \phi} p_1^2 + e^{\beta_2 \phi} p_2^2, \quad (3.5.10) \]

The condition \( \frac{\partial V_{\text{eff}}}{\partial \phi} = 0 \) determines the critical point \( \phi_c \) at the horizon,

\[ \phi_c = \frac{1}{\beta_1 - \beta_2} \ln \left( -\frac{\beta_2 p_2^2}{\beta_1 p_1^2} \right), \quad (3.5.11) \]

which makes sense only if one of the \( \beta_i \) are negative. Now, consider small perturbations of the scalar field values \( \delta \phi = \phi_c + \delta \phi \) about the critical points. For this discussion, we will ignore the back reaction of the scalar field on the attractor geometry. The scalar field equations for the perturbations take the form,

\[ 2 r^2 \partial_r \left( (r - r_h)^2 \partial_r \delta \phi \right) - \frac{\partial^2 V_{\text{eff}}}{\partial \phi^2} \bigg|_{\phi_c} \delta \phi = 0, \quad (3.5.12) \]

where we have expanded the effective potential about the critical point. For the simple model we consider the double derivative evaluated at the critical point is,

\[ \frac{\partial^2 V_{\text{eff}}}{\partial \phi^2} \bigg|_{\phi_c} = -\beta_1 \beta_2. \quad (3.5.13) \]

Substituting the above, the fluctuation equations become,

\[ (r - r_h)^2 \partial_r^2 \delta \phi + 2(r - r_h) \partial_r \delta \phi + \frac{\beta_1 \beta_2}{2 r^2} \delta \phi = 0, \quad (3.5.14) \]

The solutions for the fluctuations are easily determined as,

\[ \delta \phi = C \left( \frac{r - r_h}{r} \right)^{\frac{1}{2} \left( 1 - \beta_1 \beta_2 / (r_h - 1) \right)} . \quad (3.5.15) \]
We see that there is a regular solution which vanishes as one approaches the horizon,

\[
\delta \phi = C_+ \left( \frac{r - r_h}{r} \right)^{\frac{1}{2}} \left( \sqrt{1 - 2 \beta_1 \beta_2 / r_h} - 1 \right),
\]

(3.5.16)

and becomes constant asymptotically provided \(\beta_1 \beta_2 < 0\). Thus the existence of a constant solution at infinity allows one to vary the scalar values by changing the constant \(C_+\). While at the horizon, the fluctuations vanish and scalar values are attracted to a fixed value \(\phi_c\). Note that under the requirement \(\beta_1 \beta_2 < 0\), the double derivative of the effective potential (3.5.13) is positive which implies that the attractor geometry corresponds to an absolute minimum of the effective potential.

Using these conditions [95], have shown by perturbative analysis including backreaction that the near horizon attractor solution is stable under scalar perturbations about the attractor values. In chapter 7, we do the scalar perturbation analysis for black brane solutions in gauged supergravities and determine analogous conditions for stability.

### 3.6 Summary

In this chapter, we studied the attractor mechanism in supergravity theories. We saw that requiring maximal supersymmetry near the horizon led to an extremization condition on the central charge. The moduli values at the horizon are completely determined in terms of the charges carried by the black hole. The BPS nature of the extremal solution required the ADM mass of the black hole to be same as the central charge, which then determined the extremal black hole entropy in terms of black hole charges. Later we saw that regularity near the horizon is sufficient to determine the moduli in terms of the charges and that the effective potential approach agrees with the conditions obtained from supersymmetry. We then discussed a simple magnetically charged extremal black hole solution in a non-supersymmetric Einstein-Maxwell-Dilaton theory and the conditions for a stable attractor solution.