1.1 Introduction

Occurrence of failures in everything is an inevitable phenomenon. For example: a washing machine fails, a car battery goes dead, a floppy disk drive goes bad, a T.V. remote control quits functioning, an electric mixer, motor, refrigerator, automobile etc. fail. Causes of failures include wear out after “normal” life, bad design, faulty construction or manufacturing processes, human error, poor maintenance, inadequate testing and inspection, improper use and lack of protection against excessive environmental stress.

One may feel to be successful but it also tends to prove failure ultimately. What do we do? Feel utterly desperate or fight it with renewed vigour, what do we get? if not absolute success, a certain minimisation of failures. Minimisation of failures and improvement in the operational use of the system and increase in the available operating time can be achieved by reliability and maintainability.

Reliability of a unit (or a product) is the probability that it will give satisfactory performance for a specified period under specified operating conditions. The process of increasing reliability, no doubt, involves cost but it saves money and prevents possible losses.

Growth of reliability has been motivated by various factors including the increased complexity and sophistication of systems, public awareness of and insistence on product quality, new laws and regulations concerning product liability, government contractual requirements to meet reliability performance specifications and profit considerations resulting from the high cost of failures, their repairs and warranty programs.

Life testing, electronics and missiles reliability problems started to receive a great deal of attention both from statisticians and engineers in early 1950. The airlines set up an organization called Aeronautical Radio, Inc. (ARINC), which, among other functions, collected and analysed defective tubes. ARINC achieved notable success in improving the reliability of a number of tube types. Davis, DJ (1952) discussed an analysis of some failure data.
Epstein, B and Sobel, M (1954) published Life testing which laid the foundation of classical reliability analysis. Epstein and Sobel (1955), Epstein (1958) worked in the field of sequential life tests in exponential case. After these papers, the exponential failure distribution acquired a unique position in life testing and reliability analysis.

One unit system under different failures and repair possibility has been extensively studied in the field of reliability by a large number of researchers under various assumptions. Gopalan and Muralidhar (1991) have discussed Cost analysis of a one unit repairable system subject to on-line preventive maintenance and/or repair. Tuteja and Taneja (1993) analysed Profit analysis of a one-server one-unit system with partial failure and subject to random inspection. Tuteja et al (2001) have analysed cost benefit analysis of a system where operation and sometimes repair of main-unit depends on sub-unit. Taneja et al (2001) have discussed Reliability and profit analysis of a system with an ordinary and an expert repairman wherein the latter may not always be available. Taneja et al (2004) have analysed Profit evaluation of a system wherein instructions imply perfect repair. Taneja et al (2004) discussed Profit analysis of a single unit programmable logic controller (PLC). S.M Rizwan et al (2005) have given the concept of accident during inspection.

Said, E.L., M. Kh. and M.S. El-Sherbeny, (2005) have analysed Profit analysis of a two unit cold standby system with preventive maintenance and random change in units. but they have not consider the post repair and post inspection.

Keeping this in view, the present problem aims at studying single-unit system with post inspection, post repair, preventive maintenance replacement. A single repair facility is used to repair and post repair the failed unit. After the repair, the unit is sent for inspection to decide whether the repair is satisfactory or not.

In case the repair is found unsatisfactory then unit is again sent for post inspection and post repair. The post repair is needed only when the repair of the failed unit is found unsatisfactory on inspection.
In next approach, a good number of studies have been carried out by earlier researchers. Gupta, P.P and Sharma, R.K. (1986) has compute reliability behaviour of power plant by Boolean technique and investigated the behaviour of reliability for several point of view including the applicative approaches of reliability theory. They have computed the reliability of simple complex systems by Boolean function technique but a very little work has been evaluating the reliability of complicated system which is controlled by three terminals by Boolean function Expansion Algorithm. This work has been carried out for the reliability calculation of power generator for three subsystem \( G_i \) \( (i=1,2,3) \) and corresponding terminals \( T_i \) \( (i=1,2,3) \) connected in parallel redundant (1-out of 3-G and 3-T) along with three Main switch Board of power supplier they are connected with auxiliary switch board of power supplier and finally they all diverges on master switch board.

In next investigation, we have work on inventory production system. It is obvious observation from globalised market that many managers generally optimised the significant portion of a company’s assets, inventories for used to serve a variety of functioning like coordinating operation, smoothing production, achieving economics of scale and improving customer service. Das. D, Roy. A, and Kar .S (2010) have studied a production-inventory model for a deteriorating item incorporating learning effect using genetic algorithm and Sana S.S. (2010) has developed a production-inventory model in an imperfect production process. In this work the emphasis to be given on determination of the most cost effective production quantity under stable conditions which is commonly known as the classical economic production quantity (EPQ) model with an instantaneous or non-instantaneous receipt stock problem. This production model is the modified version of General Economic Ordered quantity (EOQ) model.

Bridging the above mentioned gap in the field of reliability, the present study has investigated the reliability and profit analysis of systems.
Reliability of a device in time ‘t’ is the probability that it will not fail in a given environment before time t.

The problem of assuring and maintaining reliability has many facts, including original equipment design, control of quality during production, acceptance inspection, field trials, life testing and design modifications. Reliability competes directly or indirectly with a host of other engineering considerations chiefly cost, complexity, size and weight and maintainability.

1.2 Different Components for measuring the system performance:

1.2.1 Reliability

Quantitatively, reliability of a device in time ‘t’ is the probability that it will not fail in a given environment before time t. If T is a random variable representing the time till the failure of the device starting with an initial operable condition at t = 0, then reliability R(t) of device is given by

\[ R(t) = P[T > t] = 1 - P[T \leq t] = 1 - F(t) \]

Thus, reliability is always a function of time. It also depends on environmental conditions which may or may not vary with time. Following assumptions are made:

(i) \( R(0) = 1 \) since the device is assumed to be operable at t = 0.
(ii) \( R(\infty) = 0 \) since no device can work forever without failure.
(iii) \( R(t) \) is non-increasing function between 0 and 1.

1.2.2 Availability

When a system is often unavailable due to break downs the concerning department becomes interested to put it back into operation after each break down with proper repairs. In fact, it is concerned with availability equally as it does with reliability because of additional costs and inconvenience incurred when the system is not available. The differences between the measures reliability and availability are as follows: (i) The reliability is an interval function while the availability is a point function describing the behaviour of the system at a specified epoch.
(ii) The reliability function precludes the failure of the system during the interval under consideration, while availability function does not impose any such restriction on the behaviour of the system.

We may categorize availability as:

(i) **Instantaneous (Point wise) Availability**

This is the probability that the system will be able to operate within the tolerances at a given instant of time.

Let \( X(t) = 1 \), if the system is operable at time \( t \), and \( X(t) = 0 \), when it is not operable. The availability \( A(t) \) of the system at time \( t \) is given by

\[
A(t) = P[X(t) = 1 | X(0) = 1]
\]

(ii) **Average (Interval) Availability**

It is the expected fraction of a given interval of time that the system will be able to operate within tolerances.

Suppose the given interval of time is \((0, T)\). Then interval availability \( H(0, T) = A(T) \) for this interval is given by

\[
A(T) = \frac{1}{T} \int_{0}^{T} A(t) \, dt
\]

(iii) **Steady State (Limiting Interval) Availability OR Asymptotic**

It is the expected fraction of time in the long run that the system operates satisfactory. To obtain steady state availability we simply compute

\[
\lim_{T \to \infty} H(0, T) = \lim_{T \to \infty} A(T)
\]

**1.2.3 Mean Time to System Failure (MTSF)**

The average duration between successive system failures, i.e. MTSF is defined as the expected time for which the system is in operation before it completely fails.
Suppose the reliability function for a system is given by \( R(t) = 1 - F(t) \), where \( F(t) \) is the failure time distribution function and \( f(t) = \frac{dF(t)}{dt} \) is the failure time density function. The mean time to system failure is given by

\[
\text{MTSF} = \int_0^\infty t f(t) \, dt
\]

\[
= -\int_0^\infty t \left( \frac{dR(t)}{dt} \right) \, dt
\]

\[
= \left[ -t R(t) \right]_0^\infty + \int_0^\infty R(t) \, dt
\]

\[
= \int_0^\infty R(t) \, dt = \lim_{s \to 0} R^*(s).
\]

Let \( \phi_0(t) \) be the cumulative distribution function of the first passage time from initial state to a failed state, then

\[
R^*(s) = \frac{1 - \phi_0^*(s)}{s}
\]

Thus, we have

\[
\text{MTSF} = \lim_{s \to 0} \frac{1 - \phi_0^*(s)}{s}
\]

### 1.2.4 Mean Sojourn Time in a State

The expected time taken by the system in a particular state before transiting to any other state is known as mean sojourn time or mean survival time in that state. If \( T_i \) be the sojourn time in state \( i \), then mean sojourn time in state \( i \) is

\[
\mu_i = \int_0^\infty P(T_i > t) \, dt
\]
1.2.5 Maintainability

Maintainability is an indices associated with a system under repair. It is the probability that the system will be restored to operational effectiveness within a specified time when the maintenance action is taken in accordance with prescribed conditions. Maintenance is one of the effective ways of increasing the reliability of a system. Maintenance of a system is of two types:

(i) Preventive maintenance (PM)
(ii) Corrective maintenance (CM)

PM includes actions such as lubrications, replacement of a nut or a screw or some part of the system, refueling, cleaning, etc., while CM involves minor repairs that may crop up between inspections.

On failure of a unit, it is sent to a repair facility if available, otherwise it queues up for repair.

1.2.6 Instantaneous Hazard Rate (or Failure Rate)

It is defined as the conditional probability that the system fails during the time interval \((t, t + \delta t)\) given that it was operating during \((0, t)\). Let \(r(t)\) \(\delta t\) = probability that the device has life time between \(t\) and \(t + \delta t\), given that it has functioned up to time \(t\).

\[
= \Pr[t < T \leq t + \delta t | T > t]
\]

\[
= \frac{P[t < T \leq t + \delta t]}{P[T > t]} = \frac{P[T < t + \delta t] - P[T < t]}{P[T > t]}
\]

\[
= \frac{[1 - R(t + \delta t)] - [1 - R(t)]}{R(t)} = - \frac{R(t + \delta t) - R(t)}{R(t)}
\]

Now, the instantaneous failure rate or hazard rate \(r(t)\) at time \(t\) is defined as

\[
r(t) = \lim_{\delta t \to 0} - \frac{R(t + \delta t) - R(t)}{R(t)\delta t} = - \frac{R'(t)}{R(t)} = \frac{f(t)}{R(t)}
\]

where \(f(t)\) is the p.d.f. of the device life time.
It can be seen that

\[
\bar{F}(t) = \int_0^\infty f(u) \, du = R(t) = \exp \left[ -\int_0^t r(u) \, du \right] \quad \text{and} \quad f(t) = r(t) \exp \left[ -\int_0^t r(u) \, du \right]
\]

1.2.7 System Configurations

By a system, we mean an arbitrary device having several units/subsystems/components assuming that their reliabilities are known which help to predict the reliability of the whole system. It is now important that the system structures be known. Various system structures have been considered as follows:

1.2.7 (i) Series Configuration

A system having \( n \)-units is said to have series configuration if the failure of an arbitrary unit (say \( i \)-th unit) causes the entire system failure. The examples of the series configurations are:

The aircraft electronic system consists of mainly a sensor subsystem, a guidance subsystem, computer subsystem and the fire control subsystem. This system can only operate successfully if all these operate simultaneously.

Deepawali or Christmas glow bulbs where if one bulb fails the whole lead fails. The block diagram of a series system configuration is shown as follows:

Let \( R_i(t) \) be the reliability of \( i \)-th components, then the system reliability is given by

\[
R(t) = \Pr (T > t) = \Pr (\min [T_1, T_2, T_3, \ldots, T_n] > t)
\]
\[
= \prod_{i=1}^{n} P[T_i > t] = \prod_{i=1}^{n} R_i(t)
\]

where \(T_i\) is the life time of the ith unit of the system. The system hazard rate, therefore, is

\[
r(t) = \sum_{i=1}^{n} r_i(t)
\]

where \(r_i(t)\) is the instantaneous failure rate of the ith unit.

1.2.7 (ii) Parallel Configuration

In this configuration, all the units in a system are connected in parallel i.e. the failure of the system occurs only when all the units of the system fail. For example, four engined aircraft which is still able to fly with only two engines working. Block diagram representing a parallel configuration is shown in Fig. 1.2.

Suppose \(R_i(t)\) and \(T_i\) be the reliability of ith component and the life time of the ith unit in time \(t\) respectively, then the system reliability is given by

\[
R(t) = Pr(T > t) = Pr[\max(T_1, T_2, T_3, \ldots, T_n) > t]
\]
= 1−P (T_1 ≤ t, T_2 ≤ t, T_3 ≤ t,…, T_n≤ t)

if the units function independently, then

\[ R(t) = 1−[1−R_1(t)][1−R_2(t)][1−R_3(t)]…[1−R_n(t)] \]

\[ = 1 − \prod_{i=1}^{n} [1 − R_i(t)] \]

1.2.7 (iii) k-out of n redundancy

A generalization of n parallel components occurs when a requirement exists for k out of n identical and independent components to function for the system to function. Obviously k ≤ n. If k=1, complete redundancy occurs, and if k=n, the n components are, in effect, in series. The reliability can be obtained from the binomial probability distribution.

Therefore

\[ R_s(t) = \sum_{x=k}^{n} P(x) \]

Is the probability of k or more successes from among the n components.

Where \( p(x)= \binom{n}{x} R^x(1-R)^{n-x} \)

1.2.7 (iv) Standby Redundant Configuration

Redundancy is a device to improve reliability of a system. In redundant system, more units are made available than which are necessary. There are two types of redundancy:

(a) Active Redundancy

(b) Passive redundancy
(a) **Active Redundancy**

In this case of redundancy, the system has a positive probability of failure even when it is not in operation. This may happen due to the effect of temperature, environment condition etc.

Active redundancy can further be classified as hot redundancy and warm redundancy:

(i) If the off-line unit can fail and is loaded in exactly the same way as the operating unit, it is called hot standby unit.

(ii) If the off-line unit can fail and can diminish the load, it is called warm standby unit. The probability of failure for a warm standby is less than that of failure for operative unit.

(b) **Passive or Cold Standby Redundancy**

This is that form of redundancy in which the off-line unit cannot fail and is completely unloaded.

![Fig.1.3. Standby redundant configuration](image-url)
Reliability $R(t)$ of an $n$-unit standby system at any time instant $t$ is given by

$$R(t) = P\left[\sum_{i=1}^{n} T_i > t \right]$$

where $T_i$ is the life time of $i$th unit and all the $n$-units are independent.

A standby system functions as long as one of the units is available for the task on hand. A block diagram of such system is shown as in Fig. 1.3.

### 1.3 Stochastic Processes used in the analysis of redundant system

A stochastic process is a family of random variables indexed by a parameter set realising values on another set known as the state space. Both the parametric set and the state space can be either discrete or continuous.

In a stochastic process $\{X(t), t \in T\}$, where $X(t)$, $t$ and $T$ respectively are the state space, parameter (generally taken to be time) and the index set. If $T$ is countable set such as $T = \{0, 1, 2, 3, \ldots\}$, then the stochastic process is said to be a discrete parameter process and if $T = \{t : -\infty < t < \infty\}$ or $T = \{t : t \geq 0\}$, the stochastic process is said to be continuous parametric process. The state space is classified as discrete if it is countable and continuous if it consists of an interval on the real line. In the present study, we have only dealt with discrete state space continuous time parameter stochastic process.

#### 1.3.1 Markov Process

A stochastic process is said to be Markov Process if the future development is completely determined by the present state and is independent of the way in which the present state has developed. If $\{X(t), t \in T\}$ is a stochastic process such that, given the value of $X(s)$, the value of $X(t)$, $t > s$ do not depend on the values of $X(u)$, $u < s$, i.e. for $t > s$, $i \in s$

$$\Pr[X(t) = i | X(u), 0 \leq u \leq s] = \Pr[X(t) = i | X(s)]$$

Then the process $\{X(t), t \in T\}$ is a Markov process.
stochastic Processes which do not possess the Markovian property are said to be non-Markovian.

1.3.2 Markov Chain

A Markov Process with discrete state space is said to be a Markov Chain. Mathematically, a stochastic process \{X_n; n = 0, 1, 2, \ldots\} is called a Markov Chain if, for \( j, k, j_1, j_2, \ldots, j_{n-1} \in \mathbb{N} \)

\[
\Pr[X_n = k | X_{n-1} = j, X_{n-2} = j_1, \ldots, X_0 = j_{n-1}] = \Pr[X_n = k | X_{n-1} = j] = p_{jk} \text{ (say)}
\]

If the transition probabilities \( p_{ij} \) are independent of \( n \), the Markov chain is said to be homogeneous and if it is dependent on \( n \) the chain is said to be non-homogeneous.

1.3.3 Renewal Process

Suppose we have repairable system which starts operation at \( t = 0 \). If \( X_1 \) denotes the time to first failure and \( Y_1 \) denotes the time from first failure to next system operation (after repair) then \( t_1 = X_1 + Y_1 \) denotes the time of first renewal. Similarly, if \( X_2 \) denotes the time to first renewal to second failure and \( Y_2 \) denotes the time from second failure to second renewal then \( t_2 = X_2 + Y_2 \) and the time of second renewal is \( t_1 + t_2 \). In general, \( t_i = X_i + Y_i \) (inter-arrival) time between the \((i-1)\)th and \( i\)th renewal) for \( i = 1, 2, 3, \ldots \) If we define

\[
S_0 = 0, S_n = t_1 + t_2 + \ldots + t_n
\]

= epoch of \( n \)th renewal,

and \( N(t) = \) number of renewals during \((0, t]\)

then the process \{\( N(t), t > 0 \}\) is called renewal process.
1.3.4 Markov Renewal Process

Let the states of a process be denoted by the set \( E = \{0, 1, 2, \ldots \} \), and let the transition of the process occur at epochs \( t_0 (= 0), t_1, t_2, \ldots, t_n \) \((t_n < t_{n+1})\). If

\[
\Pr\{X_{n+1} = k, t_{n+1} - t_n \leq t | X_0 = i_0, \ldots, X_n = i_n : t_0, t_1, \ldots t_n\} = \Pr(X_{n+1} = k, t_{n+1} - t_n \leq t | X_n = i_n)
\]

then \( \{X_n, t_n\}, n = 0, 1, 2, \ldots \), constitutes a Markov Renewal Process with state space \( E \).

1.3.5 Semi-Markov Process

In the above, if we assume that the process is time homogeneous, i.e. \( \Pr\{X_{n+1} = j, t_{n+1} - t_n \geq | X_n = i\} = Q_{ij}(t), i, j \in s \) is independent of \( n \), then there exist limiting transition probabilities

\[
P_{ij} = \lim_{t \to \infty} Q_{ij}(t) = \Pr(X_{n+1} = j | X_n = i)
\]

Then \( \{X_n, n = 0, 1, 2, \ldots \} \) constitutes a Markov chain with state space \( E \) and transition probability matrix (t.p.m) is given by

\[
P = [p_{ij}]
\]

The continuous parameter stochastic process \( Y(t) \) with state space \( E \) defined by

\[
Y(t) = X_n, t_n < t < t_{n+1}
\]

is called a semi-Markov process.

In other words, we define the semi-Markov process is a process in which transition from one state to another is governed by the transition probabilities of a Markov process but the time spent in each state before a transition occurs is random variable depending upon the last transition made. Thus at transition instants the semi-Markov behaves just like a Markov process. However, the times at which transitions occur are governed by a different probability mechanism.
1.3.6 Regenerative Process

Regenerative stochastic process was defined by Smith (1955) and has been crucial in the analysis of complex systems. In this, we take a time point at which the system history prior to the time point is irrelevant to the system conditions. These points are called regeneration points. Let $X(t)$ be the state of the system at epoch $t$. If $t_1, t_2, \ldots$ are the epochs at which the process probabilistically restarts, then these epochs are called regenerative epochs and the process $\{X(t), t = t_1, t_2, \ldots\}$ is called regenerative process.

1.4 Transforms and Convolutions

1.4.1 Laplace Transform

Let $f(t)$ be a function of a positive real variable $t$. Then the Laplace transform (L.T.) of $f(t)$ is defined as

$$L[f(t)] = \mathcal{F}[f(t)] = \int_0^\infty e^{-st} f(t) \, dt$$

For the range of value of $s$ for which the integral exists. Here $\mathcal{F}(t)$ is called an inverse Laplace transform of $\mathcal{F}(s)$ and we write $f(t) = L^{-1}\{\mathcal{F}(s)\}$. The following are some important properties of Laplace transform:

1. $L\left[\sum_{i=1}^n c_i f_i(t)\right] = \sum_{i=1}^n c_i \mathcal{F}_i(s)$
2. $L[t^n f(t)] = (-1)^n \frac{d^n \mathcal{F}(s)}{ds^n}$
3. $L\left[\int_0^t f(u)du\right] = \mathcal{L}[F(t)] = \frac{\mathcal{F}(s)}{s}$
4. $\lim_{t \to 0} f(t) = \lim_{s \to \infty} s\mathcal{F}(s)$ (initial value theorem)
5. $\lim_{t \to \infty} F(t) = \lim_{s \to 0} s\mathcal{F}(s)$ (final value problem)
6. $\lim_{s \to 0} \mathcal{F}(s) = 1$ if $\mathcal{F}(s)$ is L.T. of a p.d.f.
1.4.2 Laplace Stieltjes Transform

Let $X$ be a non-negative random variable with distribution function

$$F(x) = \Pr[ X \leq x ]$$

then Laplace Stieltjes transform (L.S.T.) of $F(x)$ is defined, for $s > 0$ by

$$F^{**}(s) = \int_0^\infty e^{-sx} dF(x)$$

Therefore, we have

$$F^{**}(s) = \int_0^\infty e^{-sx} f(x) \, dx = f^*(s)$$

where $f(x) = \frac{dF(x)}{dx}$

1.4.3 Convolution

Let $f(t)$ and $g(t)$ be two real valued non-negative continuous functions of $t$, then the integral

$$\int_0^t f(t-u) g(u) du = \int_0^t g(t-u)f(u) du$$

$$= f(t) \otimes g(t) = L^{-1} [ f^*(s).g^*(s)]$$

is called Laplace convolution of the functions $f(t)$ and $g(t)$.

If $F(t)$ and $G(t)$ be two real valued distribution functions defined for $t \geq 0$ the resulting convolution is again a distribution function and the integral

$$\int_0^t F(t-u) dG(u) = \int_0^t G(t-u) dF(u) = F(t) \boxplus G(t)$$

is known as Stieltjes convolution of $F(t)$ and $G(t)$.
1.5 First Passage Time

Suppose that a system starts with the state j, then time taken to reach a given state k for the first time from state j is called first passage time. In general, first passage time is a measure of how long it takes to reach a given state from another state.

1.6 Repairable Systems

If, on failure, a unit is replaced by a new one, then the reliability of the system increases. In a good number of cases this will turn out to be expensive and it will be necessary to repair the failed units. Thus on the failure of a unit, it is sent to a repair facility. If no repair facility is free, then the failed units queue up for repair and the repairs are normally undertaken in First in First out (FIFO) order. We assume that the lifetime of an online unit, standby unit and the repair time of a failed unit are all independent random variables and that the distribution functions of these random variables are known and that they admit the probability density functions.

1.7 Regenerative Process

A time point at which the system history prior to the time point is irrelevant to the system conditions is called a regeneration point. Let \( X(t) \) be the state of the system at epoch \( t \). If \( t_1, t_2, \ldots \) are the epochs at which the process probabilistically restarts, then these epochs are called regenerative epochs and the process \( \{X(t), t = t_1, t_2, \ldots\} \) is called regenerative process. Regenerative stochastic processes were introduced by Smith (1955).

1.8 Profit Analysis

Availability of the system leads to revenue whereas the busy period of the repairman, expected number of visits by the repairman, expected number of replacements, etc. lead to the cost of maintenance and spares. The revenue and cost function lead to the profit function of a firm, as the profit is excess of revenue over the cost of production. The profit function takes the form

\[ P(t) = \text{Expected revenue in } (0, t) - \text{expected total cost in } (0, t) \]
In general, the optimal policies can more easily be derived for an infinite time span as compared to a finite span. The profit per unit time is expressed as

\[ \lim_{t \to \infty} \frac{P(t)}{t} \]

i.e. profit per unit time = total revenue per unit time – total cost per unit time.

For example, the profit equation may be given as

The expected total profit incurred to the system in steady-state is given by

\[ P = C_0A_0 - C_1B_0 - C_2IT_0 - C_3B_0^R - C_4P_0 - C_5B_0^R - C_6R_0 \]

Where

\[ P = \text{Profit per unit up time of the model} \]

\[ C_0 = \text{Revenue per unit up time of the system} \]

\[ C_1 = \text{Cost per unit time for which the repairman is busy in repair} \]

\[ C_2 = \text{Cost per unit time for which the repairman is busy in inspection} \]

\[ C_3 = \text{Cost per visit of the repairman} \]

\[ C_4 = \text{Cost per preventive maintenance} \]

\[ C_5 = \text{Cost per unit time for replacement} \]

\[ C_6 = \text{Cost per visit of the repairman for replacement} \]

\[ A_0 = \text{Total fraction of time for which the system is up} \]

\[ B_0 = \text{Total fraction of time for which the repairman is busy} \]

\[ IT_0 = \text{Total fraction of time for which the repairman is busy in inspection} \]

\[ PM_0 = \text{Total fraction of time for which the repairman is busy in preventive maintenance} \]

\[ B_0^R = \text{Total fraction of time for which repairman is busy in replacement} \]
$RP_0 = \text{Expected number of replacements}$

**1.9 Distribution Used**

In the present work, the failure time distribution is assumed to be an exponential distribution. The family of exponential distribution is the best known and most thoroughly explored, largely through the work of Epstein (1958) and his associates. Exponential distribution plays an important role in reliability studies. Besides a number of desirable mathematical properties, it has a very important memoryless property i.e. if the life length $T$ of a structure has the exponential distribution, previous use does not effect its future life length.

Exponential distribution is defined as follows:

A continuous random variable having the range $0 \leq t < \infty$ is said to have an exponential distribution if it has the probability density function of the form

$$f(t) = \begin{cases} \lambda e^{-\lambda t}, & 0 < t < \infty \\ 0, & t < 0 \end{cases}$$

where $\lambda$ is a positive constant. The corresponding distribution function is

$$F(t) = \begin{cases} 1-e^{-\lambda t}, & 0 \leq t < \infty \\ 0, & t < 0 \end{cases}$$

**1.10 Thesis at a Glance**

The present thesis entitled “Reliability Modelling of Some Technological System” is an attempt to investigate various type of systems e.g. one-unit system, two-unit cold standby system, complicated power system, inventory Production System etc., introducing a new concept.

The thesis comprises seven chapters which are summarised as follows:
1.10.1 Chapter 1

It is an introductory in nature which includes origin, history and definition of reliability and also explains various terms related to our work discussed in subsequent chapter of the thesis.

1.10.2 Chapter 2

The present problem aims at studying single-unit system with post inspection, post repair and preventive maintenance. A single repair facility is used to repair and post repair the failed unit. After the repair, the unit is sent for inspection to decide whether the repair is satisfactory. In case the repair is found unsatisfactory then unit is again sent for post inspection and post repair. The post repair is needed only when the repair of the failed unit is found unsatisfactory on inspection. Expressions for reliability measures are obtained by using semi-Markov processes and regenerative point technique.

1.10.3 Chapter 3

In the global and competitive environment the System Maintenance is not an easy task. The system is designed for optimal mix i.e. maximum impact at minimum cost and it increases the maintainability of the machines. In this study, optimizing the cost of reliability model for one-unit system having post repair, preventive maintenance and replacement has been presented. Mathematical Expressions to be work out with reliability measures along with the cost of maintenance for one-unit system of post repair with the help of regenerative point technique. In this research, maintenance is defined in two ways i.e. Preventive maintenance and Corrective maintenance. The cost function is developed as taking consideration for cost per unit time for preventive and corrective maintenance. Analytical study is to be presented with diagrammatic & graphical presentation with cut-off points for various rates/costs for optimization purpose.

1.10.4 Chapter 4

This work has been carried out for the reliability calculation of power generator for three subsystem $G_i$ (i=1,2,3) and corresponding terminals $T_i$ (i=1,2,3) connected in parallel redundant (1-out of 3-G and 3-T) along with common power supplier having
three different switch board they are connected with auxiliary power supplier switch board device and finally they all diverges on master switch board. Complex system configuration to be presented in Boolean algebra technique with development of mathematical modelling and expression of summation of axiomatic probability for an event has been explained. The logical Matrix has been also developed for reliability of the complex system with calculating Failure time, Mean time to failure (MTTF) for arbitrary distribution (Mainly exponential and Weibull). Numerical example along with graphs among Reliability, Failure Time and MTTF has been also given for better understanding of model. Little observation has been point out for the functioning of complex power system and its connectivity through main, common and auxiliary power supplier with corresponding terminal.

1.10.5 Chapter 5

This work has been carried on the Sensitivity measure for pair of unequal unit standby systems in which cost involved as per repair facility for maximizing the reliability of system. Repair facility has been provided by the operator in the form of availability or non-availability of repairman due to tight competition in the super market. The failure and repair policy has been treated as random variable which is directly or indirectly related to each other. The various reliability measures have been performed for better operation of model in which regenerative point technique, stochastic behaviour and markov process has been used. Attempt has been drawn to check the sensitivity behaviour on repair time distribution is general and failure time of a unit must be (negative) exponential distribution along with impact of cost and profit. The expected profit incurred to the system is fully depending on the failure of unit and cost involved in repair. The model has been pointed out with usual graphical analysis.

1.10.6 Chapter 6

In this research work, Economic Production Quantity (EPQ) model is considered where the production process is assumed to be not 100% perfect, i.e, a fraction of the produced items are defective. Storage capacity is consider limited as high rent in super market. Due to limited storage space shortage can occur. This phenomenon is incorporated in this model. Further, it is assumed that the defective items are sold at a reduced price and the selling price of fresh units is taken as mark-up our unit production cost. The model is formulated to maximize the average profit. Profit
function is optimized using constraint polynomial geometric programming to determine the value of optimal decision variables. The model is illustrated with a numerical example.

1.10.7 Chapter 7: Conclusions and scope

All the reliability models discussed in the thesis have been analysed by various measures of system effectiveness such as, mean time to system failure (MTSF), steady-state availability of the system, expected busy period of repairman for repair/replacement, expected discussion time/expected instruction time, expected number of visits by the repairman, expected number of replacement have been obtained. Profit incurred to the system has also been evaluated. Graphs showing MTSF/Availability/Profit with respect to various rates/costs/probabilities have been plotted for a particular case for each of the model. Various conclusions have been drawn on the basis of cut-off points and otherwise. This work gives the new idea and approaches for optimizing the reliability for machine based engineering like power distribution, inventory production in critical situation as well inspection, repair and replacement of particular unit of a component/system within the limited invested fund and gives the maximum profit under optimum efforts by machineman.