CHAPTER-6

RELIABILITY ANALYSIS OF INVENTORY PRODUCTION SYSTEM WITH SHORTAGE

6.1 Introduction

Production of inventory means level of materials and supplies on hand for use in manufacturing production. This will be varies on situation to situation like different from work-in-process inventory, which is the value of goods in the middle of the production process, and finished goods inventory, which is the value of products to customers. Decision in manufacturing, retail and some service industry businesses is how much inventory to keep on hand. Inventory is usually a business’s largest asset. The instant inventory levels are established; they become an important input to the budgeting system. Inventory decisions involve a delicate balance between three classes of costs: ordering costs, holding costs, and shortage costs. In the reality of business process lot of difficulties faced to collect the inventory after production in competitive environment. This is due to uncertainty in material supply as per demand by market. It is necessity to hold the material for keeping a high level of stock is a costly exercise. Hence it is obvious observation from globalised market that many managers generally optimised the significant portion of a company’s assets, inventories for used to serve a variety of functioning like coordinating operation, smoothing production, achieving economics of scale and improving customer service. Das. D, Roy. A, and Kar .S (2010) have studied a production-inventory model for a deteriorating item incorporating learning effect using genetic algorithm and Sana S.S.(2010) has developed a production-inventory model in an imperfect production process.

We also know the Production cost of a manufacturing system depends upon the combination of different production factors. These factors are (a) raw materials, (b) technical knowledge, (c) production procedure, (d) firm size, (e) quality of product and so forth, normally, the cost of raw materials is imprecise in nature. So far, cost of technical knowledge, that is, labour cost, has been usually assumed to be constant. However, because the firms and employees perform the same task repeatedly, they learn how to repeatedly provide a standard level of performance. Therefore, processing cost per unit product decreases in every cycle. Similarly part of the ordering cost may also decrease in every cycle. In the inventory control literature, this phenomenon is known as the learning effect. K.J. Chung, K.L. Hou, (2003) have developed an optimal production run time with imperfect production processes and allowable shortages, Computers & Operations Research and Sana S.S. (2010) has analysed a production-inventory model in an imperfect production process.
In this work the emphasis to be given on determination of the most cost effective production quantity under stable conditions which is commonly known as the classical economic production quantity (EPQ) model with an instantaneous or non-instantaneous receipt stock problem. This production model is the modified version of General Economic Ordered quantity (EOQ) model. Major assumption in the classical EPQ model are:- production process is perfect; no excess stock is carried and no backorders and lost sales are allowed. Here, the setup cost and the reliability of the production process along with the backorder replenishment time and production run period are the decision variables. Therefore, both are first transformed to a corresponding interval number and then using the boundary value problems of ordinary differential equation for constraint condition of problem however the single objective function for expected profit over the time cycle is changed to respective objective functions. Due to highly nonlinearity of the expected profit functions it is optimized using the geometric programming concept. The associated profit maximization problem is illustrated by numerical examples and also its sensitivity analysis is carried out. Geometric programming is an efficient and effective method to solve linear or mathematical programming problem with the terms in power are of functional form in the objective function and constraints. The applications of geometric programming techniques can be mostly view on material management. We know the EOQ model with demand independent unit cost and derived the optimal solution by employing geometric programming techniques to determine the selling price and order quantity for a retailer. The geometric programming technique can be analyzed by two EOQ model based on basic inventory models under total cost minimization and profit maximization. The initiative has been taken to modify the EPQ model with considering the production process is not cent percent perfect. That means possibility to produced defective items. Storage capacity is consider limited as high rent in supermarket. Due to limited storage space shortage can occur. This phenomenon is incorporated in this model. Further, it is assumed that the defective items are sold at a reduced price and the selling price of fresh units is taken as mark-up our unit production cost. The another new initiative of this paper is the introduction of reliability production function having significant impact on inventory calculation at frees unit of demand and process reliability of general power function. The model is formulated to maximize the average profit. Profit function is optimized using constraint posynomial geometric programming to determine the value of optimal decision variables. The model is illustrated with a numerical example. The basis assumption of model is pointed out in the next section.
6.2 Basic Assumptions:
1. The unit production cost is a continuous function of demand $D$ which is instantaneous as per supply.
2. Limitation on space hence shortage are occurred.
3. Production of inventory is not always perfect.
4. Defective items are sold at reduced cost.
5. Major decision variable are treated as per cycle length $T$, process reliability $r$, and set up cost $c_0$.
6. Selling price is depends on production of fresh units is make up $(m)$ of production cost
7. The total fresh units are greater than the market demand.

The overall assumption is related to this concept “The total cost of interest and depreciation per production cycle is inversely related to set-up cost and directly related to process reliability”. Assumption explained the fact that when we reduce the costs of production set-up and scrap and rework on shoddy products, substantial investment is required in improving the flexibility and reliability is required in improving the flexibility and production process. Consequently the total cost of interest and depreciation per production cycle of the modern flexible production process is much higher than that of conventional inflexible process. In reality, this relationship should be discrete but a continuous function is used as an approximation which is needed to simplify the subsequent mathematical analysis.

6.3 General Condition
1. If we consider the Selling price is $S_1$ and $S_2$ of fresh and defective units respective is make up $(m$ and $m_1)$ of production cost. Hence condition must be follows as: $S_1=mP$, $m>1$. $S_2=m_1P$, $0<m_1 \leq 1$.
2. If $r$ is the Reliability of production($R_p(t)= r; t>0$) then the total fresh units must be greater than the demand i.e. $rK>D$.
3. If the continuous demand function $(D)$ related to two shape parameters $\alpha$ & $\beta$ under unit production cost $(P)$ must follows the form $P=\alpha D^{\beta}$, where $\beta (>0)$ and $\alpha (>0)$.
4. As per assumption of model the total cost of interest and depreciation per production cycle $Y(c_0,r)$ is inversely related to set-up cost $c_0$ and directly related to process reliability $r$ according to following general power function

$$Y(c_0,r) = a c_0^{-b} r^c \quad \text{.... (6.1)}$$

Where $a$, $b$, $c_0$ are positive constant.
After using Maxima & Minima concept of differentiation and with the support of above condition (especially on production reliability and set-up cost) is written below which is the total cost of interest and depreciation per production cycle for a production process

\[ \frac{\partial Y(c_0, r)}{\partial c_0} < 0 \quad \text{and} \quad \frac{\partial Y(c_0, r)}{\partial r} > 0 \]

6. The reliability of production process is \( r \) which is incorporated with the interest and depreciation cost can be represented by unreliability \( (1-r) \) of production process i.e.

\[ Y(c_0, r) = a c_0^{b} (1-r)^{c}, \quad \text{where} \ a, b, c (\geq 0) \]

so that \( Y(c_0, r) \rightarrow \infty \) as \( r \rightarrow 1 \) to reflect the fact the process will never be 100% reliable.

Several type of incurred cost like interest and depreciation cost, set-up cost gives the desired relation between them and process reliability but all they are depends on production process and quality of product. The additional decision variable for the reliability or unreliability complies a unique closed optimal solution which is obtained by using differential calculus and the concept of boundary value problem of differential equation.

6.4 General Notations:

For the better understanding of paper here we have to summarise the notation used in entire research paper but all most all the notation already used in inventory and reliability control system.

- \( K \): Production rate for per day and it is constant
- \( D \): Demand rate per day
- \( P \): Production cost per unit time
- \( h \): Mean holding cost per unit quantity per day
- \( r \): Reliability of the production process (a decision variable)
- \( T \): Duration of each cycle
- \( T_1 \): Production Period
- \( Y(c_0, r) \): Total cost of interest and depreciation for a production process per production cycle
- \( w \): Required storage quantity per unit time
- \( W \): Available storage area
- \( S \): Maximum shortage level
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$B$ Backorder cost per unit item

$P$ Unit production cost

$c_0$ Set up cost per cycle (a decision variable)

$S_1$ Selling price of fresh units

$S_2$ Selling price of defective units

After the compilation of all the above discussed concept and method the mathematical model of boundary value problems has been formulated which is useful for formulation the mathematical programming problems especially for constraints.

6.5 Model Formulation

If we consider the manufacturing system having reliability $R_p(t)=r$ under the above illustrated conditions. The differential equation of the system is given by follows with suitable initial and boundary condition. This model itself explain the production of inventory under reliable condition of set-up of cost and the total cost of interest and depreciation for a production process per production cycle under imitated space.

Boundary Condition of Differential Equation:

$$
\frac{dq(t)}{dt} = \begin{cases} 
  rK - D & 0 < t \leq T_1 \\
  -D & T_1 < t \leq T_2 \\
  -D & T_2 < t \leq T 
\end{cases}
$$

\[\text{.... (6.2)}\]

where $q(0)=0$, $q(T_1)=q_1$, $q(T_2)=0$, $q(T)=S$.

On solving above equation, we get

\[q(t) = \begin{cases} 
 (rK - D)t & 0 < t \leq T_1 \\
 -D(T_2 - t) & T_1 < t \leq T_2 \\
 -D(T - t) + S & T_2 < t \leq T 
\end{cases}
\]

\[\text{.... (6.3)}\]

Using the relation $q(T_2)=0$, we get

$$
T = \frac{rK}{D}T_1 + \frac{S}{D}
$$

\[\text{.... (6.4)}\]

and using the relation $q(T_1)=q_1$, we get

$$
T_2 = \frac{rK}{D}T_1
$$

\[\text{.... (6.5)}\]

6.6 Inventory Production Cost Calculation

Total holding cost per cycle = $h\int_0^{T_1} q(t)dt + h\int_{T_1}^{T_2} q(t)dt + h\int_{T_2}^{T} q(t)dt$
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\[ = \frac{h}{2} \left[ \frac{r^2 K^2}{D} T_i^2 - r K T_i^2 \right] \]

Shortage cost per cycle = \( B \left[ \int_{t_1}^{T} -q(t)dt \right] \)

\[ = \frac{S^2}{2D} B \]

6.7 Profit Calculation and Objective Function

Total profit incurred in each production cycle = “selling price of fresh units + selling price of defective units - production cost - holding cost - set-up cost - cost of interest and depreciation cost - shortage cost”

Total average profit incurred in each production cycle = \( \frac{1}{T} \) [selling price of fresh units + selling price of defective units - production cost - holding cost - set-up cost - cost of interest and depreciation cost - shortage cost]

\[ Z = \frac{1}{T} [m r PKTi + m_i (1 - r) PKTi - PKTi - \frac{r h KT_i^2}{2} \left( \frac{r K}{D} - 1 \right) - c_o - a c_o^b r^c - \frac{BS^2}{2D}] \]

\[ Z = \left[ (m - m_i)P + (m_i P - \frac{h S}{2K} \frac{1}{r} - \frac{h T}{2} - \frac{P}{2}) D - \left[ \frac{(h + B)S^2}{2} \right] \frac{1}{TD} + \frac{h}{2D} \frac{T D^2}{r} - \left[ \frac{m PS}{T} + \frac{m_i PS}{r T} \right] \right. \]

\[ - \frac{m_i PS}{T} - \frac{PS}{r T} - h S - \frac{h S^2}{2 r K T} + \frac{a c_o^b r^c}{T} + \frac{c_o}{T} \] \[ \left. \right] \]

\[ \text{where} \]

\[ Z = x_1 + \frac{x_2}{r} - x_3 T - x_4 + x_5 T + x_6 \frac{T}{r} + x_7 - x_8 c_o T^{-1} \]

\[ = F(c_o, r, T) \]

\[ \text{where} \]

\[ x_1 = (m - m_i)PD - \frac{P}{2} D + h S \]

\[ x_2 = m_i P - \frac{h S}{2K}, \quad x_3 = \frac{h D}{2}, \]

\[ x_4 = \frac{(h + B)S^2}{2D} + (m - m_i)PS + 1 \]

\[ c_o E_4, x_3 = \frac{h D^2}{2K}, \quad x_6 = -m_i PS + PS + \frac{h S^2}{2K}, \]

\[ x_7 = a, x_8 = 1 \]
6.8 Objective Function of Problem

According to this, our motive “To determine \( c_0, r, T \),” hence required to have find out optimal value of \( F (c_0, r, T) \) with the restriction of space constraint by modified geometric programming.

\[
\text{Maximize} \left( F(c_0, r, T) \right) = x_1 + \frac{x_2}{r} - x_3T - \frac{x_4}{T} + x_5 \frac{T}{rT} + x_6 \frac{x_5^{c_0^a} r^c}{T} - x_8 c_0 T^{-1}
\]

Subject to constraints

\[
w \left[ TD + \frac{SD}{rK} - \frac{TD^2}{rK} - wS \right] \leq W
\]

which is equivalent to

\[
\text{Minimize} \left( F(c_0, r, T) \right) = -x_1 - \frac{x_2}{r} + x_3T + \frac{x_4}{T} - x_5 \frac{T}{rT} + x_6 \frac{x_5^{c_0^a} r^c}{T} + x_8 c_0 T^{-1}
\]

subject to

\[
w \left[ TD + \frac{SD}{rK} - \frac{TD^2}{rK} - wS \right] \leq W
\]

here \( x_1 \) is independent of the decision variables \( c_0, r, T \), so it can be neglected to derive the optimal value of the objective function. Then the problem reduces to

\[
\text{Minimize} \left( F^* (c_r, r, T) \right) = \frac{x_2}{r} + \frac{x_4}{T} - \frac{x_5}{rT} + \frac{x_5^{c_0^a} r^c}{T} + x_8 c_0 T^{-1}
\]

Subject to constraints

\[
\frac{w}{W + wS} \left[ T \frac{SD}{rK} - \frac{TD^2}{rK} \right] \leq 1
\]

6.9 Geometric Programming Technique:

Geometric programming (GP) describes a type of optimization problem that has been known since the 1970s, but recently has attracted more attention for several reasons. The first is the development of extremely efficient interior-point algorithms for solving GPs. The second is the discovery that a wide variety of digital and analogy circuit design problems can be at least approximately expressed as GPs. We start with the basics of GP, and a recent powerful extension called generalized geometric programming (GGP). We'll illustrate how GGP can be used in a variety of circuit design problems, ranging from device sizing in op-amps to joint selection of threshold voltage, supply voltage, and device sizes in digital circuit design. We consider simple problem formulations involving tradeoffs among area, power, and delay, as well as more sophisticated formulations involving statistical and robust design, and design for multiple operating modes. We'll also give examples involving hierarchical design and joint electrical/physical design. The focus will be on GP modelling, the task of approximately
formulating a design problem in GP format. We’ll also cover related topics such as fitting empirical data or functions in a form compatible with GP, and how to handle discrete constraints on some of the variables. The increased use of mathematical models in the analysis and optimization of industrial systems is one of the significant developments of modern engineering practice. Models that describe real-life systems accurately are usually too complex for solution by the algorithms available. This is especially true of problem in which the constraints are nonlinear or the objective function is more than second degree. Geometric programming is an efficient and effective method to solve nonlinear programming with the terms in power functional form in the objective function and with constraints. The applications of geometric programming techniques on inventory management are well documented in the literature.

Let the following type of Primal Geometric Programming (PGP) problem be

\[
\text{Min} \quad f(x) = \sum_{i=1}^{n} \sum_{j=1}^{N_i} \sigma_{ij} c_{ji} \prod_{k=1}^{N} x_{ki}^{a_{jk}}
\]

subject to \( g(x) = \sum_{i=1}^{n} \sum_{l=1}^{N_i} \sigma_{jl} d_{li} \prod_{k=1}^{N} x_{ki}^{a_{lk}} \leq 1 \)

Where \( \sigma_{ij}, \sigma_{jl} = +1 \text{ or } -1, \) \( c_{ji}, d_{li} > 0, \) \( x_{ki} \geq 0, \) \( a_{jk} \) and \( a_{lk} \) are constants, for \( i=1,2,\ldots,n; \)
\( j=1,2,\ldots,N_0; \) \( k=1,2,\ldots,N; \) \( l=1,2,\ldots,N_l. \)

The corresponding dual programming (DP) problem of above problem is

\[
\text{Max} \quad d\alpha = \prod_{i=1}^{n} \left[ \prod_{j=1}^{N_i} \left( \frac{c_{ji}}{\alpha_{ji}} \right)^{\sigma_{ij}^{w_{ij}}} \right] \prod_{l=1}^{N_l} \left( \frac{d_{li}^{u_{li}}}{\alpha_{li}} \right)^{\sigma_{jl}^{w_{jl}}}
\]

where \( u = \sum_{i=1}^{n} \sum_{l=1}^{N} \alpha_{li} \)

subject to the normality and orthogonality conditions

\[
\sum_{j=1}^{N_i} \sigma_{ij} \alpha_{ji} = 1
\]

\[
\sum_{j=1}^{N_i} \sigma_{ij} \alpha_{jk} \alpha_{jl} + \sum_{l=1}^{N_l} \sigma_{jl} d_{li} \alpha_{li} = 0,
\]

\( \alpha_{ji}, \alpha_{li} \geq 0, \quad i=1,2,\ldots,n; \quad j=1,2,\ldots,N_0; \quad k=1,2,\ldots,N \)

The optimal values of the variables are obtained from the relations

\[
c_{ji} \prod_{k=1}^{N} x_{ki}^{a_{jk}} = \alpha_{ji}^{*} \sqrt{d\alpha}, \quad d_{li} \prod_{k=1}^{N} x_{ki}^{a_{lk}} = \frac{\alpha_{li}^{*}}{\sum_{i=1}^{n} \sum_{l=1}^{N_l} \alpha_{li}^{*}}, \quad j=1,2,\ldots,N_0; \quad i=1,2,\ldots,n
\]
and optimal objective is $n^{\sqrt{d}c^{\alpha^{n}}}n^{\alpha^{nd}}$.

This Geometric Programming (GP) method is noted as Modified Geometric Programming (MGP) method.

**Primal Problem:**

$$
\text{Minimize} \left( F'(c_o, r, T) \right) = -\frac{x_2}{r} + x_3T + x_4 - x_5 \frac{T}{r} - x_6 \frac{c_0 r^{s-1} r'}{rT} + x_8 c_0 T^{-1}
$$

subject to

$$
\frac{w}{W + wS} \left[ TD + \frac{SD}{rK} - \frac{TD^2}{rK} \right] \leq 1
$$

The coefficient of each term of the objective function is positive. Hence this is the posynomial primal geometric programming problem.

**Dual Problem:**

This is constraint posynomial geometric programming problem with degree of difficulty (DD)= number of terms- (number of variable+1)=10-(3+1)=6

The dual problem of the above problem is

$$
\text{Maximize} \; d(w) = \left( w_1 \right)^{w_1} \left( w_2 \right)^{w_2} \left( w_3 \right)^{w_3} \left( w_4 \right)^{w_4} \left( w_5 \right)^{w_5} \left( w_6 \right)^{w_6} \left( w_7 \right)^{w_7} \left( \frac{wD}{(W + wS)w_{o1}} \right)^{w_{o1}} \left( \frac{wSD}{(W + wS)w_{o2}} \right)^{w_{o2}} \left( \frac{wD^2}{(W + wS)w_{o3}} \right)^{w_{o3}} (w_{o1} + w_{o2} + w_{o3})^{w_{o1} + w_{o2} - w_{o3}}
$$

\ldots (6.8)

\[-w_1 + w_2 + w_3 - w_4 - w_5 + w_6 + w_7 = 1 \quad \ldots (6.9)\]

\[w_1 + w_4 + w_5 + c w_6 - w_{o2} + w_{o3} = 0 \quad \ldots (6.10)\]

\[w_2 - w_3 - w_4 + w_5 - w_6 + w_{o1} - w_{o3} + w_7 = 0 \quad \ldots (6.12)\]

\[-bw_6 + w_7 = 0 \quad \ldots (6.13)\]

Thus, on solving the above normality and orthogonality conditions, we get

\[w_1 = -\left( 1 + c + \frac{1}{b} \right) w_6 + w_{o2} - w_{o3} \]

\[w_2 = \frac{w_4}{2} - \frac{w_5}{2} - \frac{c w_6}{2} - \frac{w_{o1}}{2} + \frac{w_{o2}}{2} \]
\( w_3 = \frac{1}{2} [-w_4 + w_5 - (2 + c + 1/b)w_6 + w_{01} + w_{02} - 2w_{03}] \)

\( w_7 = \frac{1}{b} w_6 \)

On substituting the above dual weights into the dual function and taking logarithms both sides and putting \( \frac{\partial d}{\partial w_4} = 0, \frac{\partial d}{\partial w_5} = 0, \frac{\partial d}{\partial w_6} = 0, \frac{\partial d}{\partial w_{01}} = 0, \frac{\partial d}{\partial w_{02}} = 0 \),

\[ \frac{\partial d}{\partial w_{03}} = 0 \]

\[ \frac{1}{2} \left[ \log x_3 - \log \left( \frac{w_4}{2} - \frac{w_5}{2} - \frac{cw_6}{2} - \frac{w_{01}}{2} + \frac{w_{02}}{2} \right) - 1 \right] \]

\[ \frac{1}{2} \left[ \log x_4 - \log \left( -w_4 + w_5 - (2 + c + 1/b)w_6 + w_{01} + w_{02} - 2w_{03} \right) - 1 \right] \]

\[ \log x_5 - \log w_4 - 1 = 0 \] \quad \ldots (6.14)

\[ -\frac{1}{2} \left[ \log x_5 - \log \left( \frac{w_4}{2} - \frac{w_5}{2} - \frac{cw_6}{2} - \frac{w_{01}}{2} + \frac{w_{02}}{2} \right) - 1 \right] + \]

\[ \frac{1}{2} \left[ \log x_4 - \log \left( -w_4 + w_5 - (2 + c + 1/b)w_6 + w_{01} + w_{02} - 2w_{03} \right) - 1 \right] \]

\[ \log x_6 - \log w_5 - 1 = 0 \] \quad \ldots (6.15)

\[ (1 + c + 1/b) \left[ \log x_2 - \log \left( -(1 + c + 1/b)w_6 + w_{02} - w_{03} \right) - 1 \right] - \]

\[ \frac{c}{2} \left[ \log x_3 - \log \left( \frac{w_4}{2} - \frac{w_5}{2} - \frac{cw_6}{2} - \frac{w_{01}}{2} + \frac{w_{02}}{2} \right) - 1 \right] - \frac{1}{2} (2 + c + 1/b) \]

\[ \left[ \log x_4 - \log \left( -w_4 + w_5 - (2 + c + 1/b)w_6 + w_{01} + w_{02} - 2w_{03} \right) - 1 \right] + \]

\[ 2w_{03} - 1] + \left[ \log x_5 - \log w_4 - 1 \right] + \frac{1}{b} \left[ \log x_6 - \log w_5 - 1 \right] = 0 \]

\[ -\frac{1}{2} \left[ \log x_5 - \log \left( \frac{w_4}{2} - \frac{w_5}{2} - \frac{cw_6}{2} - \frac{w_{01}}{2} + \frac{w_{02}}{2} \right) - 1 \right] + \frac{1}{2} \left[ \log x_4 - \log \left( -w_4 + w_5 - (2 + c + 1/b)w_6 + w_{01} + w_{02} - 2w_{03} \right) - 1 \right] + \]

\[ \log \frac{WD}{W + WS} - \log w_{01} - 1 \right] + \log (w_{01} + w_{02} + w_{03}) \]

\[ \log \left( \frac{w_{01} + w_{02} - w_{03}}{w_{01} + w_{02} + w_{03}} \right) = 0 \]

\[ -\left[ \log x_2 - \log \left( -(1 + c + 1/b)w_6 + w_{02} - w_{03} \right) - 1 \right] + \]

\[ \frac{1}{2} \left[ \log x_3 - \log \left( \frac{w_4}{2} - \frac{w_5}{2} - \frac{cw_6}{2} - \frac{w_{01}}{2} + \frac{w_{02}}{2} \right) - 1 \right] + \]

\[ \ldots (6.17) \]
\[
\begin{align*}
\log \frac{wSD}{(W + wS)K} - \log w_{02} - 1 + \log(w_{01} + w_{02} + w_{03}) + \\
\frac{w_{03} + w_{02} - w_{03}}{w_{01} + w_{02} + w_{03}} &= 0 \\
\log x_2 - \log ((1 + c + 1/b)w_6 + w_{02} - w_{03}) - 1 - \log x_1 - \\
\log \frac{1}{2}[-w_1 + w_2 - (2 + c + 1/b)w_6 + w_{01} + w_{02} - 2w_{03}] - 1 \\
- \left[ \frac{\log wD^2}{(W + wS)K} - \log w_{03} - 1 \right] - \log(w_{01} + w_{02} + w_{03}) + \frac{w_{01} + w_{02} - w_{03}}{w_{01} + w_{02} + w_{03}} &= 0 \\
\text{solving all these equation, we get the optimal values of } & \quad w_4, w_5, w_6, w_{01}, w_{02}, w_{03}\text{ as } w^*_4, w^*_5, w^*_6, \\
& \quad w^*_01, w^*_02, w^*_03. \\
\text{So the optimal feasible solution of the above problem is } \quad \begin{align*}
w^*_1 &= -(1 + c + \frac{1}{b})w^*_6 + w^*_02 - w^*_{01} \\
w^*_2 &= \frac{w^*_4 - w^*_5 - cw^*_6}{2} - \frac{w^*_01}{2} + \frac{w^*_02}{2} \\
w^*_3 &= \frac{1}{2}[-w^*_4 + w^*_5 - (2 + c + 1/b)w^*_6 + w^*_01 + w^*_02 - 2w^*_03] \\
w^*_5 &= \frac{1}{b} w^*_6 \\
\text{Putting these values into the objective function of the problem } \\
d(w^*) &= \left( \frac{x_2}{w^*_1} \right)^{-w^*_1} \left( \frac{x_3}{w^*_2} \right)^{w^*_2} \left( \frac{x_4}{w^*_3} \right)^{w^*_3} \left( \frac{x_5}{w^*_4} \right)^{-w^*_4} \left( \frac{x_6}{w^*_5} \right)^{w^*_5} \\
& \quad \left( \frac{x_7}{w^*_6} \right)^{w^*_6} \left( \frac{x_8}{w^*_7} \right)^{w^*_7} \left( \frac{wD}{(W + wS)w^*_01} \right)^{w^*_01} \left( \frac{wSD}{(W + wS)Kw^*_02} \right)^{w^*_02} \\
& \quad \left( \frac{wD^2}{(W + wS)w^*_03} \right)^{w^*_03} \left( w^*_01 + w^*_02 + w^*_03 \right)^{w^*_01 + w^*_02 + w^*_03} \\
\text{Again from the relation between primal-dual variables we got the following equations } \\
-(mP - \frac{hS}{2K})r^{-1} &= w^*_1 d(w^*) \quad \ldots (6.21) \\
\frac{hD}{2} T &= w^*_2 d(w^*) \quad \ldots (6.22) \\
\frac{(h + B)S^2}{2D} T^{-1} &= w^*_3 d(w^*) \quad \ldots (6.23)
\end{align*}
\end{align*}
\]
\[
-\frac{hD^2}{2K}Tr^{-1} = w^*_{i}d(w^*) \quad \ldots \text{(6.24)}
\]
\[
-(1-m)SP + \frac{hS^2}{2K}T^{-1}r^{-1} = w^*_{i}d(w^*) \quad \ldots \text{(6.25)}
\]
\[
e^{-b}r^{-1} = w^*_{i}d(w^*) \quad \ldots \text{(6.26)}
\]
\[
e^{-b}r^{-1} = w^*_{i}d(w^*) \quad \ldots \text{(6.27)}
\]

After solving above constraints equation under objective function, the optimal values of decision variables has been found as follows for optimum reliable produced inventory on certain limitation.

\[
c_{9}^* = \left[\frac{2w^*_{i}w^*_{j}d(w^*)}{D^3h}\right] \quad \ldots \text{(6.28)}
\]
\[
r^* = \left[\frac{2^{b+1}(w^*_{j})^h w^*_{j}^b [d(w^*)]^{2b+2}}{a(D^3h)^{b+1}}\right]^{1/c} \quad \ldots \text{(6.29)}
\]
\[
T_{1}^* = \frac{a^{1/c}D^{-2b+3}h^{-1}}{2^{b+1}k^{-1}(w^*_{j})^{b/c}(w^*_{j})^{b+1}[d(w^*)]^{2b+2}} \quad \ldots \text{(6.30)}
\]

### 6.10 Reliability Analysis on Production of Inventory

This work is the initial initiative towards finding the reliability \(R_p(t) = r^*\) under inventory production under certain condition like shortage are allowed, Produced items are not cent percentage perfect , defective items are sold on reduced cost , Selling price is depends on production of fresh units is make up \(m\) of production cost and the total fresh units are greater than the market demand. The optimum reliability (Ref. equation (6.29)) attend during inventory production has been found after solving the problem written in equation (6.8) to (6.13). The Mathematical problems has been solved with the help of primal-dual programming condition and optimality technique of geometric programming . The final optimal solutions of Reliability, Cost and production are presented in equation (6.28) to (6.30). When we explore the relationship between Reliability and Profit during inventory production they are normally distributed in nature. When profit increases readily then reliability also increases as same pattern but when profit attend at maximum level the reliability increases at significant level which is very good sign for inventory production. The overall performance can be seen by a,b,c analysis of model foe best fit. This analysis is just equivalent to always better control in this model.
6.11 **Uses of Model in Global Market**
This model can be used by the production manager for better decision making in planning for
the production and controlling the inventory. Most frequently used organization like company
of food items, photographic film, electronic components, radioactive materials, fashionable
commodities etc., that use the decision rule to improve their total operational cost in the real
world. Mainly this model is useful like the firm having large no of production under limited
space capacity.

6.12 **Sensitivity analysis by illustration**
The model is considered for a particular EPQ problem with assuming the values of K=60 units,
h=3.49 units, m=2.0, m₁=0.80, P=21.7 units, a=800; c=0.75, α=20, β=0.2, S=2 units. The
optimal value of decision variables is \( c₀ = 89.63, r* = 0.4759, T₁ = 3.205 \). The
relationship of Reliability vs. Profit can be seen as :-

**Table-6(i): Analysis of Profit vs Reliability**

<table>
<thead>
<tr>
<th>Reliability (r)</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>0.2</td>
<td>9879</td>
</tr>
<tr>
<td>0.3</td>
<td>11564</td>
</tr>
<tr>
<td>0.4</td>
<td>13765</td>
</tr>
<tr>
<td>0.5</td>
<td>12453</td>
</tr>
<tr>
<td>0.6</td>
<td>11021</td>
</tr>
<tr>
<td>0.7</td>
<td>10567</td>
</tr>
<tr>
<td>0.8</td>
<td>9879</td>
</tr>
<tr>
<td>0.9</td>
<td>8979</td>
</tr>
</tbody>
</table>
Fig.6.1 Behaviour of Profit VS Reliability

From Table-(i)/Fig.6.1 it is observed that as the reliability of the system increases profit of the organization increases afterward it decreases. This due to achieve higher reliability of the system more investment is required in quality issue so the profit of the organization decreases.

6.13 Sensitivity Analysis with a, b, c:

1. From Table-(ii) it is clear that demand is less sensitive then set-up cost with respect to parameter a. Effect of ‘a’ on demand is reverse i.e., as ‘a’ increases demand decreases whereas set-up cost have direct relation i.e., as ‘a’ increases variation is also increases and finally profit increases as ‘a’ increases.

2. From Table-(ii) it clear that demand and profit is very sensitive with respect to parameter ‘b’. As ‘b’ increases demand decreases whereas profit increases. Both demand and profit changes rapidly with respect to parameter ‘b’.

3. From Table-(ii) it is clear that demand increases whereas profit decreases with the increasing value of ‘c’. Both the value of demand and profit changes with respect to parameter ‘c’.

4. Profit is very sensitive with respect to ‘b’ in comparison of ‘c’.
Table-6(ii): Variation in parameters $a$, $b$ and $c$ on demand, set-up cost and total profit

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Percentage Variation (%)</th>
<th>Variation in Demand (%)</th>
<th>Variation in Setup Cost (%)</th>
<th>Variation in Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>-20</td>
<td>1.27</td>
<td>-8.23</td>
<td>-3.56</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>0.60</td>
<td>-5.87</td>
<td>-1.45</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>-0.54</td>
<td>3.99</td>
<td>1.34</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>-1.03</td>
<td>7.68</td>
<td>3.78</td>
</tr>
<tr>
<td>$b$</td>
<td>-20</td>
<td>17.23</td>
<td>-5.67</td>
<td>-23.45</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>13.24</td>
<td>-1.03</td>
<td>-14.54</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>-14.56</td>
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</tr>
<tr>
<td></td>
<td>20</td>
<td>-20.45</td>
<td>4.79</td>
<td>25.34</td>
</tr>
<tr>
<td>$c$</td>
<td>-20</td>
<td>9.78</td>
<td>-2.54</td>
<td>-12.34</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>6.78</td>
<td>-0.58</td>
<td>-8.97</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>-7.23</td>
<td>0.67</td>
<td>7.98</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>10.29</td>
<td>2.76</td>
<td>13.86</td>
</tr>
</tbody>
</table>

Fig.6.2 Sensitive Analysis of 'a'
Fig. 6.3 Sensitivity analysis of ‘b’

Fig. 6.4 Sensitivity analysis of ‘c’