APPENDIX - I

STOKES SETTLING VELOCITY

A particle in a still fluid experiences two forces: a force due to gravity and a force due to buoyancy. The difference between these two forces is the net gravitational force, \( F_G \).

\[
F_G = \text{force due to gravity} - \text{force due to buoyancy} = m_s g - \rho_s v_s \quad \ldots \ldots \text{A-1}
\]

where \( g \) is the acceleration due to gravity, \( m_s \) is the mass of the particle, \( \rho_s \) is its density, and \( v_s \) is the volume of the particle. As the particle sinks through the fluid, the flow of the fluid around it creates a resisting or drag force \( F_D \). When the net gravitational and resisting forces become equal, the particle reaches the terminal velocity. The Stokes settling velocity \( U_S \) of a particle is given by

\[
U_S = \frac{2}{9} \frac{g(\rho - \rho_s)}{\eta} r^2 \quad \ldots \ldots \text{A-2}
\]

where \( r \) is the radius, \( \rho \) is the density and \( \eta \) viscosity of the medium. For bigger or heavier spheres, settlement is sufficiently fast to generate turbulence in their wake such that the drag force departs from the Stokes law. In this case, the impact of drag also has to be taken into account.
Case 1: Solid Sphere

The net gravitational force acting on a sphere

\[ F_G = \frac{4}{3} \pi r^3 g (\rho_s - \rho) \]  

..... A-3

The Stokes drag on a sphere

\[ F_D = 6 \pi n r U_s \]  

..... A-4

The impact drag on a sphere is  \( \pi r^2 \rho U_s^2 \)  

..... A-5

The total drag force (Stokes + impact)

\[ F_D = 6 \pi n r U_s + \pi r^2 \rho U_s^2 \]  

..... A-6

\( U_s \) becomes terminal velocity if

\[ F_G = F_D \]

From equations A-5 and A-6 and equality of \( F_G \) to \( F_D \) for terminal velocity, \( U_s \) is obtained as

\[ U_s = \left( \frac{9 n^2}{\rho_2 r^2} + 6 n \frac{r}{\rho} \right)^{\frac{1}{2}} - \frac{3 n}{\rho r} \]

where \( B = \frac{2 g (\rho_s - \rho)}{9 n} \)

For air,

\[ n = 1.8 \times 10^{-4} \text{ g.cm}^{-1} \text{ sec}^{-1} \]

\[ \rho = 1.2 \times 10^{-3} \text{ g.cm}^{-3} \]

\[ \rho_s = 2.7 \]
or \( U_s = \frac{0.45}{r} (1 + 14.58 \times 10^6 \ r^3)^{\frac{1}{2}} - 1 \) \hspace{1cm} \text{......A-7}

Case 2: Hollow Sphere

Let \( \rho_b \), \( \rho_s \) and \( \rho \) be the bulk, solid material and fluid density respectively, \( R_i \) is the interior wall radius and \( R \) is the outer wall radius. When the filling fluid is the same as the exterior environment through which the hollow sphere settles, then

\[
\rho_b = \rho_s + (\rho - \rho_s) \left(\frac{R_i}{R}\right)^3 \quad \text{......A-8}
\]

Wall thickness \( \Delta R = R - R_i \)

\[
\left(\frac{R_i}{R}\right)^3 = 1 - \frac{3 \Delta R}{R} \quad \text{......A-9}
\]

Substituting A-9 in A-8,

\[
\rho_b - \rho = (\rho_s - \rho) \cdot \frac{3 \Delta R}{R} \quad \text{......A-10}
\]

This difference between particle and fluid appears in A-2.

Comparing with A-2,

\[
U_s = \frac{3 \Delta R}{R} \cdot BR^2 \quad \text{......A-11}
\]

In order to express this in terms of equivalent sphere of radius \( r \), volume of the wall material is given by

\[
\frac{4}{3} \pi R^3 - (R - \Delta R)^3 = \frac{4}{3} \pi r^3 \quad \text{......A-12}
\]

When \( \frac{\Delta R}{R} \ll 1 \), \( (R - \Delta R)^3 = R^3 \left(1 - \frac{3 \Delta R}{R}\right) \).
\[ r^3 = \left( \frac{3\Delta R}{R} \right) R^3 \] ......A-13

From A-11,

\[ U_s = \left( \frac{3\Delta R}{R} \right)^{1/3} \cdot Br^2 \] ......A-14

Substituting the values for B,

\[ U_s = 4.73 \times 10^6 \times \left( \frac{\Delta R}{R} \right)^{1/3} \cdot r^2 \] ......A-15

Case 3: Ellipsoid

Let \( a \) and \( c \) be the equatorial and polar axes. The settling velocity of ellipsoids depends on their orientation. A prolate ellipsoid (\( c > a \)) settling in the direction parallel to its polar axis will settle faster as it will encounter less resistance compared to an oblate ellipsoid of the same mass and orientation.

The resisting drag force for an ellipsoid settling parallel to polar axis:

\[ F_D = 6\pi U_s (0.8a + 0.2c) \] ......A-16

(Lerman, 1979).

For settling direction perpendicular to polar axis,

\[ F_D = 6\pi U_s (0.6a + 0.4c) \] ......A-17

The volume of an ellipsoid of revolution is

\[ \frac{4}{3} \pi a^2c \]
Net gravitational force acting on an ellipsoid is

\[ F_G = \frac{4}{3} \pi a^2 c (\rho_S - \rho) \]  

\( \ldots \ldots \text{A-18} \)

Equating equations A-17 and A-13, and putting \( \frac{c}{a} = \rho \),

Parallel to \( c \) : \[ U_S = \frac{5\rho}{4+\rho} \frac{a^2}{B} \]  

\( \ldots \ldots \text{A-19} \)

Perpendicular to \( c \) : \[ U_S = \frac{2.5\rho}{1.5+\rho} \frac{a^2}{B} \]  

\( \ldots \ldots \text{A-20} \)

In terms of equivalent sphere of radius \( r \), \( \frac{4}{3} \pi r^3 = \frac{4}{3} \pi a^2 c \)

\( a = r^{1/3} \). \( r \) and substituting the value of \( B \),

Parallel to \( c \) : \[ U_S = \frac{1.64 \rho^{1/3}}{4 + \rho} \times r^2 \times 10^7 \]  

\( \ldots \ldots \text{A-21} \)

Perpendicular to \( c \) : \[ U_S = \frac{8.2 \times \rho^{1/3}}{1.5 + \rho} \times r^2 \times 10^6 \]  

\( \ldots \ldots \text{A-22} \)