2.1 INTRODUCTION:

In this chapter, an attempt is made to determine the optimum lot size for replenishment, such that the supplier's net profit becomes the maximum. The model developed here considers the case of quantity discount subjected to the stipulated markup of prices. The different inventory costs are subjected to inflation. The costs of advertising and the damaged goods in inventory are considered in the total cost function along with the cost structure of Economic order quantities (EOQ) model. Two particular cost functions allowing for quantity discount are illustrated by a numerical example and the detailed sensitivity analysis is carried out for both the illustrations and conclusions are drawn on the basis of these analysis.
2.2 ASSUMPTIONS:

Following are the assumptions for this model:

(1) Lots of size $q$ are replenished for each replenishment.

(2) Demand rate is $R$ (units/year) which is a function of the unit selling price $p$. Here we have the case of the variable demand function.

(3) Shortages are not allowed to occur.

(4) Lead time is zero.

(5) Replenishing size is constant with the lot size of $q$ units per replenishment.

(6) Inflation rate is $i$ ($\$/year).

(7) Nominal interest rate is $r$ ($\$/year) ($r > i$).

(8) Initially the purchase cost per item is $C_0$ ($\$/unit/year).

(9) Unit inventory holding cost is $C_1$ ($\$/unit/year).

(10) The set up cost per order at time zero is $S_0$ ($\$/order).

(11) Advertisement cost is a function of the total revenue and it is given by $\alpha pR$ ($\$/year) ($\alpha$ is a constant, $0 < \alpha < 1$).

(12) Cost due to the damage of items is given by $\gamma C(q)R$, where $\gamma$ is the lot fraction defective and $C(q)$ represents the cost of $q$ units replenished, thus allowing for the quantity discount case.

(13) The net revenue (Profit) is a real and continuous function of the lot size $q$. 
2.3 FORMULATION OF THE MODEL:

This problem relates to a warehouse inventory system from which the units are sold in the market depending upon their market requirements. The different inventory costs are subjected to inflation and accordingly the rate of inflation and the nominal rate of interest for actual investment would be more effective. The cost of damaged goods and advertising cost are to be considered with the routine inventory costs for EOQ. The problem is to determine the optimum lot size for maximising the net return of supplier.

2.4 DERIVATION OF THE MODEL:

Under the above assumption, the model is derived as under.

\[
\text{Total inventory cost} = (\text{Purchase cost}) + (\text{Order cost}) + (\text{Storage cost}) + (\text{Advertising cost}) + (\text{Cost of damaged goods}) \quad \text{..(2.1)}
\]

\[
\text{Purchase cost} = \left[1 + \left(\frac{i}{2}\right)(1 - \frac{q}{R})\right]C_0 R \quad \text{..(2.2)}
\]

\[
\text{Order cost} = \left[1 + \left(\frac{i}{2}\right)(1 - \frac{q}{R})\right]S_0 \frac{R}{q} \quad \text{..(2.3)}
\]

\[
\text{Storage cost} = \left[1 + \left(\frac{i}{2}\right)(1 - \frac{q}{R})\right]\left(C_0 / 2\right)q(r + C_1) \quad \text{..(2.4)}
\]

\[
\text{Cost of damaged goods} = \gamma C(q) R \quad \text{..(2.5)}
\]

\[
\text{Advertising cost} = \propto p R \quad \text{..(2.6)}
\]

Hence net return (profit) of the producer is given by

\[
P = [\text{Net Revenue}] - [(\text{Purchase cost}) + (\text{Order cost}) + (\text{Storage cost}) + (\text{Advertising cost}) + (\text{cost of damaged goods})] \quad \text{..(2.7)}
\]
It is further assumed that $p = \Theta C(q)$ ...(2.8)

Where $p = \text{Market price per unit of the commodity.}$

$\Theta = \text{Mark-up parameter } (\Theta > 1)$

then

$$P = R[p - C(q)] - [1 + (1/2)[1-(q/R)][C_0 R + (S_0 R/q)$$

$$+(C_0/2)[q(r+C_1)] - \alpha pR - \gamma C(q)R$$

...(2.9)

In the above profit function, let us consider here a specific demand function given by Kotler [10], as under

$$R = k f/p^\eta$$

...(2.10)

Where $k = \text{Constant } (k > 0)$

$f = \text{Frequency of advertisement}$

$\eta = \text{Elasticity of demand } (\eta > 0)$

So that $R = \psi(q) = k f/[\Theta (C(q))^\eta]$ ... (2.11)

$$= f(\Theta C(q))$$ ... (2.12)

Hence from equations (2.8), (2.9) and (2.11)

Net Revenue (profit) is

$$P = \psi(q) C(q) (\Theta - 1) - [1 + (1/2)[1-(q/\psi(q))]]$$

$$[C_0 \psi(q) + (S_0 \psi(q)/q) + (C_0/2)q(r+C_1)]$$

$$- (\alpha \Theta - \gamma) C(q) \psi(q)$$ ... (2.13)

Using the necessary condition $dP/dq = 0$ ...(2.14)

& sufficiency condition $d^2P/dq^2 < 0$ ...(2.15)

We get the optimum lot size $q^*$ from which optimum profit $p^*$

can be determined.

**Particular Cases:**

To illustrate the above model, let us consider here the two specific forms of quantity discounts as under.
2.4.1 Case 1: Linear Quantity Discount:

The unit cost function \( C(q) \) assumes the linear quantity discount case as given by

\[
C(q) = a - bq
\] ...(2.16)

Where \( a \) & \( b \) are positive constants. Hence for the particular case when \( \eta = 1 \), the profit function \( P \) given in (2.13) is simplified as

\[
P = \left[ (k f/\theta)(\theta(1-\eta) - (1 + \gamma)) + (iS_o/2) \right]
- \left[ (1+i/2)C_o(r+C_1) - iC_o \right](q/2)
- (1+i/2)S_o kf/[\theta(aq-bq)]
- (1+i/2)C_o kf/[\theta(a-bq)] +
iC_o(r+C_1)\theta(aq^2-bq^3)/4kf
\] ...(2.17)

Using the relations (2.14) for this case we get

\[
\sum_{j=0}^{n}\beta_j q^{j} = 0
\] ...(2.18)

Where

\[
\begin{align*}
\beta_0 &= \frac{[3b^3\theta^2iC_o(r+C_1)]}{4kf} \\
\beta_1 &= \frac{[2\theta^2iC_o(r+C_1)ab^2]}{kf} \\
\beta_2 &= \frac{b^2\theta[iC_o(1+i/2)C_o(r+C_1)]}{2kf} \\
\beta_3 &= \frac{[7a^2b^3iC_o(r+C_1)\theta^2]}{4kf} \\
\beta_4 &= \frac{[a^3\theta^2iC_o(r+C_1)]}{2kf} \\
\beta_5 &= \frac{[ab\theta(iC_o(1+i/2)C_o(r+C_1))]}{2kf} \\
\beta_6 &= \left[ iC_o(1+i/2)C_0(r+C_1) \right]/2
\end{align*}
\]

\[
-(1+i/2)C_0 kf b
\]

Solving the above six degree equation we can determine the optimum lot size \( q^* \) and hence the optimum profit \( p^* \) is obtained from equation (2.18).
2.4.1.1 Application:

To illustrate the above model, let us consider here a numerical example as under.

**Constants:** \(k = 10, f = 50, a = 50, b = 0.0001, \alpha = 0.1, \gamma = 0.2\).

**Parameters:** \(C_0 = 1, r = 0.1, C_1 = 0.05, S_0 = 0.1, i = 0.05\)

**Mark up parameter** \(\theta = 2, 3, 4\).

With the above hypothetical values, the polynomial equation (2.18) is solved to determine the optimum value of \(q\) such that \(P\) is maximum. The optimum solution is as given in table 2.1.

**Table 2.1**

**Optimum Solution**

<table>
<thead>
<tr>
<th>Mark up parameter (\theta)</th>
<th>Optimum lot size (q^*)</th>
<th>Optimum profit (p^*) in Rs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>69</td>
<td>143.08</td>
</tr>
<tr>
<td>3</td>
<td>46</td>
<td>245.38</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>296.54</td>
</tr>
</tbody>
</table>

2.4.1.2 SENSITIVITY ANALYSIS:

The sensitivity analysis is carried out in two ways as under.

(a) When all the parameters change their values simultaneously by 5\% and 10\% respectively. (This may be called as total sensitivity analysis under an inflationary or deflationary situation.)

(b) When only one of the parameter changes its value
by 5% AND 10% respectively while the other parameters remain unchanged (This may be called the partial sensitivity analysis.)

Results obtained by the total sensitivity analysis and the partial sensitivity analysis are presented in table (2.2) and table (2.3) respectively.

**TABLE - 2.2**

**TOTAL SENSITIVITY ANALYSIS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>VALUE OF P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>(-0.25)</td>
</tr>
<tr>
<td>2</td>
<td>142.72</td>
</tr>
<tr>
<td>3</td>
<td>245.14</td>
</tr>
<tr>
<td>4</td>
<td>296.36</td>
</tr>
</tbody>
</table>

(Note: The figures shown in parenthesis indicate percentage change in the expected net profit as compared to the corresponding solution given in table - 2.1).
### TABLE - 2.3

PARTIAL SENSITIVITY ANALYSIS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value of one parameter increasing by 5%</th>
<th>Value of one parameter increasing by 10%</th>
<th>Value of one parameter decreasing by 5%</th>
<th>Value of one parameter decreasing by 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_0 )</td>
<td>( 3 ) \quad 245.15 \quad 244.92 \quad 245.61 \quad 245.84</td>
<td>( 3 ) \quad (-0.09) \quad (-0.19) \quad (0.09) \quad (0.19)</td>
<td>( 4 ) \quad 296.36 \quad 296.19 \quad 296.71 \quad 296.88</td>
<td>( 4 ) \quad (-0.06) \quad (-0.12) \quad (0.06) \quad (0.11)</td>
</tr>
<tr>
<td>( r )</td>
<td>( 3 ) \quad 245.30 \quad 245.22 \quad 245.46 \quad 245.54</td>
<td>( 3 ) \quad (-0.03) \quad (-0.07) \quad (0.03) \quad (0.07)</td>
<td>( 4 ) \quad 296.48 \quad 296.42 \quad 296.59 \quad 296.65</td>
<td>( 4 ) \quad (-0.02) \quad (-0.04) \quad (0.02) \quad (0.04)</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>( 3 ) \quad 245.34 \quad 245.30 \quad 245.42 \quad 245.46</td>
<td>( 3 ) \quad (-0.02) \quad (-0.03) \quad (0.02) \quad (0.03)</td>
<td>( 4 ) \quad 296.51 \quad 296.48 \quad 296.56 \quad 296.59</td>
<td>( 4 ) \quad (-0.01) \quad (-0.02) \quad (0.01) \quad (0.02)</td>
</tr>
<tr>
<td>( S_0 )</td>
<td>( 3 ) \quad 245.30 \quad 245.30 \quad 245.30 \quad 245.30</td>
<td>( 3 ) \quad (0.004) \quad (0.004) \quad (0.0) \quad (0.0)</td>
<td>( 4 ) \quad 296.54 \quad 296.54 \quad 296.54 \quad 296.54</td>
<td>( 4 ) \quad (0.0) \quad (0.0) \quad (0.0) \quad (0.0)</td>
</tr>
<tr>
<td>( i )</td>
<td>( 3 ) \quad 245.48 \quad 245.58 \quad 245.27 \quad 245.14</td>
<td>( 3 ) \quad (0.04) \quad (0.08) \quad (-0.04) \quad (-0.10)</td>
<td>( 4 ) \quad 296.61 \quad 296.68 \quad 296.45 \quad 296.35</td>
<td>( 4 ) \quad (0.02) \quad (0.05) \quad (-0.03) \quad (-0.06)</td>
</tr>
</tbody>
</table>

(Note: The figures shown in Parenthesis indicate percentage change in the expected net profit as compared to the corresponding solution given in Table 2.1)
2.4.2 Case: II Hyperbolic Quantity Discount

Let us consider here the hyperbolic quantity discount case given by

\[ C(q) = b + \frac{d}{q} \quad \ldots (2.19) \]

Where \( b \) & \( d \) are constants. Hence for the particular value of \( \eta = 1 \), the profit function \( P \) given in (2.13) is simplified as

\[
P = \left[ (k f/\Theta)\{(\Theta(1-\alpha) - (1 + \gamma) \} + (iS_o/2) \right] + Bq
\]

\[
- (1 + i/2) S_o k f/\{(\Theta(bq + d)\} - [(1 + i/2)C_o k f/\Theta]\]

\[
[q/(bq + d)] + [iC_o(r + C_1) \Theta (bq^2 + bq)/4 k f \quad \ldots (2.20)
\]

Where \( B = (C_o/2)(1 - (1 + i/2)(r + C_1)) \)

The problem is to obtain the optimum value of \( q \) such that \( P \) is maximum.

The necessary condition gives

\[
\sum_{j=0}^{3} \beta_j C_j = 0 \quad \ldots (2.21)
\]

Where \( A = [iC_o(r + C_1) \Theta^2]/4 k f \)

\[
\beta_0 = 2 A b^3
\]

\[
\beta_1 = b^2(5 d A + B \Theta)
\]

\[
\beta_2 = 2 b d (2 d A + B \Theta)
\]

\[
\beta_3 = A d^3 + B d \Theta^2 +
\]

\[
(1 + i/2)k f(S_o b - C_o d)
\]

provided that the sufficiency condition

\[
d^2P/dq^2 < 0
\]

is also satisfied.

Solving the above cubic equation, we can determine the optimum lot size \( q^* \) and hence the optimum profit \( P^* \) is obtained from equation (2.21).
2.4.2.1 APPLICATION:

To illustrate the above model, let us consider here a numerical example as under.

**Constants**: \( k = 50, f = 50, b = 10, d = 50, \)
\( \alpha = 0.1, \gamma = 0.2. \)

**Parameters**: \( C_0 = 1, r = 0.1, C_1 = 0.05, \)
\( S_0 = 0.1, i = 0.05. \)

**Mark up Parameter** \( \theta = 2, 3, 4, 5, 6. \)

With the above hypothetical values, the polynomial equation in (2.21) is solved to determine the optimum value of \( q \) such that profit \( P \) is maximum.

The optimum solution obtained is as given in table 2.4

**TABLE 2.4**

**OPTIMUM SOLUTION**

<table>
<thead>
<tr>
<th>Mark up Parameter ( \theta )</th>
<th>Optimum Lot Size ( q^* )</th>
<th>Optimum Profit ( P^* ) in ( \text{Rs} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1718.89</td>
<td>622.37</td>
</tr>
<tr>
<td>3</td>
<td>1142.52</td>
<td>1165.09</td>
</tr>
<tr>
<td>4</td>
<td>854.35</td>
<td>1436.45</td>
</tr>
<tr>
<td>5</td>
<td>681.45</td>
<td>1599.26</td>
</tr>
<tr>
<td>6</td>
<td>566.20</td>
<td>1707.80</td>
</tr>
</tbody>
</table>

2.4.2.2 Sensitivity Analysis:

The total and partial sensitivity analysis for this model are given in table 2.5 and table 2.6 respectively.
**Table 2.5**

**TOTAL SENSITIVITY ANALYSIS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>VALUE OF P*</th>
</tr>
</thead>
<tbody>
<tr>
<td>All parameters increasing simultaneously by 5%</td>
<td>628.66</td>
</tr>
<tr>
<td>All parameters decreasing simultaneously by 5%</td>
<td>(-1.01)</td>
</tr>
<tr>
<td></td>
<td>(-0.769)</td>
</tr>
<tr>
<td></td>
<td>(0.209)</td>
</tr>
</tbody>
</table>

(Note: The figures shown in parenthesis indicate percentage change in the expected net profit compared to the corresponding solution as given in Table 2.4)

**Table 2.6**

**PARTIAL SENSITIVITY ANALYSIS**

<table>
<thead>
<tr>
<th>Chang:Mark up:</th>
<th>VALUE OF P*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of one parameter increasing</td>
<td>628.75</td>
</tr>
<tr>
<td></td>
<td>(-1.025)</td>
</tr>
<tr>
<td>Value of one parameter decreasing</td>
<td>(-0.22)</td>
</tr>
<tr>
<td></td>
<td>(-0.44)</td>
</tr>
<tr>
<td>r</td>
<td>2</td>
</tr>
<tr>
<td>----</td>
<td>-----</td>
</tr>
<tr>
<td></td>
<td>r^2</td>
</tr>
<tr>
<td>3</td>
<td>1165.086</td>
</tr>
<tr>
<td></td>
<td>(-0.021)</td>
</tr>
<tr>
<td>4</td>
<td>1436.436</td>
</tr>
<tr>
<td></td>
<td>(-0.001)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c_1</th>
<th>2</th>
<th>622.3688</th>
<th>622.370</th>
<th>622.613</th>
<th>622.619</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r^2</td>
<td>(-0.039)</td>
<td>(-0.04)</td>
<td>(0.039)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>3</td>
<td>1165.086</td>
<td>1165.86</td>
<td>1165.311</td>
<td>1165.440</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.019)</td>
<td>(-0.021)</td>
<td>(0.019)</td>
<td>(0.030)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1436.449</td>
<td>1436.447</td>
<td>1436.451</td>
<td>1436.453</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.0001)</td>
<td>(-0.0002)</td>
<td>(0.0001)</td>
<td>(0.0002)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>s_0</th>
<th>2</th>
<th>622.369</th>
<th>622.369</th>
<th>622.625</th>
<th>622.638</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r^2</td>
<td>(-0.041)</td>
<td>(-0.043)</td>
<td>(0.041)</td>
<td>(0.043)</td>
</tr>
<tr>
<td>3</td>
<td>1165.067</td>
<td>1165.043</td>
<td>1165.113</td>
<td>1165.137</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.002)</td>
<td>(-0.004)</td>
<td>(0.002)</td>
<td>(0.004)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1436.445</td>
<td>1436.445</td>
<td>1436.445</td>
<td>1436.445</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>i</th>
<th>2</th>
<th>622.762</th>
<th>622.911</th>
<th>621.978</th>
<th>621.827</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r^2</td>
<td>(0.063)</td>
<td>(0.087)</td>
<td>(-0.063)</td>
<td>(-0.087)</td>
</tr>
<tr>
<td>3</td>
<td>1165.440</td>
<td>1165.766</td>
<td>1164.752</td>
<td>1164.444</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.058)</td>
<td>(-0.029)</td>
<td>(-0.058)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1436.666</td>
<td>1436.881</td>
<td>1436.235</td>
<td>1436.019</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.030)</td>
<td>(-0.015)</td>
<td>(-0.030)</td>
<td></td>
</tr>
</tbody>
</table>

(Note: The figures shown in parenthesis indicate percentage change in the expected net profit compared to the corresponding solution given in table 2.4).
2.5 CONCLUSIONS:

(A) From the tables 2.1 and 2.4 giving the original solutions for the two cases, it may be concluded that as the mark-up parameter $\theta$ increases, the optimum profit $p^*$ also increases and the optimum lot size $q^*$ decreases. This can also be seen from the graphs G/2.1 to G/2.4.

(B) From the total sensitivity analysis done in the tables 2.2 and 2.5, it can be seen that the optimum profit decreases when all the parameters increase their values simultaneously as compared to the original solution corresponding to different values of the parameters and exactly reverse phenomena is observed when all the parameters decrease their values simultaneously as compared to the original solution.

(C) From the partial sensitivity analysis carried out in the tables 2.3 and 2.6, we can summarise as under.

When the parameter $C_0, r, C_1$, and $S_0$ increase their values (one by one) while all other remaining parameters have their fixed values, the optimum profit decreases as compared to the original solution and exactly reverse phenomena is observed for decreasing the value of a single parameter only while keeping other parameters as fixed. When the inflation rate $i$ alone increases, the optimum profit also increases as compared to the original solution given in 2.1 and 2.4 and the optimum profit is decreased when $i$ alone decreases as compared to the original solution. The two sensitivity parameters are $C_0$ and $i$ respectively in order, whereas $S_0$ is found to
be more or less stagnant parameters.

Thus it may be concluded that the inflation rate should be increased to earn a higher profit or the interest rate should be decreased. Similarly it appears from our analysis that the total sensitivity is found to be better in relation to the partial sensitivity analysis from the point of view of the original solutions given in table 2.1 and 2.4.
Optimum profit $P^*$ in $\mathbb{R}$ versus Mark-up parameter $\theta$.

Optimum Lot Size $q^*$ VERSUS Mark-up parameter $\theta$. 