CHAPTER VIII

OPTIMUM DECISION MAKING FOR A PRODUCTION PLANNING PROBLEM UNDER ELS INVENTORY MODEL SUBJECT TO DIFFERENT COSTS AND MARK-UP OF PRICES.

8.1 INTRODUCTION:

In this chapter, an attempt is made to formulate the profit function for a production planning problem which is subjected to the routine costs of production, inventory, advertising and the damaged goods of the items. For this profit maximisation approach, Economic lot size (ELS) model is considered which also takes into account the different types of quantity discount for the bulk purchase of items. The model derived is illustrated by numerical examples along with sensitivity analysis.
8.2 ASSUMPTIONS:

Following are the assumptions for this model.

(1) Lots of size q are replenished for each replenishment.

(2) Demand rate is $R$ (units/year) which is a function of the units selling price $p$ (Rs.). Here we have the case of a variable demand function.

(3) Shortages are not allowed to occur.

(4) Lead time is zero.

(5) Production rate $\lambda$ (units/year) is finite ($\lambda > R$).

(6) Replenishing size is constant.

(7) Unit inventory holding cost $c_1$ (Rs./unit/year).

(8) The unit replenishment cost is $C_3$ Rs. per order which is fixed.

(9) Advertisement cost is a fraction of the total revenue and is given by $\alpha_1 pR$ (Rs./year) ($\alpha_1$ is a given constant, $0 < \alpha_1 < 1$).

(10) Cost due to the damaged goods is given by $\alpha_2 C(q)R$, where $\alpha_2$ is lot fraction defective and $C(q)$ represents the cost of $q$ units replenished, thus allowing for the quantity discount case.

(11) The net revenue (profit) is a real and continuous function of the lot size $q$.

(12) The production firm produces only a single commodity, having its output $Q$ by employing the linear production function model which consists of only two variable input factors of production viz-capital investment $K$ and labour employed $L$. 
8.3 FORMULATION OF THE MODEL:

The formulation of the model can be described briefly as under: A manufacturer is implementing the frequent advertisement efforts to promote the sales of the produced units. The units stored in a warehouse may get damaged. The units that are produced by the firm assume to take a linear type of production function where the input factors of production are employed in some given fixed proportion. A part of the produced goods are consumed by the external demand of the consumers and the goods are stored in the warehouse, thus there will be a bufferstock ready for any situation to be faced with. The amount of goods to be replenished depends upon the production size and due to the above assumption of fixed proportion of input factors of production, the lot size to be replenished can be expressed in terms of one of the decision variables (which is the input factor of production).

8.4 DERIVATION OF THE MODEL:

Taking into account, the advertisement cost, the damaged goods cost and other inventory costs in the case of ELS model the profit function is derived as under.
Production cost = $CQ$ ... (8.1)

Here $Q = \alpha K + \beta L \quad (\alpha > 0, \beta > 0)$ ... (8.2)

Let $K/L = m$ or $K = mL$ ... (8.3)

So that $Q = (\alpha m + \beta)L$ ... (8.4)

also let $q = a + bQ$ Where $q < Q$ ... (8.5)

= $a + AL$ ... (8.6)

Where $A = b(\alpha m + \beta)$ & $A > 0$

Hence production cost $= C(q - a)/b$ ... (8.7)

= $CA/L/b$ ... (8.8)

Advertising cost $= \alpha_1pR$ ... (8.9)

Let $p = \theta C(q) \quad (\theta > 1)$ ... (8.10)

Cost of damaged goods $= \alpha_2C(q)R$ ... (8.11)

Total Inventory Cost (TIC) $= (C_1q/2)(1 - R/\lambda) + C_3R/q$ ... (8.12)

Hence Total Cost (TC) = Production cost + (Advertising cost)

+ (cost of damaged good)+TIC ... (8.13)

= $[C(q - a)/b] + \alpha_1pR + \alpha_2C(q)R$

+ $(C_1q/2)(1-R/\lambda) + C_3R/q$ ... (8.14)

Total Revenue (TR) $= R[p - C(q)]$

= $\psi(q)(p - C(q))$ ... (8.15)

= $[((\theta - 1)/\theta)sf]$ ... (8.16)
Net Revenue (profit) \( P = TR - TC \)

i.e. \( P = \psi(q)(p - C(q)) - \left[ \frac{C(q - a)}{b} \right] + \alpha_1 p \psi(q) + \alpha_2 C(q) \psi(q) + \left( C_1 q/2 \right) \left( 1 - \frac{\psi(q)}{\lambda} \right) + C_3 \psi(q)/q \) \( \ldots(8.17) \)

Where \( R = \psi(q) = f(\theta C(q)) \) \( \ldots(8.18) \)

In the above profit function, let us consider here a specific demand function given by Kotler [10], as under

\[ R = s f/p^\eta \] \( \ldots(8.19) \)

where

- \( s = \text{constant} \ (s > 0) \)
- \( f = \text{Frequency of advertisement} \)
- \( \eta = \text{Elasticity of demand} \ (\eta > 0) \)

So that \( R = \psi(q) = s f/\theta C(q) + \ldots(8.20) \)

Substituting equation (6.20) into equation (6.17) we get the profit function as

\[
P = \left( \frac{\theta - 1}{\theta} \right) s f - \left[ \frac{C(q - a)}{b} \right] + \alpha_1 s f + \left( \alpha_2 s f/\theta \right) + \left( C_1 q/2 \right) \left( 1 - \frac{s f}{\lambda \theta C(q)} \right) + C_3 s f/\theta C(q)/q \]

\( \ldots(8.22) \)

An attempt is made to investigate the optimum value of the input factors \( L^* \) & \( K^* \), optimum amount of production \( Q^* \), and optimum order quantity \( q^* \), so as to maximize the profit \( P \) of the system as whole.

The necessary condition for maximisation of profit is
\[
\frac{dP}{dL} = 0 \quad \text{...(8.23)}
\]

and sufficiency condition is \( \frac{d^2P}{dL^2} < 0 \) \( \text{...(8.24)} \)

**Particular Cases:**

To illustrate the above model, let us consider here the two specific forms of the quantity discounts as under.

**8.4.1 Case I Linear Quantity Discount:**

The unit cost function \( C(q) \) assumes the linear quantity discount case as given by

\[
C(q) = g - h \, g \quad (g > 0, \, h > 0, \, g > h) \quad \text{...(8.25)}
\]

\[
\Rightarrow P = \left\{ (\theta - 1)/\theta \right\} s f - [(C A L/b) + s f(\alpha_1 + \alpha_1/\theta)
+ C_3 s f/\theta(g(a + AL) - h (a + AL)^2)
+ \{C_1 (a + AL)/2} \{1 - s f/\lambda \theta(g - h(a+AL)^2))\}]\quad \text{...(8.26)}
\]

Now

\[
\frac{dP}{dL} = - [(CA/b) + (C_1 A/2) - C_3 sf [gA-2h(a+AL)A]/\theta (g(a+AL)-h(a+AL)^2)^2
+ \left[C_1 sf[A(g-h(a+AL))] - C_1 sf(a+AL)(-hA)]/2\lambda \theta(g-h(a+AL))^2\right.\]
\]

From condition (8.23) we get a polynomial equation of 4th degree as below.

\[
\sum_{j=0}^{4} \beta_j L^{4-j} = 0 \quad \text{...(8.24)}
\]

where \( \beta_o = h^2 A^4 \)

\[
\beta_1 = 2 A^3(g - 2ah)
\]

\[
\beta_2 = A^2[g^2-6ah(g-ah)-bC_1 sf g/\lambda \theta(2C+bC_1)]
\]

\[
\beta_3 = 2A[a(g-2ah)(g-ah) - bsf(C_1 ag-2C_3)/(2C+bC_1)]
\]

\[
\beta_4 = a^2(g-ah)^2-bsf(C_1 a^2 g + 2 \lambda C_3 (g - 2ah))/\lambda \theta(2C + bC_1)
\]
& \frac{d^2P}{dL^2} < 0 \text{ Leads to }
(2\lambda C[g^2-3h(a+AL)[g-h(a+AL)]]-C_1gh(a+AL)^3)/\{g-h(a+AL)^3\} (8.29)

Solving equation (8.28) we get the optimum labour input L* from which the necessary optimum capital investment K* & net profit P* can be computed.

The model derived above makes use of the mark-up concept for pricing which becomes extremely beneficial to the producer. The numerical example along with its sensitivity analysis in two ways namely total & partial sensitivity analysis becomes very helpful to express the derived results.

8.4.1.1 Application:

To illustrate the above model, let us consider here a numerical example as under.

Constants : s=50, f=40, g=50, h=0.1, a=1, b=0.1,
\lambda=0.3, \lambda_1=0.1, \lambda_2=0.05, \beta=0.4, m=2.

Parameters : C=0.5, C_3=10, C_1=1, \lambda=5000.

Mark-up parameter: \theta = 2, 3, 4, 5.

With the above hypothetical values, the polynomial equation (8.28) is solved to determine the optimum value of L such that P is maximum. The optimum solution is as given in table 8.1.
From table 8.1 it can be seen that as the mark-up parameter $\theta$ increases $L^*, K^*, Q^* \& q^*$ decreases but the optimum profit $P^*$ increases. This is also represented by means of plotting $P^*$ against $\theta$ as shown in graph No.G/8.1

8.4.1.2 Sensitivity Analysis:

The model can also be tested for its sensitivity by changing the values of all the parameters by 5% and 10% simultaneously (total sensitivity analysis) or by varying only one of the parameters at a time while all other remaining parameters are kept fixed (partial sensitivity analysis).
### TABLE 8.2

**TOTAL SENSITIVITY**

<table>
<thead>
<tr>
<th>Mark-up parameter</th>
<th>Value of $p^*$</th>
<th>Mark-up parameter</th>
<th>Value of $p^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Theta$ by 5%</td>
<td></td>
<td>$\Theta$ by 5%</td>
</tr>
<tr>
<td></td>
<td>$\Theta$ by 10%</td>
<td></td>
<td>$\Theta$ by 10%</td>
</tr>
<tr>
<td>2</td>
<td>685.19</td>
<td>682.10</td>
<td>691.36</td>
</tr>
<tr>
<td></td>
<td>(-0.449)</td>
<td>(-0.898)</td>
<td>(0.440)</td>
</tr>
<tr>
<td>3</td>
<td>1048.11</td>
<td>1045.63</td>
<td>1053.05</td>
</tr>
<tr>
<td></td>
<td>(-0.235)</td>
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</tr>
<tr>
<td>4</td>
<td>1230.79</td>
<td>1228.69</td>
<td>1235.00</td>
</tr>
<tr>
<td></td>
<td>(-0.171)</td>
<td>(-0.341)</td>
<td>(0.170)</td>
</tr>
</tbody>
</table>

(Note: The figures shown in the parenthesis are percentage change in the expected net profit as compared to the corresponding solution given in Table 8.1)

From total sensitivity analysis given in Table 8.2, it may be concluded that when all the parameters increase simultaneously the optimum profit decreases as compared to the original solution and exactly reverse situation is observed when all the parameters decrease simultaneously.
### Table 8.3

**PARTIAL SENSITIVITY**

<table>
<thead>
<tr>
<th>Change in Parameter</th>
<th>Value of Parameter</th>
<th>Value of Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>by 5%</td>
<td>by 10%</td>
<td>by 5%</td>
</tr>
<tr>
<td>( \Theta )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>686.93</td>
<td>685.70</td>
</tr>
<tr>
<td></td>
<td>(-0.196)</td>
<td>(-0.375)</td>
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<tr>
<td>( C )</td>
<td>1049.61</td>
<td>1048.67</td>
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<tr>
<td></td>
<td>(-0.092)</td>
<td>(-0.182)</td>
</tr>
<tr>
<td>4</td>
<td>1232.09</td>
<td>1231.31</td>
</tr>
<tr>
<td></td>
<td>(-0.0657)</td>
<td>(-0.1290)</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>1050.45</td>
<td>1050.52</td>
</tr>
<tr>
<td></td>
<td>(-0.012)</td>
<td>(-0.025)</td>
</tr>
<tr>
<td>4</td>
<td>1232.79</td>
<td>1232.69</td>
</tr>
<tr>
<td></td>
<td>(-0.0089)</td>
<td>(-0.0170)</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>1049.28</td>
<td>1048.91</td>
</tr>
<tr>
<td></td>
<td>(-0.124)</td>
<td>(-0.159)</td>
</tr>
<tr>
<td>4</td>
<td>1231.73</td>
<td>1230.59</td>
</tr>
<tr>
<td></td>
<td>(-0.0945)</td>
<td>(-0.1874)</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>1050.58</td>
<td>1050.58</td>
</tr>
<tr>
<td></td>
<td>(0.0)</td>
<td>(0.0)</td>
</tr>
<tr>
<td>4</td>
<td>1232.90</td>
<td>1232.90</td>
</tr>
<tr>
<td></td>
<td>(0.0)</td>
<td>(0.0)</td>
</tr>
</tbody>
</table>

(Note: The figures shown in the parenthesis are percentage change in the expected net profit as compared to the corresponding solution given in table 8.1).
From table 8.3, it can be summarised that as any one parameter increases its value while all other remaining parameters are kept fixed, the optimum profit decreases as compared to the original solution, and exactly reverse phenomena is observed when any one parameter decreases its value while the rest of the parameters are kept fixed. It should also be noted that $C$ and $C_3$ are the most sensitive parameters.

8.4.2 Case II Hyperbolic Quantity Discount:

Let us consider here the hyperbolic quantity discount case given by
\[ C(q) = d + \frac{e}{q} \quad (d > 0, \ e > 0) \quad \ldots (8.30) \]
Hence $p = \theta(d + \frac{e}{q})$
Hence the profit function $P$ given in (8.21) reduces to
\[ P = \left\{ \frac{(\theta - 1)}{\theta} sf - \left[ (\alpha_1 + \alpha_2/\theta) sf + (C_1 a/\theta + (C_1 aL/2) \right] (C_3 sf/\theta(d(a+AL)+e) - C_1 sf(a^2+2aAL+L^2)/2\theta \lambda(d(a+AL)+e) \right\}. (8.31) \]

An attempt is made to investigate the optimum values of the input factors $L^*$ & $K^*$, optimum amount of production $Q^*$ and the optimum order quantity $q^*$, so as to maximise the profit.

The necessary condition to maximise the profit is $dP/dL = 0 \quad \ldots (8.32)$
Which gives

\[
((2C+bC_1)A/2b)-[C_3 sf dA/\theta (d(a+AL)+e)]^2-C_1 sf [(d(a+AL)+e)(2aA+2L)]
\]
\[-(a^2+2aAL+L^2)(dA)/2\theta A (d(a + AL) + e)^2 = 0
\]

i.e. \[
[d^2 A^2 \theta (2C+bC_1)-bdC_1 sf/\lambda ]L^2+2[A\theta (2C+bC_1)(ad^2+ed)-bC_1 sf(ad + e)/A\lambda ]L + \theta (2C+bC_1)(ad+e)^2-2bdC_3 sf-
\]
\[bC_1 sf(a^2d+2ae)/\lambda \] = 0  \hspace{1cm} \ldots(8.33)

That is \[M_1 L^2 + M_2 L + M_3 = 0 \hspace{1cm} \ldots(8.34)\]

Where \[M_1 = d^2 A^2 \theta (2C+bC_1)-[bdC_1 sf/\lambda ]\]
\[M_2 = 2[A\theta (2C+bC_1)(ad^2+ed)-bC_1 sf(ad + e)/A\lambda ]\]
\[M_3 = \theta (2C+bC_1)(ad+e)^2-2bdC_3 sf-
\]
\[bC_1 sf(a^2d+2ae)/\lambda \]

Here solving equation (8.34) we get

\[L = \frac{-M_2 \pm \sqrt{M_2^2 - 4M_1 M_3}}{2M_1} \hspace{1cm} \ldots(8.35)\]

and the sufficiency condition \(d^2 P/dL^2 < 0\) ensures to take

\[L^* = \frac{-M_2 + \sqrt{M_2^2 - 4M_1 M_3}}{2M_1} \hspace{1cm} \ldots(8.36)\]

\(P^*\) can be determined from equation (8.3) and (8.31) respectively.

8.4.2.1 Application :

To illustrate the above model, we consider here a numerical example as under.

Constants : \(\alpha = 0.1, \beta = 0.5, m=5, a=1, b=0.1, d=1, e=2, \alpha_1=0.1\)
\(\alpha_2=0.2, s=100, f=20.\)

Parameters : \(C=2, C_1=1, C_3=1, \lambda = 5000.\)

Mark-up parameter : \(\theta = 2, 3, 4, 5.\)
With the above hypothetical values, the quadratic equation (8.34) is solved to determine the optimum value of \( L \) so that \( P \) is maximum. The optimum solution for the above numerical example is as given in Table - 8.4.

**Table - 8.4**

<table>
<thead>
<tr>
<th>Mark-up</th>
<th>Optimum</th>
<th>Optimum</th>
<th>Optimum</th>
<th>Optimum</th>
<th>Optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>parameter</td>
<td>Labour</td>
<td>Capital</td>
<td>Out-put</td>
<td>Lot size</td>
<td>Profit</td>
</tr>
<tr>
<td>( \theta )</td>
<td>( L^* )</td>
<td>( K^* )</td>
<td>( Q^* )</td>
<td>( q^* )</td>
<td>( P^* )</td>
</tr>
<tr>
<td>2</td>
<td>63.1205</td>
<td>315.6025</td>
<td>63.1205</td>
<td>7.312</td>
<td>405.65</td>
</tr>
<tr>
<td>3</td>
<td>36.241</td>
<td>181.205</td>
<td>36.241</td>
<td>4.624</td>
<td>837.86</td>
</tr>
<tr>
<td>4</td>
<td>24.195</td>
<td>120.975</td>
<td>24.195</td>
<td>3.420</td>
<td>1063.10</td>
</tr>
<tr>
<td>5</td>
<td>16.981</td>
<td>84.905</td>
<td>16.981</td>
<td>2.698</td>
<td>1202.04</td>
</tr>
</tbody>
</table>

From Table 8.4, it may be concluded that with the increase in the mark-up parameter \( \theta \), the optimum profit \( P^* \) also increases and the optimum value of the lot size \( q^* \), labour \( L^* \) and capital \( K^* \) decreases. Which is also apparent from the graph G/8.2.

8.4.2.2 Sensitivity Analysis :

The model can also be tested for its sensitivity by changing the value of all the parameters by 5% and 10% simultaneously or by varying only one of the parameters at a time while all other remaining parameters are kept fixed.
TABLE - 8.5

TOTAL SENSITIVITY

<table>
<thead>
<tr>
<th>Mark-up parameter</th>
<th>Value of $p^*$</th>
<th>All parameters increasing: simultaneously</th>
<th>All parameter decreasing: simultaneously</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value of $p^*$</td>
<td>by 5% : by 10%</td>
<td>by 5% : by 10%</td>
</tr>
<tr>
<td>2</td>
<td>395.97</td>
<td>386.21 : 415.36</td>
<td>425.01</td>
</tr>
<tr>
<td></td>
<td>(-2.396)</td>
<td>(-4.792) : (2.394) : (4.773)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>829.76</td>
<td>821.45 : .845.97</td>
<td>854.08</td>
</tr>
<tr>
<td></td>
<td>(-0.967)</td>
<td>(-1.959) : (0.968) : (1.936)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1056.25</td>
<td>1049.41 : 1069.94</td>
<td>1076.79</td>
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<tr>
<td></td>
<td>(-0.644)</td>
<td>(-1.288) : (0.643) : (1.288)</td>
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</tbody>
</table>

(Note: The figures shown in the parenthesis are percentage change in the expected net profit as compared to the corresponding solution given in table 8.4).

From the above sensitivity analysis, it is apparent that as the mark-up parameter increases, the optimum profit also increases when all their parameters decrease their values simultaneously and vice versa.
TABLE - 8.6
PARTIAL SENSITIVITY

<table>
<thead>
<tr>
<th>Chang:Marking</th>
<th>Value of $p^*$</th>
<th>Value of one parameter increasing by 5%</th>
<th>Value of one parameter decreasing by 5%</th>
<th>Value of one parameter increasing by 10%</th>
<th>Value of one parameter decreasing by 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter: Parameter</td>
<td>θ</td>
<td>2</td>
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<td>393.83</td>
<td>412.19</td>
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<td>(-1.50)</td>
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<tr>
<td>C</td>
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<td>831.06</td>
<td>841.61</td>
<td>845.63</td>
</tr>
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<td></td>
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<td></td>
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<td>(-0.81)</td>
<td>(0.45)</td>
</tr>
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<td>4</td>
<td>1060.76</td>
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<tr>
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<td></td>
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<td>(0.02)</td>
<td>(0.04)</td>
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<td>C₃</td>
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<td>4</td>
<td>1062.84</td>
<td>1062.61</td>
<td>1063.39</td>
<td>1063.72</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(-0.02)</td>
<td>(-0.05)</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>

(Note: The figures shown in the parenthesis are percentage change in the expected net profit compared to the corresponding solution given in table 8.4).
From the above partial sensitivity analysis, it appears that as the parameter $C_1$ alone increases (while other parameters do not change) the optimum profit also increases as compared to the original solution for increasing the mark-up parameter $\theta$.

Similarly when $C_1$ alone decreases the optimum profit also decreases as compared to the original solution.

When only one of the parameters (i.e) either $C$ or $C_3$ or $\lambda$ increases while the other parameters have their steady values, the optimum profit decreases as compared to the original solution for the changing value of the mark-up parameter $\theta$. Similarly exactly reverse phenomena is observed when $C$ (or $C_3$ or $\lambda$) alone decreases. The parameter $C$ is the most sensitive one. The next sensitive parameters in order are $C_3$, $C_1$ and $\lambda$ respectively.
Optimum profit $P^*$ versus Mark-up parameter $\theta$