CHAPTER 1
Introduction

Many real world network design problems can be formulated as graphs where the possible solutions are constrained Minimum spanning trees (CMST), for example, the leaf-constrained Minimum Spanning Tree (MST), degree constrained MST and delay constrained MST. Such problems are more often than not, NP-hard. One of the major CMST problem arising in the field of network design and other related areas of computer science is the Bounded Diameter Minimum Spanning Tree (BDMST) problem, where one seeks a minimum spanning tree on the underlying graph $G$, such that the number of edges on the path between any two nodes in the tree is bounded by a limit, say $D$. Computation of an unconstrained MST is solvable in polynomial time, BDMST problem on the other hand, is known to be \textit{NP}-hard for $4 \leq D < n - 1$, where, $n$ is the number of nodes in the graph $G$. It has been shown to be not only \textit{NP}-hard but also approximate-hard, in that, there is no polynomial time algorithm which can guarantee to find a solution having cost within $\log(|V|)$ of the optimum, where $V$ is the set of vertices in the graph.

Recently, a lot of work has been done on optimal generation of BDMSTs and a number of approaches have been suggested in literature to solve the BDMST problem with varying degree of optimality. The interest in the field of BDMST stems from the fact that BDMST finds application in a variety of domains in computer science and communication. One of the major areas of application of BDMSTs is ensuring quality of service in network design. BDMST problem may also be encountered in the field of mutual exclusion algorithms where the underlying communication infrastructure is based on a tree structure. Another area of application of BDMSTs can be found in Linear Lightwave Networks where multi-cast calls are sent from each source to multiple destinations. Besides these fields, BDMST may find application in textual information retrieval and vehicular routing. This list is
by no means an exhaustive account of the applications to which BDMST may apply, but it gives a fair idea of the interest of the academic community in solving BDMST problem to optimality.

Fueled by the numerous application areas in which BDMST arises as a subproblem, many approaches to solve the same have been given in literature. Exact approaches are exhaustive which restricts their applicability to small sized instances. To solve moderate to large-sized instances often heuristics are applied. Heuristic techniques surrender the guarantee to find optimum to the guarantee of finding a good solution in reasonable time. Usually, exact and heuristic approaches are highly problem specific. The third class of solution approaches is metaheuristics which guide some subordinate problem-specific heuristic towards better exploration of the search space. These approaches may be applicable in scenario when the instance size is large or the landscape is ill-defined.

Concentrating on a single metaheuristic like Genetic Algorithm (GA) is not likely to yield results which are competitive with the state-of-the-art customized metaheuristics. A carefully designed combination of GA with heuristics and/or local search procedures can provide a more efficient search behavior both in terms of quality of the solutions obtained and in terms of computational complexity. A typical genetic algorithm works on an initial population that has been generated randomly. But drawing from the observation that a good initial population of GA accelerates the speed of its convergence to yield good solutions whereas a poor initial populations results in a poor search behavior of GA, the hybridized metaheuristic based solution strategy proposed in this work – Guided Genetic Algorithm for Tree construction \([G-GAT]\) has been designed to work on good initial population which has been generated using a fast construction heuristic \(DRGH\) proposed in this study.

Further, it has been observed that the strength of a GA lies in exploring the search space efficiently; but slow convergence behavior plagues its performance; whereas local search procedures have been found to be efficient at exploiting the identified regions of the search space but are poor at exploration, as a result the search may get trapped in local
optima. *G-GAT* has therefore been designed as a comprehensive platform which hybridizes a quasi greedy heuristic *DRGH* with the strength of GA and the search behavior of this combination is further enhanced by the exploitation capacity of two local search procedures *Arc_Ex_Mut* and *level_Ex_Mut* which have been proposed in this thesis. These local search procedures start from an initial solution, a predefined neighborhood around them is investigated, and if a better solution can be identified, it becomes the new incumbent solution. Thus, the central idea is to focus the search for better solutions on regions of search space in the proximity of already identified good solutions.

We have been able to separate crossover operator from the genetic algorithm and harness its power by suggesting a more exploitative search strategy. An insight has been developed on the effects of increased arity of the crossover operator on the overall performance of the Genetic Algorithm. Further in this work, we have proposed a diversity management mechanism that imposes a filter/cutoff point when the solutions in *G-GAT* or hybrid GAs in general, can be submitted to the more expensive local search phase.

*G-GAT* is a steady-state hybrid GA model, which incorporates a construction heuristic and neighborhood based local search procedures to tackle the issues of diversity, premature convergence and generation of invalid solutions when GAs are applied to constrained MSTs. Here the optimization ability of *G-GAT* and its constituents is compared against the other good performing approaches on a suite of standard BDMST test instances from the OR-Library. The simulation results show that the proposed strategy significantly improves the solution quality and generates robust and near optimal solutions while requiring only a modest amount of computational effort.

Chapter 2 provides the detailed literature review of existing exact, heuristic, metaheuristic and hybridized approaches to solve the BDMST problem. In Chapter 3, the motivation for enhancing the genetic algorithm by hybridizing it with other heuristics to solve the BDMST problem has been given. Chapter 4 provides the algorithms and data structures for implementing the DRGH and justifying its need. Chapter 5 introduces the neighborhood based local search operators and the optimal strategy to navigate through
these neighborhoods. Chapter 6 provides a detailed flow of G-GAT by incorporating the strategies developed in chapter 4 and 5 and further introduces the multiparent crossover operators along with the data structure DVRRT necessary for its implementation. Chapter 7 compares G-GAT and its constituents against well known strategies from literature and provides the discussion of results for a large set of test instances of BDMST problem. Chapter 8 provides the conclusions and suggestions for future research.