CHAPTER 1

INTRODUCTION
1.1. Ionizing and non-ionizing radiation

Radiation is the transport of energy by electromagnetic waves or atomic particles. Figure 1.1 shows that, radiation is classified into two main categories, ionizing and non-ionizing, depending on its ability to ionize matter [1]. Ionization is the ejection of one or more electrons from an atom or a molecule to produce a fragment with a net positive charge i.e. a positively charged ion.

![Diagram of radiation分类](image)

**FIG. 1.1:** Classification of radiation.

Radiation with energy less than that required to produce ions in the irradiated material, is called *non-ionizing radiation*. Examples for this kind are ultraviolet radiation (except the high energy end of the UV-spectrum), sound waves, visible light, infrared radiation, microwaves and radio waves.

*Ionizing radiation* carries enough energy to break chemical bonds and to separate electrons from the parent atoms and molecules, thereby creating ions in the irradiated material. Ionizing radiation consists of directly ionizing and indirectly ionizing radiation. *Directly ionizing radiations* are charged particles (electrons, positrons, protons, alpha particles, heavy ions etc) with sufficient energy to ionize/excite atoms or molecules. *Indirectly ionizing radiations* are uncharged particles (photons, neutrons) that set directly ionizing radiations (charged particles) in motion or that can initiate nuclear transformations.

Directly ionizing radiation deposits energy in the medium through direct Coulomb interactions with orbital electrons of atoms in the medium. Indirectly ionizing radiation deposits energy in the medium through a two-step process. In the
first step, a charged particle is released in the medium (photons release electrons or positrons, neutrons release protons or heavier ions). In the second step, the released charged particles deposit energy to the medium through direct Coulomb interactions with orbital electrons of the atoms in the medium.

Depending on their origin, the indirectly ionizing photon radiations fall into one of the following four categories:

1. **Gamma rays**: resulting from the nuclear transitions.
2. **Characteristic X-rays**: resulting from the electron transitions between inner atomic shells.
3. **Bremsstrahlung** (continuous X-rays): resulting from electron–nucleus interactions.
4. **Annihilation radiation**: resulting from positron–electron annihilation.

Radiations having a wide range of energies form the electromagnetic (EM) spectrum. In EM spectrum, the energy of the radiation increases from bottom to top as the frequency rises (figure 1.2). The spectrum has two major divisions: ionizing radiation and non-ionizing radiation. The classification of radiation as "ionizing" is essentially a statement that, it has enough quantum energy to eject an electron. The threshold energy for ionization is situated somewhere in the ultraviolet region of the EM spectrum. So, gamma rays and X-rays are ionizing radiation (figure 1.2). All forms of nuclear radiations, e.g. alpha, beta particle are also ionizing radiations because of their extremely high energies. Ionizing radiation can produce a number of physiological effects, such as those associated with risk of mutation or cancer, which non-ionizing radiation cannot directly produce at any intensity. The net result of the absorption of non-ionizing radiation, generally, leads to heat in the sample and produces some chemical changes.
1.2. Interaction of gamma rays and X-rays with matter

Gamma rays and X-rays are high-energy electromagnetic waves. According to quantum theory, the apparently continuous electromagnetic waves are quantized and consist of discrete packets of energy called quanta or photons. Thus, a beam of gamma rays (or X-rays) of wavelength $\lambda$ consists of a stream of particle-like photons, each having an energy $E$ given by

$$E = h\nu = \frac{hc}{\lambda} \tag{1.1}$$
where $h$ is Planck’s constant. In practical units, one has $hc = 12.398 \text{ keV} \cdot \text{Å}$.

Gamma ray and X-ray photons have no electric charge, no rest mass and differ only in their origin of production. X-rays are emitted in atomic transitions, whereas gamma rays are emitted only after the emission of an alpha or beta particle. When a nucleus emits an $\alpha$ particle or a $\beta$ particle, the daughter nucleus will be left in an excited state. This excited nucleus makes transition to lower energy state with the emission of gamma ray. These rays can pass through many kinds of materials, including human tissue. Very dense materials, such as lead, are commonly used as shielding to slow or stop these photons.

It is well known that all matter is comprised of atoms. But sub-atomically, matter is made up of mostly empty space. Therefore, when gamma ray (or X-ray) photons pass through a material, they are primarily moving through free space, but may have a chance to encounter with the nucleus or an electron of an atom. Photons of low to moderate energy tend to interact with orbital electrons, whereas those of higher energy tend to interact with atomic nuclei, with some interesting mechanisms and effects. Since the encounters of photons with atomic particles are by chance, a given photon has a finite probability of passing completely through the medium it is traversing. The probability that a photon will pass completely through a medium depends on numerous factors including the photon energy, the medium composition and thickness. The more densely packed the medium atoms, the more likely that the photon will encounter an atomic particle. In other words, more the number of subatomic particles in a material (high-Z), greater is the likelihood that interactions will occur. Similarly, the thicker the material, through which a photon must cross, the more likely is the chance of an encounter. When a gamma ray or X-ray beam passes through a matter, three possible fates await each photon,

1. It can penetrate the section of matter without interacting,
2. It can interact with the matter and be completely absorbed by depositing its energy, and
3. It can interact and get scattered or deflected from its original direction and deposit part of its energy.
There are a number of processes by which gamma photons can interact with matter. Fano [2, 3] has made a systematic catalogue of the possible processes, by which gamma rays may interact with matter:

<table>
<thead>
<tr>
<th>Kind of interaction</th>
<th>Effect of interaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Interaction with atomic electron</td>
<td>(a) Complete scattering</td>
</tr>
<tr>
<td>2. Interaction with nucleons</td>
<td>(b) Elastic scattering (coherent)</td>
</tr>
<tr>
<td>3. Interaction with electric field surrounding nuclei or electrons</td>
<td>(c) Inelastic scattering (Incoherent)</td>
</tr>
<tr>
<td>4. Interaction with the meson field surrounding the nucleons</td>
<td></td>
</tr>
</tbody>
</table>

In general, there are 12 different processes by combining columns 1 and 2. Thus in theory, there are 12 different processes by which gamma rays can be absorbed or scattered (Appendix–I). Many of these processes are quite infrequent and some have not yet been observed. A summary and brief description of the interaction processes of gamma rays are given in Table 1.1 [4].

Gamma rays have many modes of interaction with matter (Table 1.1). Note that the effects described here, are also of relevance to the interaction of X-rays with matter. Throughout the thesis, the term photon refers to both gamma ray and X-ray. The interactions of these photons with matter are independent of the mode of origin of the photons and depend only upon their energy. On passing through matter, an X-ray or gamma ray beam undergoes attenuation, i.e. its intensity decreases gradually by absorption and scattering. Absorption refers to the case in which an incident photon gives up all of its energy. Scattering refers to those photons that have undergone a change in direction after interaction with atoms of matter. Other photons which neither absorbed nor scattered simply pass through matter. This process is known as transmission.
TABLE 1.1 Process by which gamma radiation interacts with matter.

<table>
<thead>
<tr>
<th>Process</th>
<th>Kind of interaction and approximate energy range of maximum importance</th>
<th>Other name and Z-dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absorption by the atomic electrons</td>
<td>Complete absorption by bound the atomic electron. Dominates at low energy (1 keV – 500 keV). Decreases as energy (E) increases.</td>
<td>Photoelectric-absorption, Photoeffect. Proportional to Z^4 (at low energy), and to Z^5 (at high energy).</td>
</tr>
<tr>
<td>Scattering from electrons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coherent</td>
<td>With bound atomic electrons. E &lt; 1 MeV. Greatest at small scattering angles.</td>
<td>Rayleigh scattering. Electron resonance scattering. Proportional to Z^2 (small angles) to Z^3 (large angles).</td>
</tr>
<tr>
<td>Incoherent</td>
<td>With free electrons. Independent of energy.</td>
<td>Thomson scattering. Proportional to Z.</td>
</tr>
<tr>
<td>Absorption by the nucleus</td>
<td>Complete absorption by the nucleus. Emits gamma or nucleons. Dominates in the energy range 10 MeV– 30 MeV.</td>
<td>Nuclear photoeffect. Nuclear photodisintegration.</td>
</tr>
<tr>
<td>Scattering with nucleus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coherent</td>
<td>With material as a whole. Dependent on nuclear energy levels, important only in very narrow resonance range.</td>
<td>Mössbauer effect. Nuclear resonance scattering.</td>
</tr>
<tr>
<td>Incoherent</td>
<td>With nucleus as a whole. Dependent on nuclear energy levels. Narrow resonance maxima at low energies. Broad maximum in the range of 10 MeV– 30 MeV.</td>
<td>Nuclear resonance scattering. Proportional to Z^2/A^2.</td>
</tr>
<tr>
<td>Incoherent</td>
<td>With individual nucleons. E &gt; 100 MeV.</td>
<td>Nuclear Compton scattering.</td>
</tr>
<tr>
<td>Interaction with Coulomb Field</td>
<td>In Coulomb field of nucleus. Threshold at about 1 MeV. Dominates at high E (i.e. E &gt; 5 or 10 MeV). Increases as E increases.</td>
<td>Pair production. Elastic pair production. Proportional to Z^2.</td>
</tr>
<tr>
<td>Pair-production</td>
<td>In Coulomb field of electron. Threshold at about 2 MeV. Increases with E.</td>
<td>Triplet production. Inelastic pair-production. Proportional to Z.</td>
</tr>
<tr>
<td>Interaction with mesons</td>
<td>With the meson field of the nucleus. E ≥ 150 MeV.</td>
<td>Delbrück scattering. Proportional to Z^2.</td>
</tr>
</tbody>
</table>
Major processes by which the gamma rays interact with matter are photoelectric absorption, Rayleigh (coherent) scattering, Compton (incoherent) scattering, pair production and triplet production. Other processes are minor effects and are of interest in special cases. For photon energies below 1 MeV, the three major interaction processes are photoelectric absorption (< 0.5 MeV), Rayleigh scattering and Compton scattering (0.5–5 MeV). Above 1 MeV (> 1.02 MeV, usually > 5 MeV) nuclear-field "pair" production and atomic-field "triplet" production starts to appear, and they become the dominant mode of interaction as the energy increases (> 10 MeV).

1.3. Attenuation coefficients and interaction cross-sections

An important quantity characterizing the penetration and scattering of gamma or X-rays in a medium is the linear attenuation coefficient, \( \mu \). This quantity may be defined as the probability per unit path length that a photon will interact with the medium. Consider a slab of material of thickness, \( t \), located in between a narrowly collimated source of monochromatic photons (gamma rays or X-rays) and a narrowly collimated detector, as indicated in figure 1.3. In a layer \( dx \) within the slab there will occur a reduction of the intensity, \( I \), of the photon beam due to attenuation of the beam. The attenuation occurs due to the absorption (photoelectric absorption and pair production) and scattering (coherent and incoherent) of photons. The resulting fractional reduction of the intensity, probability of interaction, can be written

\[
\frac{dI}{I} = -\mu dx
\]  

(1.2)
 Integrating this equation one obtains

\[ I = I_0 \exp \left[ - \int_0^t \mu(x) \, dx \right] \]  

(1.3)

where \( I_0 \) is the incident intensity of photons. For a homogeneous medium this reduces to the following exponential law:

\[ I = I_0 e^{-\mu t} \]  

(1.4)

It is apparent that \( \mu \) has dimensions of an inverse length (cm\(^{-1}\)). Equation (1.4) is sometimes called the Lambert-Beer’s law. The ratio \( I/I_0 \) represent the fraction of photons transmitted.

The photon interaction can also be characterized by their \textit{mean free path} (\( \lambda \)), which is defined as the average distance traveled by a photon in the medium before an interaction takes place. For a photon traversing in a medium which has linear attenuation coefficient \( \mu \), the probability of interaction in any short distance \( dx \) is \( \mu dx \). Then the probability that a photon can travel a distance \( x \) without any interaction is given by \( \exp(-\mu x) \), equation (1.4). Thus, the mean free path, \( \lambda \), can be calculated as:

\[ \lambda = \frac{\int_0^\infty xe^{-\mu x} \, dx}{\int_0^\infty e^{-\mu x} \, dx} = \frac{1}{\mu} \]  

(1.5)
Therefore, the mean free path is simply the reciprocal of the linear attenuation coefficient. It follows from equation (1.4) that a slab of thickness one mean free path, $\lambda$, reduces the intensity to $I_0/e$ or 0.37$I_0$, where $e$ is the base of the natural system of logarithms.

The thickness of the attenuator that reduces the photon beam intensity to half of its original value i.e. $1/2 I_0$, is called the half-value layer, HVL, and is given by

$$t_{1/2} = \frac{\ln 2}{\mu} = \frac{0.693}{\mu}$$  \hspace{1cm} (1.6a)

Similarly the tenth-value layer, TVL, is defined as the thickness of the attenuator that reduces the photon beam intensity to one tenth of its original value

$$t_{1/10} = \frac{\ln 10}{\mu} = \frac{2.303}{\mu}$$  \hspace{1cm} (1.6b)

1.3.1. Mass attenuation coefficient, $\mu/\rho$

Linear attenuation coefficients are convenient for engineering application, but they are not usually tabulated because of their dependence on the absorber density, $\rho$, i.e. on the physical state of the material (solid, liquid and gas). For example, at a given energy the linear attenuation coefficients of ice, water and steam are all different. Although the materials have the same chemical composition, they represent different phases having different densities. Since the density of a given material can vary widely, a coefficient more accurately characterizing a given material is the density-independent mass attenuation coefficient. Therefore, for the purpose of tabulation it is common practice to use the mass attenuation coefficient. The ratio of the linear attenuation coefficient to the density is called the mass attenuation coefficient, $\mu/\rho$. If $\mu$ is in cm$^{-1}$ and $\rho$ is in g/cm$^3$, then $\mu/\rho$ will be in the customary units of cm$^2$/g. Mass attenuation coefficient values are normalized with respect to material density, and therefore do not change with changes in density.

$$\mu(\text{cm}^{-1})/\rho(\text{g/cm}^3) = \mu/\rho(\text{cm}^2\text{g}^{-1})$$  \hspace{1cm} (1.7a)
The *mass attenuation coefficient* is defined as a measure of probability per unit mass per unit area (or per unit area mass, see equation (1.7b)) for interactions that occurs between the incident photons and matter. The $\mu/\rho$ values for gamma ray and X-ray photons are required for variety of applications in the diverse fields such as radiography, X-ray and gamma ray fluorescence studies, geophysical prospecting, radiation biophysics, nuclear diagnostics (computerized tomography), radiation protection, nuclear medicine, radiation dosimetry etc.

Alternatively, the attenuation law, equation (1.4), can be written in terms of the mass attenuation coefficient, $\mu/\rho$, which is a more fundamental parameter:

$$I = I_0 \exp\left\{-(\mu/\rho)\rho t\right\}$$

(1.7b)

The product $\rho t$ is called the *mass thickness* or *area mass*. It has dimensions of mass per unit area. If $\rho$ is in g/cm$^3$ and $t$ is in cm, then the mass thickness $\rho t$ will be in the customary units of g/cm$^2$. The thickness of absorbers used in radiation measurements is often expressed in mass thickness rather than physical thickness, because it is a more fundamental quantity in this context. For a slab of material the mass thickness is measured as the mass of the slab divided by the area.

**Mixtures and compounds:** For a chemical compound or a mixture (assumed to be homogeneous), the mass attenuation coefficient, $\mu/\rho$, can be calculated from the mass attenuation coefficients of its constituent elements, $(\mu/\rho)_i$, according to the simple mixture law (also called the Bragg additivity law):

$$\mu/\rho = \sum_i w_i (\mu/\rho)_i$$

(1.8)

where $w_i$ is the fraction by weight of the $i^{th}$ constituent element present in a compound or mixture. For any chemical compound, $w_i$ is given by:

$$w_i = \frac{n_i A_i}{\sum_i n_i A_i}$$

where $n_i$ and $A_i$ are the number of formula units and the atomic weight of the $i^{th}$ constituent element. Equation (1.8) is called the *mixture rule* [5]. It is based on the idea that radiation interacts with atoms individually and the atoms do not influence each other’s interaction probability.
The limitation of the mixture rule is that it treats the complex medium under consideration as a mixture of various atomic constituents. Thus it does not account for any variation in the atomic wave functions, which occur due to changes in the molecular, chemical or crystalline environment of the atom. This in fact may seriously affect the accuracy of the results obtained by the application of the mixture rule. This is true only when the incident photon energy is in the vicinity of the edge energies of the constituent elements of the compound [5, 6].

A companion coefficient to $\mu/\rho$, of particular interest in medical and biological applications is the mass energy-absorption coefficient, $\mu_{\text{en}}/\rho$, and is discussed in the section 1.3.2.

Mass attenuation coefficients can be measured experimentally or obtained from existing tabulations. Hubbell [7] published tables of mass attenuation coefficients and the mass energy-absorption coefficients for 40 elements and 45 mixtures and compounds over the energy range from 1 keV to 20 MeV. Recommended theoretical values of the mass attenuation coefficients and mass energy-absorption coefficients for elements having atomic numbers from 1 to 92 and for 48 additional substances of dosimetry interest have been compiled by Hubbell and Seltzer [8]. A convenient alternative to manual calculations by the mixture rule using tabulated data is to generate attenuation data using a computer, which can save a lot of manual work and of course time. For this purpose, Berger and Hubbell [9] developed a computer program, XCOM, for calculating mass attenuation coefficients and cross-sections for any element, compound and mixture, at energies from 1 keV to 100 GeV. The program has undergone number of updates and is now available in a web version. This well-known and widely used program was enhanced and transformed to the Windows platform by Gerward et al. [10, 11] under the name WinXCom.

In the present work, mass attenuation coefficients and interaction cross-sections of the elements and materials were generated using the computer program WinXCom [10, 11], which has the same underlying cross-section database as the Hubbell and Seltzer tabulation [8]. The use of WinXCom saves the user from
interpolation work and makes it possible to export the cross-sectional data to a predefined MS Excel template, thereby greatly facilitating subsequent graphical or numerical data treatment. For mixtures, XCOM requires the weight fractions of the constituent elements as input. In contrast, WinXCom is able to calculate mass attenuation coefficients directly for any mixture of a given set of chemical compounds. WinXCom has a great advantage in the present study, where mass attenuation coefficients are required for several mixtures of the compounds SiO\textsubscript{2}, Na\textsubscript{2}O, CaO, P\textsubscript{2}O\textsubscript{5} for the selected bioactive glasses such as SiO\textsubscript{2}-Na\textsubscript{2}O-CaO-P\textsubscript{2}O\textsubscript{5}, SiO\textsubscript{2}-CaO-P\textsubscript{2}O\textsubscript{5} and SiO\textsubscript{2}-CaO.

1.3.2. Mass energy-absorption coefficient, $\mu_{en}/\rho$

The amount of energy that is actually deposited in the medium is important in evaluating biological effects, explaining the response of a radiation detector and other applications. For this purpose, one can define a linear energy-absorption coefficient, $\mu_{en}$ (with units of inverse length, e.g. cm\textsuperscript{-1}). This coefficient includes only the energy absorbed in the medium from photoelectrons, Compton electrons, and the electron-positron pair. But, energy carried away by scattered Compton photons, annihilation radiation (not subsequently absorbed in the medium), and bremsstrahlung radiation is not included.

A more detailed version of the energy-deposition quantity is the mass energy-absorption coefficient, $\mu_{en}/\rho$, and can be described more clearly through the use of an intermediate quantity called mass energy-transfer coefficient. Mass energy-transfer coefficient, $\mu_{d}/\rho$, is the mass attenuation coefficient multiplied by the fraction of energy of the interacting photons which is transferred to charged particles as kinetic energy. Hence $\mu_{d}/\rho$ is a measure of the average fractional amount of incident photon energy transferred to charged particles as kinetic energy due to all types of interactions of photons with matter.

The mass energy-absorption coefficient, $\mu_{en}/\rho$, takes into account the fraction of the kinetic energy that is subsequently lost in radiative energy-loss processes (bremsstrahlung, annihilation in flight etc) as the charged particles (electrons or
positrons) slow to rest in the absorbing medium. The net kinetic energy of charged particles is in turn, a more or less valid approximation to the amount of photon energy made available for the production of chemical, biological and other effects associated with exposure to ionizing radiation. Therefore, $\mu_{en}/\rho$ has an essential role in estimating the absorbed dose in medical and health physics. The $\mu/\rho$ and $\mu_{en}/\rho$ are basic quantities used in calculations of the penetration and the energy deposition by photons (gamma ray, X-ray and bremsstrahlung) in biological, shielding and other materials [8].

Mixtures and compounds: The mass energy-transfer coefficient, $\mu_{en}/\rho$, for homogeneous compounds or mixtures can be obtained in a manner analogous to that for $\mu/\rho$:

$$\mu_{en}/\rho = \sum_{i} w_{i} (\mu_{en}/\rho)_{i}$$

(1.9)

The mass energy-absorption coefficient, $\mu_{en}/\rho$, involves the further emission of radiation produced by the charged particles in traveling through the medium, and is defined as

$$\mu_{en}/\rho = (1 - g) \mu_{en}/\rho$$

(1.10)

It follows that, the mass energy-absorption coefficient is slightly smaller than the mass energy-transfer coefficient. The factor $g$ in equation (1.10) represents the average fraction of the kinetic energy of secondary charged particles that is subsequently lost in radiative, photon-emitting, energy-loss processes as the particles slow to rest in the absorbing medium.

Attix [12] suggested that simple additivity for $\mu_{en}/\rho$ as that for $\mu/\rho$ is formally incorrect. However, when the numerical values of $g$ are relatively small, the errors in $\mu_{en}/\rho$ incurred by using simple additivity schemes are usually very small, particularly for photon energies below 20 MeV [8]. Hence, the values of mass energy-absorption coefficient, $\mu_{en}/\rho$, for homogeneous compound or mixture can be calculated using simple additivity:

$$\mu_{en}/\rho = \sum_{i} w_{i} (\mu_{en}/\rho)_{i}$$

(1.11)
where \((\mu_{en}/\rho)_i\) is the mass energy-absorption coefficient of the \(i^{th}\) constituent element present in a compound or mixture. The meaning of the symbol \(w_i\) is the same as in equation (1.8). In the present work, values of \((\mu_{en}/\rho)_i\) for elements were taken from the compilation of Hubbell and Seltzer [8]. To obtain \((\mu_{en}/\rho)_i\) values at the absorption edges of all constituent elements, linear interpolation has been performed separately.

### 1.3.3 Interaction cross-sections

The concept of an interaction cross-section is very useful in radiation physics. Each atom has associated with it an area \(\sigma\), called the cross-section, which is imagined to be oriented at right angles to the incident photon beam. Cross-section is a measure of the probability of interaction of photons with matter. The area of the cross-section is so chosen that if an incident photon strikes the area \(\sigma\), interaction will take place; and if an incident photon misses the area \(\sigma\), no interaction takes place. The most common unit for interaction cross-section is the barn (b), where

\[1 \text{ b} = 10^{-28} \text{ m}^2 = 10^{-24} \text{ cm}^2.\]

One can derive an expression for the total target area presented by all interaction centers within a thin layer of area \(A'\) and thickness \(dx\). The layer is assumed to be so thin that the cross-sectional area presented by any one atom does not overlap or cover with that of any other atom as shown in figure 1.4.

![FIG. 1.4 Target areas presented by the interaction centers (atoms).](image)
It is known that the total mass of atoms in one mole of substance, i.e. the mass of the Avogadro number of atoms, is equal to its atomic weight. Hence, the mass of each atom is $A/N_A$, where $N_A$ and $A$ are Avogadro’s constant and the atomic mass, respectively.

Let $N$ be the number of atoms or interaction centers per unit volume. Then the value of $N$ can be computed from definition of density $\rho$:

$$N = \frac{\rho N_A}{A} \quad (1.12)$$

In the thin layer of volume $dV = A'dx$, there are altogether $NdV = NA'dx$ atoms. The total shaded area of figure 1.4, representing the target area for interaction, is then $\sigma NA'dx$. The probability that any one incident photon will interact in this thin layer is simply the ratio of the target area for interaction, $\sigma NA'dx$, to the total area $A'$ of the layer. It follows that the fractional reduction in intensity is

$$\frac{dI}{I} = -\sigma N dx \quad (1.13)$$

This equation is directly comparable with equation (1.2), and thus the linear attenuation coefficient $\mu$ is given by

$$\mu = \sigma N \quad (1.14)$$

Using equation (1.12), the mass attenuation coefficient $\mu/\rho$ can be written

$$\frac{\mu}{\rho} = \frac{\sigma N_A}{A} \quad (1.15)$$

It follows that $\mu/\rho$ is a truly atomic property, since it depends on the interaction cross-section, the atomic mass and Avogadro’s constant. If one uses molar mass, $M$, in place of atomic mass, $A$, one can deduce the following relation, which is valid for a molecule:

$$\frac{\mu}{\rho} = \frac{\sigma N_A}{M} \quad (1.16)$$

A useful property of interaction cross-sections is that they can be added. Thus, if the medium contains different kinds of atoms, the total linear attenuation coefficient can be written
$
\mu = \sum_i \sigma_i N_i
$  

(1.17)

where $\sigma_i$ is the interaction cross-section, and $N_i$ is the number of atoms per unit volume of the $i^{th}$ constituent. The mixture rule, equation (1.8), can easily be derived from equation (1.17). Another consequence of the additivity of cross-sections is that they consider the sum of contributions from various interaction processes as will be seen in the following paragraph.

Only six interaction processes have any real significance in radiation physics: photoelectric absorption, Compton scattering, Rayleigh scattering, pair production, triplet production and photonuclear process. The total photon interaction cross-section, $\sigma$, can be written as the sum of the cross-sections of the partial interaction processes:

$$
\sigma = \sigma_{\text{pe}} + \sigma_{\text{incoh}} + \sigma_{\text{coh}} + \sigma_{\text{pair}} + \sigma_{\text{trip}} + \sigma_{\text{ph},n}
$$

(1.18)

where $\sigma_{\text{pe}}$ (or $\tau$), $\sigma_{\text{incoh}}$ and $\sigma_{\text{coh}}$ are the photoelectric cross-section, incoherent (Compton) and coherent (Rayleigh) cross-sections, respectively. $\sigma_{\text{pair}}$ (or $\kappa_n$) and $\sigma_{\text{trip}}$ (or $\kappa_e$) are the cross-sections for electron-positron pair production in the field of the nucleus and in the field of the atomic electrons ("triplet" production), respectively. $\sigma_{\text{ph},n}$ is the photonuclear cross-section. The photonuclear cross-section, $\sigma_{\text{ph},n}$, is a measurable effect [13]. However, this process in which the photon is absorbed by the atomic nucleus and one or more nucleons (neutrons and/or protons) are ejected, is not readily amenable to systematic calculation and tabulation. This is due to a number of factors including its irregular dependence, both in shape and in magnitude, on both $A$ and $Z$, and its sensitivity to isotopic abundances in a given sample of an element [14–16]. Hence, $\sigma_{\text{ph},n}$ has been omitted from $\mu/\rho$ compilations up to the present, even though at its giant resonance peak between 5 and 40 MeV it can contribute between 2% (high-$Z$ elements) and 6% (low-$Z$ elements) to the total cross section $\sigma_{\text{total}}$ [6, 7].
1.4. Interaction mechanisms

The total mass attenuation coefficient for gamma rays or X-rays traveling through a medium has main contributions from photoelectric absorption, Compton scattering, Rayleigh scattering, pair production and triplet production processes. In the present work, photonuclear and other such small effects are neglected. The main interaction processes are discussed below:

1.4.1. Photoelectric absorption: (γ, e−)

Photoelectric absorption is a process in which the incident gamma ray photon interacts with a tightly bound orbital electron (core electrons) of an atom of the material. If the energy of the incident photon is equal to the binding energy of electron, then the photon is completely absorbed. This results in the removal of that electron from the atom. The freed electron is known as a photoelectron and an ion results when the photoelectron leaves the atom. Figure 1.5 illustrates the photoelectric absorption process.

If the incident photon energy is greater than the binding energy of the electron, then part of the energy of photon is used to overcome binding energy of the electron and most of the remainder is transferred to the freed photoelectron as kinetic energy. During this process the recoil of the entire residual atom conserves the momentum. In order to conserve energy and momentum, the electron must be bound to the atom. Hence this interaction is with the atom as a whole and cannot take place with free electrons. Two subsequent points should also be noted. Firstly, the ejected photoelectron will then produce secondary ionization events with its surrounding atoms in a similar manner to beta particles. The electron rapidly loses its energy and moves only a relatively short distance from its original location. Secondly, the vacancy or hole created in one of the electron orbits of the atom is quickly filled by an outer orbital electron or a free electron from the medium. This transition is accompanied by an emission of characteristic X-ray, often called a fluorescent photon. The energy of the emitted photon is equal to the difference in energy levels of the transferred electron. Some of these photons are reabsorbed by
electrons of less tightly bound shells, which results in the emission of an Auger electron.

**FIG. 1.5:** X-ray fluorescence and Auger effect following a photoelectric absorption.

It follows from the principle of conservation of energy that, the resulting photoelectron has kinetic energy $E_e$ given by:

$$E_e = E - E_B$$  \hspace{1cm} (1.19)

where $E$ is the energy of the incident photon and $E_B$ is the binding energy of the electron. The process is possible only for $E \geq E_B$, because one has to have $E_e \geq 0$. Thus the incident photon energy must exceed binding energy or ionization energy of the electron shell considered, otherwise photoelectric interaction cannot occur.

Figure 1.5 shows the basic processes involved in a photoelectric absorption [17]. Figure 1.5a shows an atom with various energy levels having binding energies $E_K, E_L, E_M$ etc., and a photon of energy $E$ is incident on it. The notation K, L, M etc., refers to electron shells with principal quantum numbers $n = 1, 2, 3$ etc. Figure
1.5b shows the ejected photoelectron leaving the K-shell of the atom with kinetic energy $E - E_K$. The illustrated process is possible only when $E \geq E_K$. Figure 1.5c shows an electron jumping from the L shell into the vacancy in K shell. This electron transfer will result in the production of a fluorescent X-ray photon, called a $K\alpha$ photon, with energy

$$E_{K\alpha} = E_K - E_L$$

There is another de-excitation process, called the Auger effect that can occur. It may happen that the ionization of an inner shell electron produces a photon which in turn gets absorbed by an outer shell electron of the same atom. Thus, as shown in figure 1.5d, the $K\alpha$ photon is immediately absorbed by an M electron, which is ejected as a so-called Auger electron. In the illustrated case, the energy of the Auger electron is

$$E_{ae} = E_K - E_L - E_M$$

This process is enhanced for absorber materials of high atomic number, $Z$. A plot of the photoelectric absorption cross-section versus energy for medium and high-$Z$ elements shows discontinuities at several characteristic energies. The discontinuities in the curve or absorption edges appear at photon energies that correspond to the binding energies of electrons in the various shells of the absorber atom. Therefore the edge lying highest in energy corresponds to the binding energy $E_K$ of the K-shell electron. For photon energies slightly above the K edge, the photon energy is just sufficient to undergo a photoelectric interaction in which a K-electron is ejected from the atom. For photon energies slightly below the edge, this process is no longer energetically possible, and therefore the interaction probability drops abruptly. Similar absorption edges occur at lower energies for the L, M etc., electron shells of the atom. For energies above the K-edge, and between the edges, (for energies less than 0.2 MeV) the photoelectric absorption cross-section is a continuous function approximately proportional to $E^{-3}$ (or in terms of wavelength, proportional to $\lambda^3$) and for large values of energy ($E > 0.5$ MeV) it is
Introduction

proportional to $E^{-1}$. The absorption edge energy $E_K$ corresponds to an absorption edge wavelength $\lambda_K$ given by [cf. Equation (1.1)]

$$\lambda_K = \frac{12.398 \text{keV} \cdot \AA}{E_K}$$

(1.22)

The photoeffect cross-section greatly depends on the atomic number $Z$ (nuclear charge) of the absorber. It is proportional to $Z^n$ where $n$ varies between 4 and 5 depending on the energy of the gamma ray photon. The cross-section decreases with increase in energy. So, the photoelectric absorption will give major contribution to the total attenuation of gamma rays for lower energy photons and high-Z materials. In the current study, photoelectric absorption contribution to the total photon attenuation cross-sections is significant only below 150 keV, but it has much less importance at high energies.

An explanation for the increase in photoelectric interactions with atomic number is that, as atomic number increases the binding energies become closer to the photon energy. The probability is greater for more tightly bound electrons. Therefore K shell electrons are most affected, about 80% of the photoelectric absorption takes place in the K shell, provided the gamma ray energy exceeds the K-electron binding energy.

Most of the early calculations of the atomic photo effect were for K-shell only, typified by the high-energy work of Pratt [18] showing the asymptotic behavior going to arbitrary high energies, and by Pratt et al. [19] in the range 200 keV to 2 MeV. Hultberg et al. [20, 21] used the Swedish BESK computer to compute K-shell cross-section including photoelectron angular distributions, for 21 elements $Z = 1$ to 100 for photon energies extending as low as 1 keV ($Z = 1$) to as high as 10 MeV ($Z = 92$). Pratt et al. [22] reviewed the development on theory of photo effect for incident photon energies above 10 keV.

Rakavy and Ron [23, 24] produced a significant advance with their atomic photo effect cross-section calculations for not only the K shell, but also for all the significantly contributing higher sub-shells ($L_{\text{I-III}}$, $M_{\text{I-V}}$, $N_{\text{I-VII}}$ and $O_{\text{I-III}}$) over the
energy range 1 keV to 2 MeV for $Z = 13$, 26, 50, 74 and 92. Other important multi-shell photo effect calculations in this time period, which also provide historical reviews of earlier work, are those by Alling and Johnson [25], Matese and Johnson [26], and by Schmickley and Pratt [27]. Interpolations from these works, along with the K-shell high-energy asymptotic behavior provided by Pratt[18], were helpful in constructing the tables of Hubbell [6], along with a large body of experimentally determined total photo effect cross-section data obtained by subtracting "known" theoretical scattering cross-sections from measured total cross-sections (attenuation coefficients).

However, Scofield [28] introduced the major advance with his systematic calculations of atomic photo effect cross-sections for all sub shells and for all elements $Z = 1$ to 101 over the photon energy range from 1 keV to 1.5 MeV. These non-relativistic calculations were based on solution of the Dirac equation for the orbital electrons moving in a static Hartree-Slater central potential. For $Z = 2$ to 54, Scofield [28] provided renormalization factors to convert his cross-section results to values expected from a relativistic Dirac-Hartree-Fock (DHF) computation.

This renormalization was performed for two subsequent compilations of $\mu/\rho$ and $\mu_{el}/\rho$ by Hubbell [7, 29] and by Hubbell et al. [30]. However, detailed comparisons [31, 32] with the extensive NBS/NIST $\mu/\rho$ measurement database tend to favor the un-renormalized $\sigma_{pe}$ over the renormalized values [28]. Hence, in subsequent compilations by Berger and Hubbell [33], and Hubbell and Seltzer [8], the unrenormalized $\sigma_{pe}$ values [26] have been used.

Scofield [34] later extended these calculations down to 0.1 keV, and these unrenormalized values are also included in the compilation by Saloman and Hubbell [31] and Saloman et al. [32], both numerically and graphically, with the NBS/NIST $\mu/\rho$ measurement data base as well as with an experiment-based compilation by Henke et al. [35]. Values of $\sigma_{pe}$ are also given in the extensive theoretical results of Chantler [36] computed within a self-consistent Dirac–Hartree–Fock framework, mentioned earlier. For the elements $Z = 1$ to 92, the
Introduction

lower-bound energy varies between 1 eV and 10 eV, and the upper-bound energy varies between 0.4 MeV and 1 MeV.

1.4.2. Compton scattering or Incoherent scattering: (\(\gamma; \gamma', e^-\))

The Compton process is appreciable only when the photon energy passes the limiting value of the photoelectric absorption process. In Compton scattering, a gamma ray photon interacts with a free or weakly bound electron (outer, least tightly bound electrons), loses some of its energy and is deflected from its original direction of travel. In this process part of the incident photon energy is imparted to the recoil electron; hence it is called inelastic scattering. In this case there is no phase relationship between photons scattered by the different electrons of the same atom and hence it is also said to be incoherent. A sketch of this interaction is given in figure 1.6 [17]. The interaction looks like a collision between the photon and a "free" electron. By the conservation of momentum and energy, the electron must recoil in a specific direction with a specific energy. The recoil electron rapidly loses its energy and moves only a relatively short distance in the medium. The scattered photon deflects off in a different direction with lower energy. The direction of the Compton photon, a "secondary photon" like the fluorescent photon in a photoelectric process, is not along the same trajectory as the initial incoming photon. This deflected or scattered photon may escape from the matter or undergo further Compton scattering or can be absorbed through the photoelectric effect within the material.

The relation between photon deflection and energy loss, assuming the electron to be initially free and at rest, can be determined from conservation of momentum and energy between the photon and the recoiling electron. Let \(E\) and \(E'\) be the energy of incident and scattered gamma ray photon, and \(E_e\) be the energy of recoil electron. By applying the law of conservation of momentum in the direction of the incident photon and at right angles to direction the incident photon:
\[ \frac{h}{\lambda} = \frac{h}{\lambda'} \cos \theta + p_e \cos \psi \quad \text{(Conservation of momentum in X-direction)} \]  

(1.23a)

\[ 0 = \frac{h}{\lambda'} \sin \theta - p_e \sin \psi \quad \text{(Conservation of momentum in Y-direction)} \]  

(1.23b)

**FIG. 1.6:** The Compton scattering. Scattering angle = \( \theta \), recoil angle = \( \psi \).

By applying the law of conservation of energy:

\[ \frac{hc}{\lambda} + E_0 = \frac{hc}{\lambda'} + E_{e'} \]  

(1.23c)

where \( \lambda \) and \( \lambda' \) are the wavelengths of the incident and scattered photons; \( p_e \) and \( E_{e'} \) are the momentum and energy of the recoiling electron; and \( \theta \) and \( \psi \) is the scattering angle and electron recoil angle, respectively. Since the recoil energy \( E_{e'} \) of the electron is greater than its initial energy \( E_0 \), equation (1.23c) immediately shows that the scattered photon has less energy i.e. a longer wavelength or lower frequency than the incident photon. Because the incident and scattered photons have different frequencies, the scattered photon is not the incident photon moving in a different direction, rather, the incident photon is annihilated and the scattered photon is created in the interaction process.
Replacing $E_0$ by $m_0c^2$ and recalling from relativity theory that $E_e^2 = (p_e c)^2 + E_0^2$, one can solve equations (1.23a-c) for $\lambda'$ and $E_e$. For $\lambda'$ one can show that

$$\lambda' = \frac{\Delta \lambda}{m_0c} = \frac{\hbar}{m_0c}(1 - \cos \theta) \tag{1.24}$$

This equation was first derived by Compton and the difference, $\lambda' - \lambda = \Delta \lambda$, is called Compton shift. The quantity $\frac{\hbar}{m_0c}$ has the dimensions of length and is called Compton wavelength of the electron. Its numerical value is 0.02426 Å. The kinetic energy $E' = h\nu' = hc / \lambda'$ of the scattered photon, solving equation (1.24), is then given by

$$E' = \frac{E}{1 + \frac{E}{m_0c^2}(1 - \cos \theta)} \tag{1.25}$$

where $E = h\nu$ is the energy of the incident photon, and $m_0c^2 = 511$ keV is the rest mass energy of the electron.

The kinetic energy of the recoil electron is equal to the difference of the energy lost by the photon and the electron binding energy. For Compton process to occur, the photon energy should be much greater than electron binding energy ($E \gg E_B$). So the kinetic energy of the recoil electron is very nearly equal to the energy lost by the photon.

$$E_e = E - E' = \frac{E}{m_0c^2}(1 - \cos \theta) \tag{1.26}$$

Two extreme cases can be identified:

1. A grazing angle scattering, i.e. $\theta \approx 0$. In this case equations (1.25) and (1.26) predict that $E' \approx E$, and $E_e \approx 0$. In this extreme, the recoil electron has very little energy and the scattered photon has nearly the same energy as the incident photon.
2. A head-on collision or backscattering collision, i.e. $\theta = \pi$. In this case, the incident photon is backscattered toward its direction of origin, whereas the electron recoils along the direction of incidence. In this extreme, the scattered photon has minimum energy and hence maximum energy is transferred to the electron. Inserting $\theta = \pi$ in equations (1.25) and (1.26) gives

$$E_{\text{min}} = \frac{E}{2E} \approx \frac{m_0c^2}{2} = 256 \text{ keV}; \text{ if } E >> m_0c^2 / 2 \quad (1.27a)$$

$$E_{\text{c, max}} = \frac{E}{m_0c^2} \approx E - \frac{m_0c^2}{2} = E - 256 \text{ keV}; \text{ if } E >> m_0c^2 / 2 \quad (1.27b)$$

In normal circumstances, all scattering angles will occur in the medium. Therefore, a continuum of energies can be transferred to the electron, ranging from zero to the maximum predicted by equation (1.27b). Klein and Nishina, assuming the electron to be initially free and at rest, formulated the basic theory of Compton scattering [37]. Departures from the Klein-Nishina theory occur at low energies because of electron binding effects. The angular distribution of the scattered radiation is predicted by the Klein-Nishina formula for the differential scattering cross-section (having dimensions of area per unit solid angle) per electron. For unpolarised photons, the free electron Compton scattering cross-section is given by Klein-Nishina (KN) formula [38]:

$$\frac{d\sigma_{\text{c, KN}}}{d\Omega} = \frac{r_0^2}{2} \left( \frac{1 + \cos^2 \theta}{[1 + \alpha(1 - \cos \theta)]^2} + \frac{\alpha^2 (1 - \cos \theta)^2}{[1 + \alpha(1 - \cos \theta)]^3} \right) \quad (1.28)$$

where $\alpha = E / m_0c^2$, $r_0 = e^2 / 4\pi\varepsilon_0m_0c^2 = 2.818 \times 10^{-15} \text{ m}$ is the classical radius of electron, and $d\Omega$ is the solid angle element. A study of equation (1.28) shows that there is a strong tendency for forward scattering at higher photon energy. As $\alpha \rightarrow 0$, equation (1.28) reduces to classical differential Thomson scattering cross-section.
\[
\frac{d\sigma_{c}^{KN}}{d\Omega} = \frac{r_0^2 (1 + \cos^2 \theta)}{2} \quad (1.29a)
\]

Integration of equation (1.28) over all angles gives the total Klein-Nishina cross-section \(\sigma_{c}^{KN}\) per electron.

\[
\sigma_{c}^{KN} = 2\pi r_0^2 \left( \frac{1 + \alpha}{\alpha^2} \left[ \frac{2(1 + \alpha)}{1 + 2\alpha} - \frac{\ln(1 + 2\alpha)}{\alpha} \right] + \frac{\ln(1 + 2\alpha)}{2\alpha} - \frac{1 + 3\alpha}{(1 + 2\alpha)^2} \right) \quad (1.29b)
\]

As \(\alpha \to 0\) (in the low-energy limit), this expression becomes equal to the classical Thomson cross-section, \(\sigma_T\):

\[
\sigma_{c}^{KN} \to \sigma_T = \frac{8}{3} \pi r_0^2 = 0.665 \text{ b/atom} \quad (1.29c)
\]

It should be noted that the extrapolation \(\sigma_{c}^{KN} \to \sigma_T\) for \(\alpha \to 0\) is valid only for free electrons. For bound electrons one has \(\sigma_{c} \to 0\) for \(\alpha \to 0\). For an atom containing Z electrons, Compton cross-section per atom is given by,

\[
\sigma_c = Z\sigma_c^{KN} \quad (1.29d)
\]

At lower energies, where electron motion and binding energies are a significant fraction of the incident photon energy, the Klein-Nishina differential cross-section can be modified by the use of an incoherent scattering function \(S(q, Z)\) in which \(q\) is the magnitude of the momentum transfer variable dependent on the incident photon energy and the deflection angle of the scattered photon, and \(Z\) is the atomic (charge) number of the nucleus of the target atom.

\[
\frac{d\sigma_c}{d\Omega} = S(q, Z) \frac{d\sigma_{c}^{KN}}{d\Omega} \quad (1.30a)
\]

The factor \(S(q, Z)\) represents the probability that an atom be raised to any excited or ionized state as a result of a sudden impulsive action, which imparts a recoil momentum \(q\) to an atomic electron. The momentum transfer \(q\) is defined by

\[
q = k' - k \quad (1.30b)
\]
where $\mathbf{k}'$ and $\mathbf{k}$ are the wave vectors of the scattered and incident rays. Since binding effects are important mainly for small momentum transfers, the approximation $\mathbf{k}' \approx \mathbf{k}$ is usually made, so that

$$q \approx 2k \sin(\theta/2) \quad (1.30c)$$

The Compton scattering is a predominant interaction process at intermediate gamma energies, approximately in the energy range $0.05 \text{ MeV} > E > 5 \text{ MeV}$. The Compton scattering cross-section decreases as incident photon energy increases and varies linearly with the atomic number, $Z$. The interaction probability depends on the electron density, which is proportional to $Z/A$ and nearly constant for all materials.

Hubbell et al. [39] tabulated the incoherent scattering functions, $S(q, Z)$, based on the nonrelativistic Hartree-Fock model for the elements $Z = 1$ to 100. The Hubbell et al. [39] $S(q, Z)$ values were pieced together from data available in the literature, including the work of Pirenne [40] ($Z = 1$), Brown [41–44] ($Z = 2$ to 6, with configuration interaction) and by Cromer and Mann [45] and Cromer [46] ($Z = 7$ to 100, from a non-relativistic Hartree-Fock model). Kahane [47] has used the Ribberfors and Berggren [48] relativistic treatment to compute and tabulate the relativistic Dirac-Hartree-Fock photon incoherent scattering functions $S(q, Z)$ for all elements $1 \leq Z \leq 110$.

The $S$-matrix theory with its claimed higher accuracy is available for Compton scattering [49]. Bergstrom and Pratt have reviewed the exact second order relativistic $S$-matrix prescriptions [50]. However, it appears that no comprehensive tabulations of incoherent scattering cross-sections, utilizing $S$-matrix formalism, are available for all $Z$ and photon energies. Therefore, compilations of $\mu/\rho$ such as XCOM PC program of Berger and Hubbell [33], the tabulations of Hubbell-Seltzer [8], and the database of Cullen et al. [51], still rely on the incoherent scattering function $S(q, Z)$ approach.

*Strictly speaking, the momentum transfer is $\eta q$. 
1.4.3. Rayleigh scattering or coherent scattering: ($\gamma, \gamma$)

Rayleigh scattering is the process in which gamma ray photons interacts with a bound orbital electrons (with the combined action of the whole atom), and the atom is neither ionized nor excited. In this process, the incident photon does not impart any energy to the atom and the electrons remains in the same energy states, hence it is called elastic scattering. There is a definite phase relationship between the scattered photons from different electrons of the same atom and hence it is also said to be coherent scattering.

In this interaction process, a photon is scattered by the atomic electron cloud as a whole and is scattered through only a small angle, with the entire atom taking up the recoil. Because of the great mass of an atom very little recoil energy is transferred to the atom. Thus, from energy and momentum conservation considerations, the deflected photon undergoes negligible loss of energy or change in wavelength. Figure 1.7 illustrates Rayleigh scattering process, in which the incident photon with $\lambda_1$ interacts with an atom and the scattered photon with $\lambda_2$ is being emitted with approximately of same wavelength and energy.

![FIG. 1.7: The Rayleigh scattering.](image)

The atomic form factor or scattering factor, $F(q, Z)$, was introduced to take into account the charge distribution of bound electrons. It is somewhat complementary to the incoherent scattering function, $S(q, Z)$. The atomic form factor represents the amplitude for the scattered wave arising from a superposition of the partial waves.
scattered from the different parts of the atomic charge distribution. The square of this form factor is proportional to the scattered intensity with momentum transfer $q$. In this process, $q$ is well represented by equation (1.30b) since it is assumed that $k' = k$. In the form factor approximation the differential Rayleigh scattering cross-section, $d\sigma_r / d\Omega$, for the initially unpolarized photons and averaged over final polarization states, is equal to the product of the classical Thomson scattering cross-section i.e. the low-energy form ($\alpha \rightarrow 0$) of the Klein-Nishina formula, equation (1.29a) and the square of the atomic form factor.

$$\frac{d\sigma_r}{d\Omega} = \frac{1}{2} r_0^2 (1 + \cos^2 \theta) F^2(q, Z)$$  \hspace{1cm} (1.31a)

The square of the atomic form factor takes into account the electronic charge distribution and expresses the probability that the recoil momentum is taken up by the atom as a whole. Atomic form factor is a function of the momentum transfer and the atomic number $Z$.

$$F(q, Z) = \int \rho(r) \exp(i\mathbf{k} \cdot \mathbf{r}) \, d^3r$$  \hspace{1cm} (1.31b)

where $\rho(r)$ is the charge density of electrons at a distance $r$ from the nucleus, and $h\mathbf{k} = q$ is the momentum transfer which is usually expressed in $mc$ units.

Rayleigh scattering is never the dominant photon interaction process, but in the energy region just below the photoelectric K-shell absorption edge it can account for up to 10% of the total attenuation cross-section. Rayleigh scattering is possible only at low photon energies (< 50 keV). The atomic cross-section for Rayleigh scattering is proportional to $E^{-2}$ and $Z^2$ at small angles and $Z^3$ at large angles.

Young [52] has given a summary of this and other photon scattering work by Strutt [Lord Rayleigh]. Kane et al. [53] gave an extensive review on elastic scattering. For compilations of $\mu/\rho$ in the medical and biological region of interest, the coherent scattering cross-section $\sigma_{coh}$ has been computed by numerical integration of the Thomson formula [54] weighted by $F^2(q, Z)$. 
Theoretical values of $F(q, Z)$ were compared graphically with available measurements in the review and compilation by Hubbell et al. [39]. Although relativistic Hartree-Fock values were available at that time, the $F(q, Z)$ values tabulated for $7 \leq Z \leq 100$ were taken from the non-relativistic Hartree-Fock results [55, 56]. For $Z = 1$, the $F(q, Z)$ values in Hubbell et al. [39] were computed from the 'exact' formula of Pirenne [40], and for $Z = 2$ to 6 were taken from the configuration interaction calculations by Brown [41, 42, 44, 57]. Thus, in the Hubbell et al. [39] compilation, both $S(q, Z)$ and $F(q, Z)$ are tabulated for all $Z = 1$ to 100 over the range $0.005 \, \text{Å}^{-1} \leq q \leq 10^9 \, \text{Å}^{-1}$, and both $\sigma_{\text{incoh}}$ and $\sigma_{\text{coh}}$ are tabulated for all $Z = 1$ to 100 over the photon energy range 100 eV to 100 MeV.

In the XCOM $\mu/\rho$ data set [33], and in the Hubbell and Seltzer [8] tabulation, the values of $\sigma_{\text{coh}}$ are taken from the relativistic compilation of Hubbell and Øverbø [58]. For these computations relativistic theoretical values of $F(q, Z)$ were pieced together from Pirenne [40] for $Z = 1$, and for the other elements, over the different ranges of $q$ and $Z$, from Doyle and Turner [59], Cromer and Waber [60] and from Øverbø [61, 62]. Somewhat higher accuracy is anticipated from the relativistic Hartree-Fock-Slater modified atomic form factor (MFF) calculations by Schaupp et al. [63] for $F(q, Z)$ for $Z = 1$ to 100, $0 \, \text{Å}^{-1} \leq q \leq 100 \, \text{Å}^{-1}$.

Current theoretical efforts towards improved values of the coherent scattering cross-section $\sigma_{\text{coh}}$ are focused on use of the second-order relativistic $S$-matrix formalism [64–66]. This formalism is capable of revealing anomalous scattering, particularly in the vicinity of absorption edge energies.

1.4.4. Pair production or elastic pair production: ($\gamma; e^-, e^+$)

The creation of an electron-positron pair ($e^-, e^+$) when a gamma photon interacts with the coulomb field of a nucleus is called pair production. In this process, the incident photon is completely absorbed and in its place an electron-positron pair appears.

$$\gamma \rightarrow e^- + e^+$$
Pair production is an example of materialization of energy. This interaction has a threshold of 1.022 MeV (i.e. twice the electron rest mass energy: \(2m_0c^2\)), because that is the minimum energy required to create the electron and positron. If the photon energy exceeds 1.022 MeV, the excess energy is shared between the electron and positron as kinetic energy. The total kinetic energy of the resultant particles is equal to the incident photon energy minus the rest mass energy of the two particles which have been created.

The electron and positron from pair production are rapidly slowed down in the absorber. After losing its kinetic energy, the positron will eventually encounter one of the atomic electrons (free electron), and these two particles will annihilate each other (positron interaction), converting their mass directly into energy which produces two gamma photons of energy equal to the electron rest energy, roughly 0.511 MeV. These two 0.511 MeV photons travel exactly in opposite directions (180°) away from each other. These lower energy gamma photons may interact further by Compton scattering or the photoelectric effect, or they may escape.

Pair production is impossible for gamma rays with energy less than 1.022 MeV. Above this threshold, the probability of the interaction increases rapidly with energy. At high energies, above several MeV, it becomes the dominant mode of interaction. The cross-section for pair production, \(\sigma_{\text{pair}}\), varies approximately as the square of the nuclear charge \(Z\), and is significant in high-\(Z\) materials.

\[
i.e. \quad \sigma_{\text{pair}} \sim Z^2
\]  

(1.32a)

The \(\sigma_{\text{pair}}\) calculation [30] begins with the Bethe and Heitler [67] Born-approximation unscreened pair-production cross-section as an initial approximation, to which Coulomb screening corrections and radiative corrections are applied. The differential Bethe-Heitler unscreened \(\sigma_{\text{pair}}\) cross-section has been cast in forms, suitable for computation, by Bethe and Maximon [68], Davies et al. [69] and by Maximon [70].

The Coulomb correction for the Hubbell et al. [30] computations was pieced together from the low-energy results of Øverbø et al. [71, 72], the intermediate-
energy results of Øverbø [73] and the high-energy results of Sørensen [74, 75], which is the high energy limit go to the Davies et al. [69] extreme relativistic Coulomb correction. Screening corrections were pieced together from the near-threshold results of Tseng and Pratt [76, 77] and the intermediate- and high-energy work of Øverbø [78]. The Øverbø work [78] used the Jost et al. [79] expression for nuclear-field pair production in the Born approximation for small nuclear recoil, but without the extreme high energy approximation. This expression required values of the atomic form factor $F(q, Z)$, for which Øverbø [78] used the relativistic $F(q, Z)$ values pieced together from Doyle and Turner [80], Cromer and Waber [60] and Øverbø [61, 62], later published as systematic tabulations by Hubbell and Øverbø [58]. The radiative corrections [81, 82] of the order of $1/137$ and associated with the emission and re-absorption of virtual photons and with the emission of both soft and hard real photons, were obtained from Mork and Olsen [83].

1.4.5. Triplet production or inelastic pair production: ($\gamma; 2e^-, e^+$)

As second order effect, gamma photon interacts with the Coulomb field of an orbital/free electron, to create an electron, a positron and frees the original electron. This is called triplet production. The threshold energy for this process is $4m_0c^2$ (2.044 MeV). In this process, three particles share the available energy. The probability increases with increase in incident photon energy. The triplet production cross-section, $\sigma_{\text{trip}}$, is proportional to $Z$. Figure 1.8 illustrates the pair and triplet production process.

![Diagram of pair and triplet production](image)

**FIG. 1.8:** The pair and triplet production.
The cross-section for triplet production, $\sigma_{\text{trip}}$, in the field of one of the atomic electrons varies as $Z$ times the square of the unit charge, or

\[ \text{i.e. } \sigma_{\text{trip}} \sim Z \]  

(1.32b)

This cross-section is usually called the "triplet" cross-section, since the atomic electron involved in this process is also ejected from the atom, giving rise to a trident signature including the created electron and positron, when observed in a cloud chamber.

For $Z = 1$ (hydrogen) $\sigma_{\text{trip}}$ is approximately equal to $\sigma_{\text{pair}}$, and it becomes progressively less important for higher $Z$ materials, according to

\[ \sigma_{\text{trip}} / \sigma_{\text{pair}} \sim \frac{1}{Z} \]  

(1.32c)

Both $\sigma_{\text{pair}}$ and $\sigma_{\text{trip}}$ are extensively reviewed, calculations are performed, and tabulations of these cross-sections are provided for all elements $Z = 1$ to 100 over the photon energy range 1 MeV to 100 GeV by Hubbell et al. [30]. Values from this publication are still used in current $\mu$/\rho compilations, for example in Berger and Hubbell [33] (XCOM), Hubbell and Seltzer [8] and in Cullen et al. [51].

The starting point for these computations of $\sigma_{\text{trip}}$ is again the Bethe and Heitler [67] Born-approximation, now requiring the retardation effect due to the recoil of the target atomic electron. This effect is included in the unscreened formula of Borsellino [84], improved by including higher terms by Ghizzetti [85]. Corrections for exchange could be obtained as a ratio between the results by Haug [86] to the Borsellino–Ghizzetti results which neglected this effect. A scheme for including screening is given by Wheeler and Lamb [87] who presented some results computed using Thomas–Fermi [89, 90] statistical-atomic-model values of $S(q, Z)$. In the Hubbell et al. [30] computations, screening corrections were obtained by replacing the Thomas-Fermi $S(q, Z)$ values in the Wheeler-Lamb formula by the $S(q, Z)$ values in Hubbell et al. [39], based on the configuration-interaction $Z = 2$ to 6 values of Brown [41–44] and non-relativistic Hartree–Fock $Z = 7$ to 100 values of Cromer and Mann [45] and Cromer [46].
1.5. Relative Importance of Photon Attenuation Processes

The relative probability of one of the three most important individual interactions to occur is a function of the incident photon energy $E$ and the atomic number $Z$ of the absorber and is shown in figure 1.9 [1, 91]. The left curve represents the region where the atomic cross-section for the photoelectric effect and Compton effect are equal ($\sigma_{pe} = \sigma_{Comp}$) and the right curve is for the region where the atomic Compton cross-section equals the atomic pair production cross-section ($\sigma_{Comp} = \sigma_{pair}$). The two curves display the points in the $(E, Z)$ diagram for which $\sigma_{pe} = \sigma_{Comp}$ or $\sigma_{Comp} = \sigma_{pair}$ and thus delineate the regions of photoelectric effect predominance at low photon energies and large $Z$, Compton effect predominance at intermediate energies and for all $Z$, and pair production predominance at high photon energies and large $Z$. For example, a 100 keV photon will interact with lead ($Z = 82$) predominantly through the photoelectric effect and with oxygen ($Z = 8$) predominantly through the Compton effect. A 10 MeV photon, on the other hand, will interact with lead predominantly through pair production and with oxygen predominantly through the Compton effect.

![Relative importance of the three major types of gamma ray interactions.](image)
1.5. Effective atomic number and electron density

The study of ionizing radiations, gamma rays and X-rays, and their interaction with different materials has gained great importance and finds wide application in areas such as medicine, agriculture, industry, radiation sterilization, medical radiation dosimetry, radiation shielding, science and technology, space research programs, radiometric gauging and process control, security screening etc [49, 93-95].

The atomic number, \( Z \), is a ubiquitous parameter in radiation physics, and in nuclear and atomic physics which occurs in almost any formula. For a complex medium the effective atomic number is a convenient parameter, in some cases, for representing X-ray and gamma ray interactions, for example: in designs of radiation shielding or in calculations of absorbed dose in radiotherapy. However, as stated by Hine [96], for gamma photon interactions a single number cannot represent the "effective" atomic number of a multielement material, composed of several elements, uniquely across the entire energy region. Instead, one defines the so-called effective atomic number, \( Z_{\text{eff}} \). For each of the different processes, by which X-rays or gamma rays can interact with matter, the various atomic numbers in the material have to be weighted differently. Accordingly, the effective atomic number, \( Z_{\text{eff}} \), and effective electron density, \( N_{e,\text{eff}} \), are not true constants for a given material, but vary with energy depending on the interaction processes involved. The effective atomic number is closely related to the electron density. The \( Z_{\text{eff}} \) and \( N_{e,\text{eff}} \) represent the "effective" number of electrons per atom and per unit mass of the material, respectively. The \( Z_{\text{eff}} \) concept is to describe the properties of the composite materials in terms of an equivalent element i.e. a compound or a mixture may be considered as a single element with an effective atomic number, \( Z_{\text{eff}} \) [97].

The parameter \( Z_{\text{eff}} \) is very useful in choosing a substitute composite material in place of an element or a material at a given energy depending on the requirement. The energy absorption in the given medium can be calculated by means of well-established formulae if certain constants such as \( Z_{\text{eff}} \) and \( N_{e,\text{eff}} \) of medium are known. Among the parameters determining the constitutive structure of an
unknown object or material, one should especially note the effective atomic number, $Z_{\text{eff}}$. In fact, the value of this parameter can provide an initial estimation of the chemical composition of the material. A large $Z_{\text{eff}}$ generally corresponds to inorganic compounds and metals, while a small $Z_{\text{eff}}$ ($\leq 10$) is an indicator of organic substances. For many applications, for example in radioisotope monitoring, cross-section studies of absorption, scattering and attenuation of electromagnetic radiation, testing of multi-component, heterogeneous and composite materials etc., this parameter is of principal significance.

The scattering and absorption of gamma rays and X-ray are related to the density and atomic numbers of an element. For a material composed of several elements, it is related to the effective atomic number and the effective electron density. The effective atomic number also finds its utilization in the computation of some other useful parameters, namely the effective dose and buildup factor.
References


