The present investigations have been carried out towards the fulfillment of the requirements for the award of a Ph.D. degree in Physics of V.B.S. Purvanchal University, Jaipur (U.P.), India under the supervision of Dr. R.S. Chauhan, Reader and Head, Department of Physics, R.P.G. College Jamuhai, Jaipur (U.P.), India and Co-supervision of Dr. K.S. Upadhyaya, Reader and Head, Department of Physics, K.N. Govt. P.G. College Gyanpur, Sant Ravidas Nagar, Bhadohi (U.P.), India.

The thesis deals with some investigations in hypothesis of path integral duality. It has been divided into four chapters. The first chapter is introductory. So, we have formulated and discussed some of the techniques and results which are relevant for our subsequent investigations. Hence, we have presented, Bosonic fields, relation to the Hilbert formalism, Euclidean path integrals, diagrams and determinants, and Fermionic fields.

In chapter II, One immediate consequence of this result is the interpretation in terms of the “zero-point length” mentioned in the introduction. We know that the kernel $K (x, y, \tau \mid g)$ has an expansion of the form
(1) \[ K(x, y, c | g) = \left( \frac{1}{4\pi c} \right)^{\frac{D}{2}} \exp \left( -\frac{(x - y)^2}{4c} \right)[1 + .......] \]

where the ellipsis represents metric-dependent corrections.

Using Eq. (2.84) in Eq. (2.83) we can write our propagator as

\[
G_p(x, y | g) = \int_0^\infty dc \; e^{-m^2 c} \left( \frac{1}{4\pi c} \right)^{\frac{D}{2}} \]

(2) \[
\exp \left( -\frac{(x - y)^2 + 4L_p^2}{4c} \right) x[1 + .......] .
\]

Thus the net effect of our modification is to add a “zero-pint length” \( 4L_p^2 \) to \( (x - y)^2 \) in the exponent, modifying the leading singular behaviour of the original propagator. In other words, the modification of the path integral based on the principle of duality leads to results which are identical to adding a “zero-point length” in the spacetime interval.

We wish to argue that the connection shown above is non-trivial; we know of no simple way of guessing this result. The standard Feynman propagator of quantum field theory can be obtained either through a lattice regularization of a path integral or from Schwinger’s
proper time representation. By adding a “zero-point length” in the Schwinger’s representation we obtain a modified propagator. Alternatively, using the principle of duality, we could modify the expression for the path integral amplitude on the lattice and obtain in the continuum limit- a modified propagator. Both these constructions are designed to suppress energies larger than Planck energies. However, there is absolutely no reason for these two expressions to be identical. The fact that they are identical suggests that the principle of duality is connected in some deep manner with the spacetime intervals having a "zero-point length". Alternatively, one may conjecture that any approach which introduces a minimum length scale in spacetime (like in string models) will lead to some kind of principle of duality. This conjecture seems to be true in conventional string theories though it must be noted that the term duality is used in a somewhat different manner in string theories.

The second obvious point, of course, is the improved ultraviolet behavior in the theory. For example, this ultraviolet finiteness allows a renormalization procedure to be carried out without the need for regularization in $\lambda \phi^4$ theory and QED. Renormalized coupling constants now have no divergent pieces and depend on the
Planck length. In this sense, the Planck length acts as a natural cutoff, as to be expected.

The third issue is related to anomalies (like the trace anomaly) in curved spacetime. The conventional calculations do depend on the need to regularize the expressions in one way or the other. With ultraviolet finiteness it is not clear whether the anomalies will survive or not.

There is another implication of this result which requires study. To begin with a Planck length cutoff is equivalent to changing the density of states at high energies. The number of quantum states accessible to field theoretic systems becomes effectively finite. In the case of a black hole—for example—the number of microstates will be finite and will lead to a finite value of its entropy.

In chapter III, we have presented the quantum gravitational corrections to three radiative corrections: to the vacuum polarization, electron self energy, and vertex function by using the modified propagator i.e. using the principle of path integral duality, in the first order of $\alpha$ in quantum electrodynamics. In the conventional quantum electrodynamics evaluations the divergent terms are observed into the physical parameters like mass, charge and spin, where as the modified
propagator works as a regulator to the ultra-violet divergences. The modified propagator introduces two kinds of regulators in the radiative correction evaluations, which are logarithmic and power law. In the three radiative corrections the power law regulator is $O \left( L_p^2 m^2 \right)$ and is very small. The three renormalization factors have the logarithmic correction $O \left( \ln \left( L_p m \right) \right)$. The value of $L_p^2 m^2 \approx 10^{-45}$, and $O \left( \ln (m L_p) \right) \approx 0.1$. The renormalization factors $z_1$, $z_3$ are found different in this case. The use of modified propagator to the field theoretic evaluation is a more proper approach towards the removal of the divergences in field theory based on the relevant physics, rather than a formalistic approach based on improper mathematical manipulations.

In the last chapter, we have presented the contour of integration in path-integral approach to quantum cosmology. We have described a general technique for the approximate evaluation of the path integral for spatially homogeneous mini super space models. In this technique the path integral reduces to a single ordinary integration over the lapse after some trivial functional integrals. Then we have studied the lapse intergration contours in detail by finding the steepest-descent paths. By selecting different complex contours, different solutions to the wheeler-Dewitt equation may be obtained. The method may also be very useful for generating and studying the complex
solutions to the Einstein equations that inevitably arise as saddle points. One may apply this method to a class of anisotropic minisuperspace models, such as Bianchi type I and III, and the Kantowski-Sachs model. We may regard the amplitude evaluated here as a candidate wave function of the Universe and determine the extend to which the contours satisfy the criteria:

1. The integral should converge, which implies that the contour should be complex.

2. The wave function generated should satisfy the Wheeler-Dewitt equation and momentum constraints.

3. The wave function should be consistent with a prediction of classical spacetime when the Universe is large.

4. In the limit that space time becomes classical i.e. at the saddle points of the functional integral over metrics satisfying the conditions (3), conventional quantum field theory for $\varphi$ in those classical backgrounds should be recovered. This implies a further restriction on the nature of the dominating saddle points, namely, they should have $\text{Re} \left( g^{1/2} \right) > 0$. 
5. Coleman [69] has suggested that the extend the cosmological constant is a variable assumed, the contour should be consistent with the vanishing of the low-energy effective cosmological constant. The real part $A_g$ of the action of a complex solution, loosely speaking, gives a measure $e^{-A_g}$ on possible values of $\Lambda$. Again this implies that the dominating saddle points must satisfy $\text{Re}(g^{1/2}) > 0$.

Every chapter has been divided in sections following decimal system: section (2.5) means fifth section of chapter second. On the same line the equations in different chapters are also numbered i.e. eq. (1.6) means, sixth equation of chapter one. At last references are given.