CHAPTER 6

TWO-LAYERED CASSON MODEL OF BLOOD FLOW INSIDE TAPERED ARTERY WITH ASYMMETRIC STENOSIS

6.1 INTRODUCTION

The body fluid blood is made up of a concentrated suspension of different particles in a complex aqueous solution and as such blood may be divided into two parts viz. (i) the suspending medium (plasma) and (ii) the cells, consisting the erythrocytes (RBC), leukocytes (WBC) and platelets (thrombocytes). Erythrocytes, leukocytes and platelets are the main constituents of blood. It has been pointed out that plasma behaves as a Newtonian fluid (Schlichting, 1968) whereas the whole blood exhibits the non-Newtonian property (Biswa, 2000; Fung 1981). The Red Blood cells (RBC) are more than thousand times numerous than the White Blood cells (WBC) and much larger than platelets. For this reason, the flow properties of blood mainly involve the RBC. When the flow of blood to a part of the body is reduced, the oxygen supply to that part of the body is cut off and cells begin to die.

It is seen that the abnormal growth of stenosis at the arterial wall is mostly non-symmetrical. Keeping in view of this, here we have considered non-symmetrical stenosis formation at the vessel wall. However, symmetric stenosis growth is a particular case of this asymmetric stenosis. Stenosis, a generic medical term which means narrowing of an artery, tube or orifice, is an abnormal growth in the innermost arterial wall. It may develop at various locations of the cardiovascular systems under diseased conditions. Among the different systems present inside a body, one of the most important systems is obviously Cardiovascular system (CVS). The human cardiovascular system (CVS) consists of three primary components viz. (i) Cardio-the heart which is a double pump, (ii) Blood (iii) Blood vessels-arteries, veins and capillaries (Guyton 1970). Heart ceaselessly pumps blood which flows through the blood vessels, supplies oxygen to different organs and tissues and, removes metabolic waste products and regulates physiological conditions throughout the body. In this way, the flowing blood is working through the CVS, which is a closed system. Owing to the intermittent pumping by the heart, there produces a pressure difference i.e., the
difference in diastolic and systolic pressures, called the pressure pulse which is checked at our wrist by the physicians.

It has been reported by many authors theoretically that body fluid blood while flowing through circulatory channels leaves a peripheral layer (ppl) of plasma and a core region of red cell suspension which is supported by experimental results. It seems therefore rational to consider a cell free layer near the vessel wall and a core region consisting of red cell suspension. In view of this fact, in the present study we have considered a two-fluid model or a two-layered model with the ppl i.e., a cell poor region near the vessel wall and a core region of cells and, investigated the behaviour of flow variables in this constricted artery channel.

The study of blood flow through tapered tubes is important not only for an understanding of the flow behavior of the marvelous body fluid blood in arteries, but also for the design of prosthetic blood vessels (How and Black, 1987). In the blood flow modeling, A number of studies of blood flow in particular both theoretical (Vand, 1948; Bloch, 1962; Brum, 1975; Nubar, 1967) and experimental (Bugliarello and Hayden, 1962; Bennett, 1967) have done in the presence of slip at the flow boundaries. It seems that consideration of a velocity slip at the vessel wall will be quite rational in blood flow modeling. In most of the abovementioned literatures, constant velocity slip has been considered at the wall. In my present study, slip velocity has been taken at the innermost wall which varies with radial distance due to non-symmetric stenosis.

6.2 PROBLEM FORMULATION

Consider two layered blood flow through tapered non-symmetric stenosed artery with red cell suspension (Casson fluid) in the core region and peripheral layer by plasma (Newtonian fluid) is modeled. The radius of the non-tapered portion is $R_0$ and radius of the core region is $\alpha R_0$ with Casson’s viscosity $K_c$ and Newtonian viscosity $\mu$ in the peripheral layer.
Fig 6.1.- Schematic diagram of two-layered Casson fluid model in a tapered artery with asymmetric stenosis.

Where $R(z)$ and $R_1(z)$ are the critical distances from stenosis in peripheral and core region respectively. The geometries of two-layered tapered artery with asymmetric stenosis have been given in Chapter 3 (equation (3.3.1-- 3.3.2)).

The constitutive equation for Casson fluid is furnished by the Casson’s equation (Fung1981) in the form

$$\sqrt{\tau} = \sqrt{\tau_y} + \sqrt{\eta \gamma}, \quad (6.2.1)$$

where $\tau$, $\tau_y$, $\eta$ are shear stress, yield stress and co-efficient of viscosity of Casson’s fluid respectively and, $\gamma = \frac{\partial u}{\partial y}$ is the strain rate.

The above equation can be re-written as

$$\gamma = \frac{1}{K_c} \left( \sqrt{\tau_{rz}} - \sqrt{\tau_y} \right)^2, \quad \tau_{rz} \geq \tau_y, \quad (6.2.2)$$

$$= 0, \quad \tau_{rz} \leq \tau_y, \quad (6.2.3)$$

where $K_c$ is the Casson’s viscosity $\tau_{rz}$ is the shear stress.
In order to obtain strain rate ($\gamma$) of the blood at the artery walls, with the formula used for Newtonian fluid flow, the governing equation as

$$C + \frac{\mu}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) = 0,$$

(6.2.4)

where, $C = -\frac{dp}{dz}$ and $\mu$ is the Newtonian viscosity.

Whose integral results in

$$u(r) = \frac{Cr^2}{4\mu} + A \ln r + B,$$

(6.2.5)

where $A$ and $B$ are integrating constants. Their values can be determined with the help of the usual boundary conditions:

Applying the symmetry ($u(0)$ is finite) and zero slip ($u_s = 0$ at $R(z)$) condition, we obtained the result as

$$u(r) = \frac{CR^2(z)}{4\mu} \left[ 1 - \left( \frac{r}{R(z)} \right)^2 \right], \quad 0 \leq r \leq R(z).$$

(6.2.6)

Also, shear stress component at any distance $r$ from the tube axis is given by (Schlichting 1968)

$$\tau_{rz} = \mu \frac{\partial u}{\partial r} = \mu \gamma.$$

(6.2.7)

With the help of equation (6.2.3), the equation (6.2.4) becomes

$$\tau_{rz} = -\frac{Cr}{2} = \frac{r}{2} \frac{dp}{dz}.$$

(6.2.8)

This implies that, $\tau_{rz} \propto r$
The expression for wall shear stress \( \tau_w \) obtained from the formula

\[
\tau_w = \tau_{rz} \bigg|_{r=R(z)} = -\frac{C}{2} R(z). \tag{6.2.9}
\]

But, when the critical distance from the tube axis \( r = r_c \) then the shear stress becomes the yield stress \( \tau_y \) (Fung 1981). Where \( r_c \) is the critical radius of the tube.

In order to find out the yield stress \( \tau_y \), we have obtained the yield stress by replacing \( r \) by \( r_c \) in the equation (6.2.8) as

\[
\tau_y = -\frac{C r_c}{2} = \frac{r_c}{2} \frac{dp}{dz}. \tag{6.2.10}
\]

As a result, there are arise two cases:

i) If the shear stress \( \tau_{rz} \) at a distance \( r \) is not higher than the yield stress \( \tau_y \)

\[ \tau_{rz} \leq \tau_y \]

or corresponding to \( r \leq r_c \), blood will not flow.

ii) If shear stress is greater than yield stress i.e. \( \tau_{rz} \geq \tau_y \) or corresponding to \( r \geq r_c \), than the blood will flow.

Now, we transform the equations (6.2.2--6.2.6) with the help of equations (6.2.9—6.2.10), for core region \( (0 \leq r \leq R_1(z)) \) as

\[
\frac{d u_2(r)}{dr} = \frac{1}{K_c} \left[ \sqrt{\frac{Cr}{2}} - \sqrt{\frac{C r_c}{2}} \right]^2, \quad r_c \leq r \leq R_1(z), \tag{6.2.11}
\]

\[
= 0, \quad 0 \leq r \leq r_c. \tag{6.2.12}
\]
6.3 SOLUTIONS

The Integral is obtained from the above equation (6.2.11) becomes

\[ u_2(r) = \left( -\frac{C}{4K_c} \right) \left( r^2 + 2r_c - \frac{8}{3} r_c r^3 \right) + A_3, \quad r_c \leq r \leq R_1(z). \]  
(6.2.13)

From equation (6.2.12) we have seen that \( \gamma = 0 \) implies that

\[ \frac{du_c}{dr} = 0. \]  
(6.2.14)

For identifying axial velocity in the central region of radius \( r_c \), we have written \( u_c \) instead of \( u_2(r) \)

On integration the equation (6.2.14) becomes

\[ u_c = \text{constant} = A_2 \text{ (say)}, \quad 0 \leq r \leq r_c. \]  
(6.2.15)

The expression for axial velocity for ppl layer is obtained from the equation (6.2.6) as

\[ u_i(r) = -\frac{Cr^2}{4\mu_i} + A_1 \ln r + B_1, \]  
(6.2.16)

where \( A_1, A_2, A_3 \text{ and } B_1 \) are integrating constants.

In order to obtain the above constants, we employed the following boundary conditions:

(i) \( u_i = 0, \text{ at } r = R(z) \) (zero slip condition)

(ii) \( u_2 = u_c, \text{ at } r = r_c \) (no flow)

(iii) \( u_2 - u_i = u_s, \text{ at } r = R_1(z) \) (slip at the interface)

(iv) \( u_i \text{ is finite at } r = 0 \) (symmetry condition)
where $u_1$ and $u_2$ are the axial velocities in the peripheral and core region respectively and $u_s$ is the slip at the interface in the core region.

As a result, applying the above boundary conditions, we have obtained the expression for axial velocity in ppl layer as

$$u_1(r) = \frac{C}{4\mu_1} \left( R^2(z) - r^2 \right), \quad R_1(z) \leq r \leq R(z), \quad (6.2.18)$$

for core region as

$$u_2(r) = u_s + \frac{C}{4K_c} \left( \left( R^2(z) - r^2 \right) + 2r_c \left( R_1(z) - r \right) - \frac{8}{3} \sqrt{r_c} \right) \left( \sqrt{R_1^3(z)} - \sqrt{r^3} \right) + \frac{C}{4\mu_1} \left( R^2(z) - R_1^2(z) \right), \quad R_1(z) \leq r \leq R(z), \quad (6.2.19)$$

and

$$u_2(r) = u_c, \quad 0 \leq r \leq r_c, \quad (6.2.20)$$

where $u_c$ is the core velocity and it is evaluated by replacing $r$ by $r_c$ in the equation (6.2.19) and finally it takes the form

$$u_c = u_s + \frac{C}{4K_c} \left( \sqrt{R_1(z)} - \sqrt{r_c} \right)^3 \left( \sqrt{R_1(z)} + \frac{1}{3} \sqrt{r_c} \right) + \frac{C}{4\mu_1} \left( R^2(z) - R_1^2(z) \right), \quad 0 \leq r \leq r_c, \quad (6.1.21)$$

The total volumetric flow rate ($Q$), defined by

$$Q = 2\pi \left[ \int_{r=0}^{r_c} ru_cdr + \int_{r_c}^{R_1} ru_2dr + \int_{R_1}^{R} ru_1dr \right], \quad (6.2.21)$$

it can be obtained with the help of equations (6.2.18—6.2.21) as
\[ Q = \pi R_i^2(z)u_s + \frac{\pi CR^4(z)}{8k_c} \left\{ 1 - \left(\frac{R(z)}{R_i}\right)^4 \right\} - \frac{\pi R_i^4(z)\left(\frac{dp}{dz}\right)}{8\mu_i} \psi(\beta), \]  
(6.2.22)

where,

\[ \psi(\beta) = \left(1 + \frac{4}{3}\beta + \frac{1}{21}\beta^4 + \frac{16}{7}\beta^7\right), \]
(6.2.23)

\[ \beta = t^2 \quad \text{and} \quad l = \frac{r_c}{R_i} \]

From equation (6.2.22), the pressure gradient term can be expressed as

\[ \frac{dp}{dz} = 8\mu_i \left[ R^4 - \left(1 - \frac{k_c}{\mu_i}\psi(\beta)\right) R_i^4 \right]^{-1} \left(Q\pi^{-1} - R_i^2 u_s\right) \]
(6.2.24)

Integrating the above equation (6.2.24) between the limits \( p = p_i \) at \( z = 0 \) and \( p = p_0 \) at \( z = L \) (L is the length of the tube).

We may obtain the pressure drop in the following form

\[ p_i - p_0 = 8k_c \int_{z=0}^{L} \left[ R^4 - \left(1 - \mu_i^{-1} k_c \psi(\beta)\right) R_i^4 \right]^{-1} \left(Q\pi^{-1} - R_i^2 u_s\right) dz. \]
(6.2.25)

The resistance to flow \( (\lambda) \) is defined by

\[ \lambda = \frac{P_i - P_0}{Q} \]
(6.2.26)

Can be computed the resistance to flow with the help of equations (6.2.22—6.2.25) as

\[ \lambda = 8k_c \left[ R_i^4 \left\{ 1 - \left(1 - \frac{k_c}{\mu_i} \psi\left(\frac{r_c}{\sqrt{\alpha R_i}}\right) \alpha^4 \right) \right\} \right]^{-1} \left(1 - Q_i^{-1}(\alpha R_i)^2 u_s\right)(L - L_0) \]
\[ +8k_c \int_{r=a}^{d} \left[ R^4(z) - \left(1 - \frac{k_c \psi(\beta)}{\mu_t} \right) R_t^4(z) \right]^{-1} \left( \pi^{-1} - Q^{-1} R_t^2(z) u_z \right) dz, \quad (6.2.27) \]

where \( Q_1 = Q \big|_{a \rightarrow R_t} \)

The average pressure gradient in the axial direction, defined by

\[ \left( \frac{dp}{dz} \right)_{av} = \frac{\int_{0}^{R} r \left( \frac{dp}{dz} \right) dr}{\int_{0}^{R} r dr}, \quad (6.2.28) \]

Can obtained as (using equation (6.2.24))

\[ \left( \frac{dp}{dz} \right)_{av} = \left( \frac{dp}{dz} \right). \quad (6.2.29) \]

Apparent viscosity \( \mu_a \) is defined by

\[ \mu_a = \frac{\pi CR^4}{8Q} \quad (6.2.30) \]

We obtained the expression for apparent viscosity with the help of equation (6.2.22) as

\[ \mu_a = \left[ \frac{8u_s}{CR^3} \left( \frac{R_t}{R} \right)^2 + \frac{\mu_t}{k_c} \left( 1 - \left( 1 - \mu_t^{-1} k_c \psi(\beta) \right) \left( \frac{R_t}{R} \right)^4 \right) \right]^{-1}. \quad (6.2.31) \]

Stress component at stenotic wall (\( \tau_w \)), interface (\( \tau_{R(z)} \)) and yield stress (\( \tau_y \)) can be obtained from the formula \( \tau_{rz} = -\frac{C}{2} r = r \left( \frac{dp}{dz} \right) \) in the following forms

\[ \tau_w = \tau_{rz} \big|_{r=R(z)} = -\mu_t \frac{\partial u_z}{\partial r} \big|_{r=R(z)} = -\mu_t \frac{\partial}{\partial r} \left( \frac{C}{4\mu_t} \left( R^2 - r^2 \right) \right) \big|_{r=R(z)}, \text{ we get} \]
\[ \tau_w = \tau_{rz} \bigg|_{r=R(z)} = -\frac{C}{2} R = \frac{R}{2} \left( \frac{dp}{dz} \right). \] (6.2.32)

Similarly, we obtained as

\[ \tau_R(z) = \tau_{rz} \bigg|_{r=R(z)} = -\frac{C}{2} R_1 = \frac{R_1}{2} \left( \frac{dp}{dz} \right), \] (6.2.33)

and

\[ \tau_y = \tau_{rz} \bigg|_{r=r_c} = -\frac{C}{2} r_c = \frac{r_c}{2} \left( \frac{dp}{dz} \right). \] (6.2.34)

A second representation of flow variables can be expressed in the following

For core region (from equation (6.2.18)), we obtain as

\[ \bar{u}_2 = \bar{u}_s + \frac{k_c}{\mu_1} \left\{ R(z)^2 - R(z)^2 \left( \frac{r}{R(z)} \right)^2 + 2r_c \left( R(z) - R(z) \frac{r}{R(z)} \right) \right\} \]

\[ -\frac{8}{3} \sqrt{r_c} \left( \sqrt{R(z)}^3 - \sqrt{\left( \frac{r}{R(z)} \right)^3} \right) \]

\[ + \left( R(z)^2 + \frac{R_1(z)^2}{R(z)} \right), \quad \frac{r_c}{R(z)} \leq \frac{r}{R(z)} \leq \frac{R_1(z)}{R(z)}. \] (6.2.35)

For peripheral layer (from equation (6.2.19)) as

\[ \bar{u}_1 = \frac{R(z)^2}{1 - \left( \frac{r}{R(z)} \right)^2}, \quad \frac{R_1(z)}{R(z)} \leq \frac{r}{R(z)} \leq \frac{R_1(z)}{R(z)}. \] (6.2.36)

and

\[ \bar{u}_c = \bar{u}_s + \frac{k_c}{\mu_1} \left( \sqrt{R(z)} - \sqrt{r_c} \right)^3 \left( \sqrt{R(z)} - \frac{1}{3} \sqrt{r_c} \right) + \]

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\[
\left( R(z) \right)^2 + \frac{r_c}{R(z)} \left( \frac{r_c}{R(z)} \right)^2 , \quad 0 \leq \frac{r_c}{R(z)} \leq \frac{r_c}{R(z)} .
\]

(6.2.37)

A second representation for flow rate, defined by

\[
\bar{Q} = \frac{Q}{Q_0}
\]

(6.2.38)

can be obtained as

\[
\bar{Q} = 2R(z) \bar{u}_s + \left\{ R(z)^4 - \frac{1 - \frac{k_c}{\mu}}{R(z)^4} \left( 1 - \frac{k_c}{\mu} \psi(\beta) \right) \right\} \left( \bar{Q} - 2R(z)^2 \bar{u}_s \right)
\]

(6.2.39)

\textit{where,} \quad \psi(\beta) = \psi \left( \frac{r_c}{R(z)} \right)

A second representation of pressure gradient can be obtained as (using equation (6.2.24)),

\[
\frac{dp}{dz} = \left[ R(z)^4 - \left( 1 - \frac{k_c}{\mu} \psi(\beta) \right) R(z)^4 \right]^{-1} \left( \bar{Q} - 2R(z)^2 \bar{u}_s \right) = \left( \frac{dp}{dz} \right)_\text{avg}.
\]

(6.2.40)

A second representation of resistance to flow (\( \bar{\lambda} \)), defined by

\[
\bar{\lambda} = \frac{\lambda}{\lambda_0}
\]

(6.2.41)

we have obtained with the help of equation (6.2.27) as

\[
\bar{\lambda} = \left\{ 1 - \left( 1 - \frac{k_c}{\mu} \psi(\sqrt{\alpha^{-1} r_c}) \right) \alpha^4 \right\}^{-1} \left( 1 - 2\alpha^2 \frac{\bar{u}_s}{Q_0} \left( \frac{L - L_0}{L} \right) \right)
\]

\[
+ \frac{1}{L} \int_{z=d}^{d+L_0} \left[ R(z)^4 - \left( 1 - \frac{k_c}{\mu} \psi(\beta) \right) R(z)^4 \right]^{-1} \left( 1 - 2\bar{u}_s \frac{R(z)^2}{\bar{Q}} \right) dz.
\]

(6.2.42)
A second representation of apparent viscosity, defined by

$$\overline{\mu}_a = \frac{\mu_a}{\mu_i}$$  \hspace{1cm} (6.2.43)

Is obtained as

$$\overline{\mu}_a = \left[ 1 + 2\frac{u_s}{R^2} \left( \frac{R_i(z)}{R(z)} \right)^2 \left( 1 - \frac{k_c}{\mu_i} \psi(\beta) \right) \left( \frac{R_i(z)}{R(z)} \right)^4 \right]^{-1}$$  \hspace{1cm} (6.2.44)

where, 

$$\overline{\mu}_a = \frac{\mu_a}{\mu_i} \quad \overline{\beta} = \frac{r_s}{R_1} \quad \overline{R} = \frac{R}{R_0} \quad \overline{\delta}_s = \frac{\delta_s}{R_0},$$

$$\overline{u}_1 = \frac{u_1}{u_0} \quad \overline{u}_s = \frac{u_s}{u_0} \quad \overline{Q} = \frac{Q}{Q_0} \quad \overline{\lambda} = \frac{\lambda}{\lambda_0},$$

$$Q_0 = \frac{\pi CR_0^4}{8\mu_i} \quad \lambda_0 = \frac{8\mu_iL}{\pi R_0^4} \quad \left( \frac{dp}{dz} \right)_0 = \frac{8\mu_iQ_0}{\pi R_0^4} \quad u_0 = \frac{CR_0^2}{4\mu_i}$$  \hspace{1cm} (6.2.45)

6.4 RESULTS AND DISCUSSIONS

In the present analysis three successive growths at lumen of an artery, slip and no-slip cases at interface for investigating two-layered in a tapering stenosed tube are employed in Fig 6.1. In the model body fluid blood is assumed to behave as a non-Newtonian fluid, inhabiting an yield property known as Casson fluid. In such a visco-inelastic fluid shear stress versus axial distance $z$ relationship is non-linear and the Casson fluid possess a finite yield stress $\tau_y$, there may arise two flow situations mainly (a) if the shear stress at a radial distance $r$ is not higher than the yield stress $\tau_y$ ($\tau_{rz} \leq \tau_y$), blood will not flow and if otherwise i.e. if the shear stress is lower its yield stress $\tau_y$ value ($\tau_{rz} \geq \tau_y$), blood will be possible. Analytical expression for flow variables are obtained and the variations of axial velocity of both regions, rate of flow, pressure gradient, shear stress and apparent viscosity at the interface of fluids in the constricted region etc. have been presented graphically (Figs 6.2-6.27)
In this two-layered blood flow there are three regions namely \(0 \leq r \leq r_c\) (critical radius), \(r_c \leq r \leq R_1\), and \(R_1 \leq r \leq R\), where \(r_c\) is a critical radius, where \(r = r_c\), then \(u(r) = u_c\) (a constant velocity) and \(\tau_{rz} = \tau_y\). (b) when \(R(z) = R_0 = R_1(z), \tau_y = 0\) (or \(r_c = 0\)) it leads to Poiseuille flow of blood (behaving as a Newtonian fluid) with slip or zero-slip at the vessel wall.

The behaviour of velocity is presented in the Fig 6.2-6.7. It attains a fixed magnitude in yield stress zone and alters parabolically in core and ppl regions. It increases with shape parameter \(n = 2\) (symmetric case) to \(n > 2\) (asymmetric stenosis). It attains the highest magnitude for mild growth and lowest value for severe stenosis. However, in all forms of stenosis development, velocity increases with a velocity slip as well as with an increasing magnitude of slip.

The variation of flow rate are shown in Figs 6.8-6.11. It is found that \(\bar{Q}(\varphi < 0) < \bar{Q}(\varphi = 0) < \bar{Q}(\varphi > 0)\). Its magnitude attained with \(n = 2\) is lower than that with \(n > 2\). Flow rate reaches the maximum for mild stenosis and the minimum for severe stenosis.

The behaviour of pressure gradient, as shown in Figs 6.12-6.19. However, pressure gradient \(C = -\frac{dp}{dz}\), indicates the highest value for mild growth and the lowest magnitude for severe stenosis. It increases with velocity slip. it is observed that \(\left(\frac{dp}{dz}\right)(\varphi < 0) < \left(\frac{dp}{dz}\right)(\varphi = 0) < \left(\frac{dp}{dz}\right)(\varphi > 0)\).

The behaviour of wall shear stress (Figs 6.20-6.26) decreases with velocity slip. it is observed that shear stress \((\varphi < 0) < \text{shear stress} (\varphi = 0) < \text{shear stress} (\varphi > 0)\). It is the lowest for mild stenosis and the highest for severe stenosis.

Apparent viscosity in Fig 6.27 is lowered with velocity slip. It attains the greatest value for severe growth and the lowest magnitude for mild formation. It is found that \(\bar{\mu}_a\) (mild case) < \(\bar{\mu}_a\) (moderate forms) < \(\bar{\mu}_a\) (severe growth).
Fig 6.2: Variation of axial velocity against radial distance for different axial locations

Fig 6.3: Variation of axial velocity against radial distance for different axial locations
Fig 6.4: Variation of axial velocity against radial distance for different axial locations

Fig 6.5: Variation of axial velocity against radial distance for different heights of stenosis
Fig 6.6: Variation of axial velocity against radial distance for different heights of stenosis

Fig 6.7: Variation of axial velocity against radial distance for different heights of stenosis
Fig 6.8: Variations of flow rate against axial distance for different tapering angle.

**Equation:**
- $u_0 = 0.00, \alpha = 0.8$
- $k_i = 2cp$

Fig 6.9: Variations of flow rate against axial distance for different tapering angle.

**Equation:**
- $u_0 = 0.05, \alpha = 0.8$
- $k_i = 2cp$
Fig 6.10: Variation of flow rate against axial distance for different tapering angle

Fig 6.11: Variation of flow rate against axial distance for different heights of stenosis
Fig 6.12: Variation of pressure gradient against axial distance for different tapering angles.

Fig 6.13: Variation of pressure gradient against axial distance for different tapering angles.
Fig 6.14: Variation of pressure gradient against axial distance for different tapering angles

Fig 6.15: Variation of pressure gradient against axial distance for different heights of stenosis $n=2$ and various tapering angles with velocity slip $\vec{u}_s = 0.00$ and $\bar{Q} = 1.00$
Fig 6.16: Variation of pressure gradient against axial distance for different heights of stenosis $n=2$ and various tapering angles with velocity slip $\bar{u}_z = 0.10$ and $\bar{Q} = 1.00$

Fig 6.17: Variation of pressure gradient against axial distance for different heights of stenosis $n=2$ and various tapering angles with velocity slip $\bar{u}_z = 0.10$ and $\bar{Q} = 0.50$
Fig 6.18: Variation of pressure gradient against axial distance for different heights of stenosis \( n = 6 \) and various tapering angles with velocity slip \( \bar{u}_s = 0.00 \) and \( \overline{Q} = 0.50 \)

Fig 6.19: Variation of pressure gradient against axial distance for different heights of stenosis \( n = 6 \) and various tapering angles with velocity slip \( u_s = 0.00 \) and \( Q = 1.50 \)
Fig 6.20: Variation of wall shear stress against axial distance for different tapering angles

Fig 6.21: Variation of wall shear stress against axial distance for different tapering angles
Fig 6.22: Variation of wall shear stress against axial distance for different tapering angles.

Fig 6.23: Variation of wall shear stress against axial distance for different heights of stenosis n=2 and various tapering angles with velocity slip $\bar{u}_r = 0.00$ and $\bar{Q} = 0.50$. 

$n = 9, \bar{k}_r = 2c_p, \bar{\alpha} = 0.8$
Fig 6.24: Variation of wall shear stress against axial distance for different heights of stenosis $n=2$ and various tapering angles with velocity slip $\bar{u}_s = 0.00$ and $Q = 1.00$

Fig 6.25: Variation of wall shear stress against axial distance for different heights of stenosis $n=2$ and various tapering angles with velocity slip $\bar{u}_s = 0.00$ and $Q = 1.50$
Fig 6.26: Variation of wall shear stress against axial distance for different heights of stenosis n=2 and various tapering angles with velocity slip $u_s = 0.10$ and $Q = 0.50$

Fig 6.27: Variation of apparent viscosity against axial distance for different heights of stenosis n=2 and various tapering angles with velocity slip and yield stress.
6.5 CONCLUSIONS

The present chapter deals with steady, laminar and one-dimensional (1-D) flow of blood in the two-layered region of an asymmetric tapered stenosed artery in Fig 6.1. In order to study the effect of velocity slip at the fluid interface, an analysis has been developed here by employing a slip condition at the interface of fluids in case of a mild, moderate and severe arterial stenosis. Analytical expression Analytical expression for flow variables are obtained and the variations of axial velocity of both regions, rate of flow, pressure gradient, shear stress and apparent viscosity at the interface of fluids in the constricted region etc. have been presented graphically (Figs 6.2-6.27)

A second form of the flow variables are obtained and their variations are shown graphically. It is found that velocity is a function of $\text{R, } \delta_s, r, z, k_c, L_0$ etc. The following observation can be recorded from the present analysis

(a) In the two-layered Casson fluid flow, there arises two region for flow (for flow situation) viz. $\tau_{\text{yz}} \leq \tau_y$ and $\tau_{\text{yz}} \geq \tau_y$ and in the former case, no flow will occur whence as in this later blood flow will be possible.

(b) For two-layered blood flow within a tapering stenosed arterial region, there are regions viz. $0 \leq r \leq r_c, r_c \leq r \leq R_1(z)$ and $R_1(z) \leq r \leq R(z)$.

(c) It includes (i) Poiseuille flow of blood (acting as non-Newtonian fluid) with wall-slip or at the vessel boundary. (ii) one-dimensional stenosed flow of both Newtonian fluid and Casson fluid with slip or zero-slip at tapering arterial wall

(d) The behaviour of Velocity is presented in the Fig (6.2-6.8). it attains a fixed magnitude in yield stress zone and alters parabolically in core and ppl regions. It increases with shape parameter $n = 2$ (symmetric case) to $n > 2$ (asymmetric stenosis). It attains the highest magnitude for mild growth and lowest value for severe stenosis. However, in all forms of stenosis development, velocity increases with a velocity slip as well as with an increasing magnitude of slip.

(e) The variations of flow rate are shown in Figs (6.8-6.11), it shows that the magnitude attained with $n = 2$ is lower than that with $n > 2$. Flow rate reaches the maximum for mild stenosis and the minimum for severe stenosis.
(f) The behaviour of pressure gradient, as shown in Figs (6.12-6.19). However, pressure gradient \( C = -\frac{dp}{dz} \), indicates the highest value for mild growth and the lowest magnitude for severe stenosis. It increases with velocity slip.

(g) The behaviour of wall shear stress (Figs 6.20-6.26) decreases with velocity slip. It is observed that shear stress \( (\phi < 0) < \) shear stress \( (\phi = 0) < \) shear stress \( (\phi > 0) \). It is the lowest for mild stenosis and the highest for severe stenosis.

(h) Apparent viscosity in Fig (6.26-6.27), is lowered with velocity slip. It attains the greatest value for severe growth and the lowest magnitude for mild formation. It is found that \( \overline{\mu_a} \) (mild case) < \( \overline{\mu_a} \) (moderate forms) < \( \overline{\mu_a} \) (severe growth).

Thus in view of the above analysis, for flow variables it could be concluded that with an employment of axial velocity slip at fluids interface in the constricted tapering channel. Damages to a diseased or occluded artery could be reduced. In this analysis, slip has been employed to accelerate the flow in the constricted two-layered region in one hand and to reduce the impedance to this unidirectional flow, on the other. However, in the analysis, some arbitrary measures of slip \( u_s = 0.5, 1, \tau_y, n = 2, 6, 9; k_c = 2c_p \), have been used and stenotic wall at rigid, non-porous and visco-elastic as well as flow is steady and uni-directional.

This theoretical model could be improved if an appropriate magnitude of a velocity slip, yield stress of blood, PPL thickness, two-dimensional flow etc. are considered in the modeling. In order to propose such an improved model of blood flow through a two-layered tapering stenosed artery, the present model could be used as base. In blood flow modeling, it has been reported that apart from possessing a finite yield stress \( (\tau_y) \), blood exhibit a Newtonian behaviour. In order to include both the characteristic of blood, Bingham plastic fluid, in a two-layered tapered constricted artery region has been dealt with in the next chapter 7.