CHAPTER 5

TWO-LAYERED NEWTONIAN MODEL OF ANNULAR BLOOD FLOW
WITH ASYMMETRIC STENOSIS

5.1 INTRODUCTION

The studies of blood flow through arteries of human circulatory systems have been attributed to discovering the cardiovascular diseases by many researchers for a long time (Fung, 1981, 1984; Puniyani and Niimi, 1998; Biswas, 2000; Pedley, 1980, etc.). The cardiovascular disease, such as stenosis or arteriosclerosis, is closely associated with the flow conditions in the blood vessels. Naturally, the Cardio Vascular System (CVS) is very complicated as well as the body fluid blood is a complex aqueous suspensions of cells. So it is important to study theoretically and experimentally, the blood flow phenomena through narrow vessels for diagnosing the Cardio vascular diseases and, also very useful to the development of pathological patterns in human physiology and other clinical purposes (Bugliarello and Hayden, 1962; Bugliarello and Sevilla, 1970; Young and Tsai, 1973; Dintenfass, 1980, 1981). The laminar flow of blood through arteries in presence of symmetric or non-symmetric stenosis plays an important role in the diagnosis and critical treatment as well as in the fundamental understanding of many cardiovascular diseases (Misra et al. 2008). Many researchers have studied, the flow behaviour of blood to understand the effects of stenosis formation along the lumen of an artery.

Theoretical and experimental studies of the circulatory disorders, causing deaths in most of the cases, have been the subject of scientific research since the investigation of Mann et al. (1990). According to the discovery that the cardiovascular disease, such as stenosis or arteriosclerosis, is closely associated with the flow conditions in the blood vessels, scientists have been focusing on this area of biomechanics. Stenosis medically means narrowing of anybody passage (tube or orifice). It is the abnormal and unnatural growth in the arterial wall thickness that develops at various locations of the cardiovascular system under certain conditions. It may result in serious consequences (cerebral strokes, myocardial infarction leading to heart failure, etc.) by reducing or occluding the blood supply. Also, it has been suggested that the deposits of the cholesterol and proliferation of connective tissue
form plaques that enlarge and restrict the blood flow. One may expect that if such an event occurs, the flow characteristics in the vicinity of the resulting protuberances may be significantly altered. Moreover, in cardiac related problems, the affected arteries get hardened as a result of accumulation of fatty substances, inside the lumen or because of formation of plaques as a result of hemorrhage. As the disease progresses, the arteries/arterioles get constricted. Consequently, the flow behavior in the stenosed artery is quite different from the case of the normal artery. Also, stresses and resistance to flow are much higher in stenosed artery in comparison to the normal ones. Having knowledge on flow parameters in the stenosed artery, such as the velocity pattern, the flow rate, and the stresses, will help bio-medical engineers in developing bio-medical device for improving the diseased condition.

5.2 PROBLEM FORMULATION

Consider two-layered flow of blood through an annular region between the constricted tapered arterial segment and a catheter co-axial to it of radius $KR_0$ ($K << 1$, $KR_0 << R_0$). At the innermost vessel wall of the artery, non-symmetric stenosis, has been considered. Here, the formulation of blood flow is considered as a Newtonian model and the flow is assumed to be steady, laminar and axi-symmetric. In this two-layered annular flow, both the ppl and core region are represented by the Newtonian fluid with different viscosities.

![Schematic diagram of two-layered catheterized tapered artery with asymmetric stenosis.](image)
Where $\mu_1$ and $\mu_2$ are the viscosities in core and peripheral region respectively. 
$R_1(z)$ and $R(z)$ are the effective radii of the tapered tube in the constricted region for core and ppl regions respectively. The geometries of two-layered tapered artery with asymmetric stenosis, have been included in preceding Chapter 3 (equation 3.3.1 and 3.3.2).

The governing equation of unidirectional Newtonian fluid flow is

$$C + \frac{\mu}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) = 0,$$

(5.2.1)

where

$$C = -\frac{dp}{dz}$$

5.3 SOLUTIONS

The integral of the equation (5.2.1) is obtained as

$$u(r) = \frac{Cr^2}{4\mu} + K\ln r + B,$$

(5.2.2)

where $K$ and $B$ are integrating constants.

The expression of axial velocity in the equation (5.2.2) can be written as, for core region

$$u_i(r) = \frac{Cr_i^2}{4\mu_i} + K_i\ln r + B_i,$$

(5.2.3)
and for peripheral layer as

\[
    u_2(r) = \frac{Cr^2}{4\mu_2} + K_2\ln r + B_2, \quad (5.2.4)
\]

where \( K_1, K_2, B_1, \) and \( B_2 \) are constants and they can be evaluated with the help of the following boundary conditions:

\[ i) \quad u_2 = 0 \quad \text{at} \quad r = R(z) \quad \text{(No-slip at outer wall)} \]

\[ ii) \quad u_1 - u_2 = u_s \quad \text{at} \quad r = R_i(z) \quad \text{(Slip at interface region)} \quad (5.2.5) \]

\[ iii) \quad u_1 = 0 \quad \text{at} \quad r = KR_0 \quad \text{(Zero-slip at catheter wall)} \]

\[ iv) \quad \mu_1 \frac{\partial u_1}{\partial r} = \mu_2 \frac{\partial u_2}{\partial r} \quad \text{at} \quad r = R_i(z) \quad \text{(Stresses are equal at interface)} \]

where \( u_1 \) and \( u_2 \) are velocities of blood at the core region and ppl layer respectively and, \( u_s \) is the slip velocity at the interface of the stenotic region.

As a result of employing conditions (5.2.5) in the solutions (5.2.3-5.2.4), the expressions for axial velocity in core region and ppl layer are

\[
    u_1(r) = \frac{C}{4\mu_1} \left( K^2R_0^2 - r^2 \right) + \frac{\mu_2 H_1}{H_2} \ln \left( \frac{r}{KR_0} \right), \quad (5.2.6)
\]

and

\[
    u_2(r) = \frac{C}{4\mu_2} \left( R^2(z) - r^2 \right) + \frac{\mu_1 H_1}{H_2} \ln \left( \frac{r}{R(z)} \right), \quad (5.2.7)
\]

where

\[
    H_1 = u_s + \frac{C}{4} \left\{ R_1^2(z) \left( \frac{1}{\mu_1} - \frac{1}{\mu_2} \right) - \left( \frac{(KR_0)^2}{\mu_1} - \frac{R^2(z)}{\mu_2} \right) \right\}, \quad (5.2.8)
\]
\[ H_2 = \mu_2 \ln \left( \frac{R_1(z)}{KR_0} \right) - \mu_1 \ln \left( \frac{R_1(z)}{R} \right), \tag{5.2.9} \]

The volumetric flow rate \( (Q) \), may be defined as

\[ Q = Q_1 + Q_2 \tag{5.2.10} \]

where

\[ Q_1 = 2\pi \int_{r=KR_0}^{R_1} r \ u_1(r) \ dr, \tag{5.2.11} \]

and

\[ Q_2 = 2\pi \int_{r=R_1}^{R} r \ u_2(r) \ dr, \tag{5.2.12} \]

The expressions of \( Q_1 \) and \( Q_2 \) can be computed with the help of equations (5.2.6—5.2.7) as

\[ Q_1 = \frac{\pi \mu_2}{2H_2} \left( u_s + \frac{C}{4} H \right) \left[ 2R_1^2(z) \ln \left( \frac{R_1(z)}{KR_0} \right) - \left( R_1^2(z) - (KR_0)^2 \right) \right] - \frac{\pi C}{8 \mu_1} \left( R_1^2(z) - (KR_0)^2 \right)^2, \tag{5.2.13} \]

and
\[
Q_2 = \frac{\pi C}{8\mu_2} \left( R^2(z) - R_1^2(z) \right)^2 - \frac{\pi \mu_1}{2H_2} \left( u_s + \frac{C}{4} H \right) \left\{ 2R_1^2(z) \ln \left( \frac{R(z)}{R_1(z)} \right) + \left( R^2(z) - R_1^2(z) \right) \right\}, \tag{5.2.14}
\]

where

\[
H = R_1^2(z) \left( \frac{1}{\mu_1} - \frac{1}{\mu_2} \right) - \left( \frac{(KR_0)^2}{\mu_1} - \frac{R^2(z)}{\mu_2} \right). \tag{5.2.15}
\]

The total volumetric flow rate can be obtained (using equations (5.2.13—5.2.14)) as

\[
Q = \frac{\pi C}{8} \left[ \frac{1}{\mu_2} \left( R^2(z) - R_1^2(z) \right)^2 - \frac{1}{\mu_1} \left( R_1^2(z) - (KR_0)^2 \right)^2 + \frac{HH_3}{H_2} \right] + \frac{\pi u_s}{2H_2} H_3, \tag{5.2.16}
\]

where

\[
H_3 = 2R_1^2(z)H_2 - \mu_1 \left( R^2(z) - R_1^2(z) \right) - \mu_2 \left( R_1^2(z) - (KR_0)^2 \right). \tag{5.2.17}
\]
The expression for pressure gradient term can be obtained from equation (5.2.16) as

\[
\frac{dp}{dz} = -8 \left[ \frac{Q}{\pi} \frac{H_3 u_s}{2 H_2} \right] \frac{1}{\mu_2} \left( R_2(z) - R_1(z) \right)^2 - \frac{1}{\mu_1} \left( R_1(z) - (K R_0)^3 \right)^2 + \frac{H H_3}{H_2}.
\] (5.2.18)

Integrating the equation (5.2.18) between the limits

\[ p = p_i \text{ at } z = 0 \text{ and } p = p_0 \text{ at } z = L, \text{ can be found as} \]

\[
p_i - p_0 = 8 \left( \frac{Q}{\pi} \frac{H_3 u_s}{2 H_2} \right) \frac{1}{\mu_2} \left( R_2(z) - R_1(z) \right)^2 - \frac{1}{\mu_1} \\
\left[ \left( R_1^3(z) - (K R_0)^3 \right)^2 + \frac{H H_3}{H_2} \right]^{-1} (L - L_0) + 8 \int_{z=0}^{z=L} \left( \frac{Q}{\pi} \frac{H_3 u_s}{2 H_2} \right) dz.
\] (5.2.19)

The resistance to flow \( \lambda \), defined by

\[
\lambda = \frac{p_i - p_0}{Q}
\] (5.2.20)

Can be obtained with the help of equations (5.2.16) and (5.2.19) as

\[
\lambda = 8 \left( \frac{1}{\pi} \frac{H_3 u_s}{2 Q H_2} \right) \frac{1}{\mu_2} \left( R_2^3 - R_1^3 \right)^2 - \frac{1}{\mu_1} \left( R_1^3 - (K R_0)^3 \right)^2 + \frac{H H_3}{H_2} \right]^{-1} (L - L_0)
\]
\[ +8 \int_{z=d}^{z=a} \left( \frac{1}{\pi} - \frac{H_3 u_3}{2QH_2} \right) \left[ \frac{1}{\mu_2} \left( R^2 - R_1^2 \right)^2 - \frac{1}{\mu_1} \left( R_1^2 - (KR_0)^2 \right)^2 + \frac{HH_3}{H_2} \right]^{-1} dz. \] (5.2.21)

Shear stress at the vessel wall and interface of fluids core represented as

\[ \tau_{r(z)} = -\mu_2 \left( \frac{\partial u_2}{\partial r} \right)_{r=R(z)}, \] (5.2.22)

and

\[ \tau_{r_1(z)} = -\mu_1 \left( \frac{\partial u_1}{\partial r} \right)_{r=R_1(z)}, \] (5.2.23)

which results in (using equations (5.2.6—5.2.7))

\[ \tau_{r(z)} = \frac{C}{2} R - \frac{\mu_1 \mu_2}{R H_2} H_1, \] (5.2.24)

and

\[ \tau_{r_1(z)} = \frac{C}{2} R_1 - \frac{\mu_1 \mu_2}{R_1 H_2} H_1. \] (5.2.25)

The apparent viscosity \( \mu_a \) defined by

\[ \mu_a = \frac{\pi C}{8Q} R^4 \] (5.2.26)

can be obtained with the help of equation (5.2.16) as
\[
\mu_a = R^4 \left[ G + \frac{4H_3}{C H_2} \mu_s \right]^{-1}. \tag{5.2.27}
\]

A second representation flow variable can be computed for the core region with the help of equation (5.2.6) as

\[
\bar{u}_1 = \mu'_2 \left\{ (K R_0)^2 - r^2 \right\} + \frac{\mu'_2}{A} (\bar{u}_s + B) \ln \left( \frac{r}{K R_0} \right), \tag{5.2.28}
\]

and

\[
\bar{u}_2 = (R(z)^2 - \bar{r}^2) + \frac{1}{A} (\bar{u}_s + B) \ln \left( \frac{r}{R(z)} \right), \tag{5.2.29}
\]

where

\[
A = \mu'_2 \ln \left( \frac{R(z)}{K R_0} \right) - \ln \left( \frac{R(z)}{R_0} \right), \tag{5.2.30}
\]

and

\[
B = \mu'_2 \left\{ R(z)^2 - (K R_0)^2 \right\} + \left( R(z)^2 - R(z)^2 \right). \tag{5.2.31}
\]

A second representation of flow rate is defined by

\[
\bar{Q} = \frac{Q}{Q_0} \tag{5.2.32}
\]

which computed with the help of equation (5.2.16) as follows
\[
\overline{Q} = W + \frac{1}{A} \left( \overline{u_s} + B \right) \left( 2 \overline{R_1^2} A - B \right), \tag{5.2.33}
\]

where

\[
W = \left( \overline{R^2} - \overline{R_1^2} \right)^2 - \mu_2' \left( \overline{R_1^2} - \left( KR_0 \right)^2 \right)^2. \tag{5.2.34}
\]

A second representation of shear stress in the peripheral region is given by

\[
\overline{\tau_R(z)} = \frac{\tau_R(z)}{\left( \tau_R \right)_0}, \tag{5.2.35}
\]

and for core region is given by

\[
\overline{\tau_{R_1}(z)} = \frac{\tau_{R_1}(z)}{\left( \tau_R \right)_0}, \tag{5.2.36}
\]

where, \( \left( \tau_R \right)_0 = \frac{C}{2} R_0 \)

Which can be obtained with the help of equation (5.2.24) for ppl layer as

\[
\overline{\tau_R(z)} = \overline{R(z)} - \frac{1}{2 \overline{R(z)} A} \left( \overline{u_s} + B \right), \tag{5.2.37}
\]

and for core region (using equation (5.2.25)) as

\[
\overline{\tau_{R_1}(z)} = \overline{R_1(z)} - \frac{1}{2 \overline{R_1(z)} A} \left( \overline{u_s} + B \right). \tag{5.2.38}
\]
Another representation of apparent viscosity can be obtained by the following formula

\[ \frac{\mu_a}{\mu_2} = \frac{\mu_a}{\mu_2} \]  \hspace{1cm} (5.2.39)

and can be obtained (using equation (5.2.27)) as

\[ \frac{\mu_a}{\mu_2} = R(z)^4 \left[ W + \frac{1}{A} (u_s + B)(2R(z)A - B) \right]^{-1}. \]  \hspace{1cm} (5.2.40)

where, \( \frac{u_1}{u_0}, \frac{u_2}{u_0}, \frac{Q}{Q_0}, \frac{\mu_2}{\mu_0} \)

\[ u_s = \frac{u_s}{u_0}, \quad u_0 = \frac{CR_0^2}{4\mu_2}, \quad Q_0 = \frac{\pi CR_0^4}{8\mu_2}. \]  \hspace{1cm} (5.2.41)

5.4 RESULTS AND DISCUSSIONS

In carrying out the present work for blood flow through an annular region in (Fig.5.1) \( kR_o \leq r \leq R(z) \) between a stenotic wall \( R(z) \) and a catheterized artery (\( k << 1, \) catheter radius \( kR_0 << R_o, \) artery radius), the following estimates for the constricted region, such as stenosis length \( L_0 = 8 \) its location \( d \) in the region \( d \leq z \leq d + L_0 \) (Biswa, 2000; Chaturani and Biswas, 1983), stenosis development in asymmetric manner and maximum heights of stenosis \( \delta_s \) in dimensionless form equals to \( 1 - \sqrt{3}/2, 1 - 1/\sqrt{2} \) and \( \frac{1}{2} \) corresponding to an abnormal growth of 25, 50 and 75 percents in three respective and gradual cases of mild, moderate and severe formations at the lumen of an artery, have been used in developing the current mathematical analysis. It is already reported that knowledge of rheological and fluid dynamic properties of blood and its flow, like velocity, pressure gradient, shear stress at wall, flow rate etc., might play an important role in the fundamental understanding, diagnosis and treatment of many cardiovascular (cvs), renal and arterial diseases (Dintenfass, 1981; Punder and Punder, 2006; Cokelet, 1972). In view of this, analytical expressions for axial velocity, flow rate, pressure gradient, resistance to flow, wass
shear stress, apparent viscosity etc. have been obtained in this study and their graphical representations are shown in Figs. (5.2-5.30). It may be noticed that velocity is a function of shear viscosities ($\mu_1, \mu_2$), pressure gradient $\frac{dp}{dz}$, tube radii $R_0, R(z)$, $R_1(z)$ and a catheter $kR_0 \leq 1$, stenosis length $L_0$, its location $d$, $\delta_s, \delta_t$, axial coordinate $z$, radial coordinate $r$ and slip velocity $u_s$. This is in contrast to Poiseuille flow of blood (behaving as a Newtonian fluid) wherein velocity depends only on $R_0$, $\mu_1$ and $R$. Also, the non-uniform radii $R(z)$ and $R_1(z)$ in eqs. (5.4.3-5.4.6) depends on axial distance $z$, $R_0$ normal artery radius in an unobstructed tube.

The present model includes the following cases:

When $kR_0=0$ and $R(z) = R_1(z)$, it results in Newtonian flow model of an arterial stenosis with no-slip ($u_s=0$) and slip $u_s \neq 0$ respectively. In case $R(z)=R_1(z) = R_0$ and $u_s \geq 0$, it reduces to annular flow between co-axial cylindrical tube models of Newtonian fluid with slip and no-slip conditions.

When $R(z)=R_1(z) = R_0$, $kR_0=0$ and $u=0$, it expresses a Poiseuille flow of blood inside a uniform tube with slip or zero-slip at the $\delta_t \neq 0 \neq \delta_s$ and $\alpha \neq 1, e$ boundary. If $R(z)=R_1(z) \neq R_0$, $kR_0 \neq 0$ and $u_s \geq 0$, then it represents a two-layered annular flow of blood through a uniform artery with slip or no-slip at interface.

In the analysis, the combined influence of several parameters have been developed, in cases of uniform region. To analyze the quantitative effect of uniform artery, maximum height of stenosis $\delta_1, \delta_s$, slip velocities ($u_s \geq 0$) at the interface, Newtonian behaviour of blood, two-layered flow etc., computer codes have been developed for the numerical evaluations of the analytic results obtained for velocity, flow rate, wall shear stress and pressure gradient for parameter values $\delta_s = .15, \delta_t = .12, u_s = .00, .05, .1, Q = .5, 1, 1.5$ (Verma and Parihar2009,2010) and viscosities $\mu_1 = 1.2$ cp, $\mu_2 = 2$ cp, $\mu'_2 = .62, \alpha = .82 (< 1)$ for a full scale location from $z = d$ to $d + L_0$, and $0 \leq r \leq R_1(z)$, $R_1(z) \leq r \leq R(z)$ for PPL and core regions have been used. In the foregoing analysis, an attempt is taken up to address the variations of velocity, flow rate characteristics etc., due to such parameters.
5.4.1 Velocity profiles

A comparison of velocity profiles that have been obtained from eqs. (5.4.17-5.4.18), for slip and no-slip cases, maximum heights of stenosis and different axial locations for \( z=d+L_0/2, d+L_0/4, d+3L_0/4 \) for shape parameter \( n=2, 6, 9 \) and for other parameter values, is shown in Figs. (5.2-5.16). As tube radius \( r/R \) ranges from 0 (at tube axis) to 1 (at wall) on either side of axis, velocity decreases from a greater value at axis to a smaller one slip velocity at interface and, then to a minimum magnitude zero-slip velocity at boundary. As expected, velocity increases with slip at interface. Its values are higher for flows with slip \( (u_s>0) \) than those with no-slip \( (u_s=0) \). Also it is observed from Figs. (5.2-5.3) that \( u_1\big|_{n=9} < u_1\big|_{n=2} < u_1\big|_{n=6} \). Velocity attains the greatest magnitude (Figs. 5.2-6.16) in mild form of stenosis and its is the minimum for the case of severe growth. Although, it shows a little deviation from parabolic profile, in to the core region, its behaviour is parabolic in the peripheral region. For symmetric and asymmetric stenoses, it behaves differently. As slip velocity increases, velocity increases in all three forms of stenosis formation at an artery wall.

5.4.2 variation of flow rate

The variations of flow rate with different parameters are shown in (Figs.5.12-5.16). It decreases in magnitude from the initiation position to the stenotic throat and there after it increases to the termination position. It is seen that \( \bar{Q} \) decreases as shape parameter \( n \) decreases. The greatest magnitude is attained at the stenotic throat for \( n=2 \) (symmetric case) at \( z=d+L_0/2 \) and away from the throat for \( n>2 \) (asymmetric case) at \( z=d+3L_0/4 \). In all cases of stenosis, flow rate increases as slip velocity increases. As stenotic growth is considered from mild form to severe case, flow rate decreases from a higher magnitude to a lower one.

5.4.3 Variation of wall shear stress

Figs. (5.17-5.23) shows the variation of wall shear stress in the annular region. It decreases with velocity slip and also with an increasing magnitude of slip velocity. It decreases from mild form to severe case, but with slip in velocity, it is lowered. It decreases from a higher magnitude at the end of stenosis to the position of minimum constriction and therefore, it increases to a higher value at the other end of stenosis.
5.5.4 Variation of apparent viscosity

Apparent viscosity can be calculated from equation (5.2.40) and its variations with axial distance for different stenosis sizes and for various magnitudes of velocity slip, are shown in Figs.5.27-5.29.

It is observed from the figures that as axial coordinate ranges from either end to the throat of stenosis, \( \mu_a \) increases from a lower value at the initiation or termination of stenosis to a higher one at the minimum constricted area. The variation of apparent viscosity (Figs. 5.27-5.29) shows that it is lowered with the employment of slip in all forms of stenosis. It shows differently for shape parameter \( n=2 \) and \( n > 2 \). As \( \mu_a \) increases as tube radius decreases from either end of stenosis to its throat, therefore, it shows Inverse Fahraeus-Lindqvist Effect (IFLE).

![Graph showing variation of axial velocity against radial distance for different slip](image)

Fig5.2: Variation of axial velocity against radial distance for different slip
Fig5.3: Variation of axial velocity against radial distance for different slip

Fig5.4: Variation of axial velocity against radial distance for different slip
Fig 5.5: Variation of axial velocity against radial distance for different slip with same heights of asymmetric stenosis, $n = 2, n = 6$ and $n = 9$

\[ \mu_s = 0.62, \alpha = 0.8, \phi = 1^\circ \]

Fig 5.6: Variation of axial velocity against radial distance for different axial locations $n = 2$

\[ n = 2, \quad \mu_s = 0.05, \phi = 1^\circ \]

\[ \alpha = 0.8, \mu_s = 0.62 \]
Fig 5.7: Variation of axial velocity against radial distance for different axial locations $n = 2$

Fig 5.8: Variation of axial velocity against radial distance for different axial locations $n = 6$
Fig 5.9: Variation of axial velocity against radial distance for different axial locations $n = 6$

Fig 5.10: Variation of axial velocity against radial distance for different axial locations $n = 9$
Fig 5.11: Variation of axial velocity against radial distance for different axial locations $n = 9$

Fig 5.12: Variation of flow rate against axial distance for different tapering angles and stenoses
Fig 5.13: Variation of flow rate against axial distance for different tapering angles and stenoses

Fig 5.14: Variation of flow rate against axial distance for different tapering angles and stenoses
Fig 5.15: Variation of flow rate against axial distance for different heights of stenosis and tapering angles

Fig 5.16: Variation of flow rate against axial distance for different heights of stenosis and tapering angles
Fig 5.17: Variation of wall shear stress against axial distance along symmetric stenosis $n = 2$, for $\phi > 0, \phi = 0, \phi < 0$ respectively.

Fig 5.18: Variation of wall shear stress against axial distance along symmetric stenosis $n = 6$, for $\phi > 0, \phi = 0, \phi < 0$ respectively.
Fig 5.19: Variation of wall shear stress against axial distance along symmetric stenosis $n = 9$, for $\phi > 0, \phi = 0, \phi < 0$ respectively.

Fig 5.20: Variation of wall shear stress against axial distance for different heights of stenoses.
Fig. 5.21: Variation of wall shear stress against axial distance for different heights of stenoses.

Fig. 5.22: Variation of wall shear stress against axial distance for different heights of stenoses.
Fig. 5.23: Variation of wall shear stress against axial distance for different heights of stenoses.

Fig. 5.24: Variation of apparent viscosity against axial distance for asymmetric stenosis $n = 2, n = 6, n = 9$ with different tapering for velocity slip 0.00.
Fig 5.25: Variation of apparent viscosity against axial distance for asymmetric stenosis $n = 2, n = 6, n = 9$ with different tapering for velocity slip 0.05.

Fig 5.26: Variation of apparent viscosity against axial distance for asymmetric stenosis $n = 2, n = 6, n = 9$ with different tapering for velocity slip 0.1.
Fig 5.27: Variation of apparent viscosity against axial distance for different heights of stenoses

Fig 5.28: Variation of apparent viscosity against axial distance for different heights of stenoses
5.5 CONCLUSION

In the present chapter, steady flow of blood (a Newtonian fluid) through a catheterized stenosed vessel subject to the condition of slip at the interface of fluids/layers, obey mild, moderate and severe stenosis as well as zero-slip at the catheter and at the tube wall, has been investigated. Analytic expressions for velocity, flow rate, pressure gradient, wall shear stress, apparent viscosity etc. have been obtained in this study. It can be noticed that velocity is a function of $C$, $\mu_1$, $\mu_2$, tube radii $R_0$, $R_1(z)$ and $R(z)$ and a catheter $kR_0 \leq 1$, stenosis length $L_0$, its location $d$, heights $\delta_0$, $\delta_1$, axial $(z)$, radial $(r)$ co-ordinates and slip velocity $u_s$. This is in contrast to Poiseuille flow of blood (behaving as Newtonian fluid) wherein velocity depends only on $R_0$, $\mu_1$ and $R$. Here three gradual advances of an abnormal growth (symmetric $n=2$, $n \geq 2$) and lumen of an artery cases of a slip and no-slip at interface and a catheter boundary are dealt with. Important observation of the present analysis may be included in the following:

a) The current model includes Poiseuille flow of blood (Newtonian Fluid) with slip or zero-slip at vessel wall and non-Newtonian fluid with slip or zero-slip
at stenotic wall, annular flow models between co-axial cylindrical tubes and a
Newtonian fluid with slip and no-slip and Newtonian fluid models of blood
flow in a catheterized stenosed uniform artery with slip or zero-slip conditions.
b) The velocity increases with an axial slip and it increases to higher magnitudes
with the increasing values of slip whereas it decreases with an increase in
height of the stenosis, as expected.
c) The flow rate increases with a slip and attains the greatest magnitudes at
either end of the constricted annular region and the least value at the throat of
the stenosis. However an increase in stenosis size, the flow rate decreases.
d) wall shear stress decreases with velocity slip and also with an increasing
magnitude of slip velocity. It decreases from mild form to severe case, but
with slip in velocity, it is lowered. It decreases from a higher magnitude at the
end of stenosis to the position of minimum constriction and therefore, it
increases to a higher value at the other end of stenosis.

e) as axial coordinate ranges from either end to the throat of stenosis, \( \bar{\mu_a} \)
increases from a lower value at the initiation or termination of stenosis to a
higher one at the minimum constricted area.

It may be worth mentioning that by employing a velocity slip at interface, wall shear
stress may be reduced to a considerable extent that in term will help in reducing
damage or rupture to the arterial endothelium. Also apparent viscosity \( \bar{\mu_a} \)
increases as tube radius increases in the annular constricted region decreases, it shows Inverse
Fahrnus-Lindqvist Effect (IFLE) for cases of slip or no-slip. Therefore the present
model could explain an anomaly in blood flow. Also the reduction in wall shear
stress, pressure drop and apparent viscosity and enhancement in velocity and flow rate
as a result of introducing at the axial velocity at the slip at interface, may be exploited
for better functioning of the diseased arterial system and pressure flow-relationship in
a uniform stenosed artery. Hence one may look forward for such device (drugs or
tools) which could produce slip and used them for treatment and cure of PPL and
arterial diseases as well as for rupture or damage to the arterial endothelium. Further
the existing experimental work on blood flow through stenosed vessels, consider only
pressure drop. It could be a matter of interest and important to determine the wall
shear stress slip at interface in two-layered flow, velocity and flow rate etc. in the
stenosed flow as well in the annular flow in a catheterized stenosed artery. Such investigation may be useful in determining the growth, development and progression of an arterial stenosis and investigating the pressure-flow relations and the behaviour of flow variables in the two-layered annular region caused by the invention of a catheter in an arterial stenosis and that in turn may be useful for better understanding of stenotic and arterial diseases like angina, pectoris, myocardial infarction, stroke, thrombosis and Hbss etc. In blood flow modeling, it has been reported that body fluid blood posses a finite yield stress and exhibit a non-Newtonian behaviour. In order to take account of both the characteristic of blood a Casson fluid in a two-layered uniform stenosed artery have been consider in the next chapter 6.