A supply chain (SC) consists of all parties involved directly or indirectly, in fulfilling a customer’s request. The SC includes not only the manufacturer and suppliers, but also transporters, warehouses, retailers and even customers themselves. SC design, planning, and operation decisions play a significant role in the success or failure of a firm, [Chopra et. al. (2010)]. Procurement and distribution of goods are the functions which go in tandem in SC; therefore, for minimizing the costs of a SC and to have significant gains, it becomes important to integrate procurement and distribution decisions.

In the recent past, the concept of ordering optimum quantity i.e. economic order quantity (EOQ) with discounts on ordered quantity and freight in procurement and distribution SC gained wide acclaim by reducing the buyer’s inventory cost and improving the supplier’s profit simultaneously as a subject of study and reflection. These studies reveal appealing results on quantity discounts, freight discounts and integration of both discounts, which are discussed further. Rosenblatt and Lee (1985), discussed the quantity discount problem from the view of a supplier, assuming a retailer always uses an EOQ of the given model. It has been claimed that sometimes quantity discounts may be beneficial for both supplier and retailer. Jeuland (1983) proposed a quantity discount schedule as a way of coordinating a channel; the work explains that quantity discount schedule would fix retailer’s and manufacturer’s profit to some extent. Moorthy (1987), however, states that there is nothing exceptional about quantity discounts for channel co-ordination while they are special in price discrimination. Maqbool and Srikanth (1987) analyzed the seller’s decision to offer quantity discounts to the buyer. They ranged the order sizes and prices which reduce costs for buyer and seller. Weng (1995) and Karabati and Sayin (2008) showed that


centralized channel-wise profits can be achieved in a decentralized system by different quantity discount schemes.

Freight rate discounts depending on shipment size has become common practice. Many companies rely on the third party carrier for transportation of goods from supply location to retail or consumption point, [Carter and ferrin (1996)]. De Jong and Ben-Akiva (2007), employed a discrete choice model to describe the decision process for transportation choice, and applied it to freight traffic in Norway and Sweden. The authors developed a multi-nominal logit model that focused on the choice of shipment size, number of segments in the transport chain, use of consolidation and distribution centres for road, rail, water and air transport, and mode choice for each segment. These decisions were incorporated into the model as a minimization of the full logistics cost function which included order, transport, and inventory related costs. Caputo et al. (2005) developed a decision support system to aid in mode and carrier selection, for Long Range Direct Shipping (LRDS) in the European Union. LRDS is a type of vehicle transportation where goods are shipped from the manufacturer to the final customer without intermediate warehouses. This results in a decision problem that is highly complex since customer assignment and order aggregation are an important part of the problem. Burks et. al. (2004) noted that truckload (TL) firms have almost three times as many large trucks as less than truckload (LTL) firms and operate more than three times the annual total mileage, a consequence of the fact that TL firms use their vehicles more intensely (mean annual mileage) than LTL firms. In TL transportation, the cost is a fixed for one truck up to a given capacity. However in some cases the weighted quantity may not be large enough to substantiate the cost associated with a TL mode. In such situation, a LTL mode may be used. LTL is defined as a shipment of weighted quantity which does not fill a truck, and the transportation cost is taken on the basis of per unit weight. However, the usage of combination of both modes is not discussed by the author. Pooley (1993) explored the carrier selection problem of choosing either a LTL or a multiple-stop TL motor carrier to deliver products. Specifically, the paper presented a methodology for analyzing the cost implications of different carrier selection decisions. Shippers and carriers can use this methodology to analyze how changes in input variables may affect a shipper's...
carrier selection decision. Carrier selection and mode choice are addressed as procurement of transportation services by Caplice and Sheffi (2003). The authors developed an approach for procuring TL motor carrier transportation services based on economies of scope in transportation. The paper described a combinatorial auction run by the shipper to determine the minimum cost allocation of its lanes to carriers. The process is structured so that carriers bid on bundles of lanes, i.e. a conditional package bid which reflects the firm's cost based on volume and lane assignment.

Coordination of different entities plays a major role in minimizing the total cost of SC. Cheung and Lee (2002) modeled forced shipment co-ordination in order to have full TL shipments. Corbett and Groote (2000) considered coordinating the SC when the buyer has some private information. Mendoza and Ventura (2008) developed an unconstrained integrated inventory-transportation model to decide optimal order quantities for an inventory system over a finite horizon. In the present article, we are considering a SC of a multi brand garment retail stores of India where garments are procured from multiple sources, a distributor transports the garments to various stores of a buyer, i.e. integration of procurement and distribution thereby reducing the total cost of a retailer.

In this chapter we consider multi source multi-destination procurement-distribution supply chain in which models are proposed to compute optimal ordered quantities for the integrated procurement-distribution problem with all unit discounts on quantity ordered and shipping that discounted quantity with two modes of transportation, namely ‘truckload’ (TL) and ‘less-than-truckload’(LTL) in Section I. A finite planning horizon and deterministic demand is assumed to make ordering decisions. Section II develops coordinated quantity and freight discount policy for perishable products under uncertain cost and demand information i.e. fuzzy nature. The fuzzification grants authenticity to the model in the sense that it allows vagueness in the whole setup which brings it closer to reality. A fuzzy optimization problem is converted to crisp mathematical programming problem using membership function. Cases are in both sections are provided to validate the procedure.
4.1 PROCUREMENT DISTRIBUTION COORDINATION FROM MULTIPLE-SOURCE TO MULTIPLE-DESTINATION

4.1.1 Problem Description & Formulation

This section discusses the current problem in a garment company and formulates a mathematical model based on the description.

4.1.1.1 Problem Description

According to Panthaki (2008), the Garment Industry of India is a Rs. 1 trillion industry. Geographically, men’s garments are largely produced in western and southern India while production of ladies garments predominates in Northern India. Eastern part of India specializes in children garments. The industry manufactures over 100 different types of garments which includes overcoats/raincoats, suits, ensembles, jackets, dresses, skirts, shirts, blouses, T-shirts, jerseys/pullovers, jeans, pants, babies garments as well as accessories like shawls/scarves, handkerchiefs, gloves and parts of garments. Globally, it is led by a coordination of retailers, transporters, merchandisers, buyers, and suppliers; each plays an important role in a network of SCs. The peculiar characteristics of garment SC are short product life cycle and impulse purchasing. These factors bring high pressure to big garment retailers like Pantaloon Retail Ltd, Fashion at Big Bazaar, Reliance Trends etc. to manage their SCs. Moreover in today’s competitive environment, markets are becoming global, dynamic, and customer-driven as customers are demanding more variety, better quality and service. In order to ensure growth, it has become mandatory for garment retail SC to be adaptive and anticipative that would only be possible if the garment companies are capable of responding quickly the demanded product at the right time and at the right place. Therefore Indian companies in this sector have to be more responsive which can be only achieved by establishing a high level of collaboration with other partners of the SC. Thus there is an urgent need to understand the importance of collaboration in a SC perspective and develop a set of strategies to manage them.

In a current study, a garment company is surveyed for its integrated procurement and distribution policies of four retail stores(RS1, RS2, RS3 and RS4) in a city. Company is growing at the rate of 30% per year with premium brands like Louis Philippe, Van Heusen, Allen Solly, Allen Solly women’s wear, Peter England, Element, Byford, &
SF Jeans. Stores procure primarily shirts and trousers like Jeans, Pants, Shirts and T-shirts quarterly from two warehouses (WH1 & WH2) of a supplier, whose carrying cost is borne by the company. Company appreciates and understands the importance of business relationships with its suppliers, therefore, rarely breaks contractual agreements, hence is offered by discounts on bulk purchase. Also, goods are transported from supplier to retail stores through various modes i.e. TL, LTL and combination of both in kgs., where, per unit weight of Jeans, Pants, Shirt and T-shirt are 0.60 kgs., 0.70 kgs, 0.80 kgs. and 0.50 kgs. respectively. Each truck carries 1,500 kgs. of garments and cost per kg of shipping the garments is $2.0. The cost – benefit measurements are very important in any business, the same is the case in the current study that Garment Company desires to minimize the cost incurred on procurement and distribution SC. The data pertaining to various costs during the procurement and distribution to reduce their costs is given in Appendix B (Table 4.1.1 – 4.1.5).

4.1.1.2 Assumptions

The assumptions of this research are essentially the same as those of EOQ model except for the transportation cost. The section considers a single stage system with finite planning horizon. The demand is dynamic and no shortages are allowed. Lead times are assumed to be zero for both modes of transportation available, namely truck load (TL) and less-than truckload (LTL). Initial inventory of each product is zero at the beginning of the planning horizon and the holding cost is independent of the purchase price and any capital invested in transportation.

4.1.1.3 Sets

- Product set with cardinality \( P \) and indexed by \( i \).
- Period set with cardinality \( T \) and indexed by \( t \).
- Quantity discount break point set with cardinality \( L \) and indexed by small \( l \).
- Source set with cardinality \( J \) and indexed by \( j \).
- Destination set with cardinality \( M \) and indexed by \( m \).
4.1.1.4 Decision Variables

- **\(X_{ijmt}\)**: Amount of product \(i\) ordered during period \(t\) transported from \(j^{th}\) source to \(m^{th}\) destination.
- **\(R_{ijmlt}\)**: If the \(i^{th}\) ordered quantity from \(j^{th}\) source to \(m^{th}\) destination during \(t^{th}\) period falls in \(l^{th}\) price break, then the variable takes value 1, otherwise zero.
- **\(I_{ijmt}\)**: Inventory level for \(i^{th}\) product at \(m^{th}\) destination at the end of period \(t\).
- **\(IN_{im}\)**: Inventory level at the beginning of planning horizon for product \(i\) at destination \(m\).
- **\(\delta_{jmt}\)**: Total products ordered (transported in weights) from \(j^{th}\) source to \(m^{th}\) destination during \(t^{th}\) period.
- **\(\alpha_{jmt}\)**: Total number of truckloads from \(j^{th}\) source to \(m^{th}\) destination during \(t^{th}\) period.
- **\(y_{jmt}\)**: Amount in excess of truckload capacity (in weights) from \(j^{th}\) source to \(m^{th}\) destination during \(t^{th}\) period.
- **\(u_{jmt}\)**: Reflects usage of either both TL and LTL policies or only TL policy
  \[u_{jmt} = \begin{cases} 1, & \text{if considering TL and LTL both} \\ 0, & \text{otherwise} \end{cases}\]

4.1.1.5 Parameters

- **\(C\)**: Total inventory cost
- **\(\beta_{jmt}\)**: Fixed freight cost for each truck load transported from \(j^{th}\) source to \(m^{th}\) destination during \(t^{th}\) period
- **\(D_{imt}\)**: Demand for item \(i\) in period \(t\) for destination \(m\)
- **\(h_{ijmt}\)**: Inventory holding cost per unit of item \(i\) from \(j^{th}\) source at \(m^{th}\) destination in period \(t\)
- **\(w_i\)**: Per unit weight of item \(i\)
- **\(a_{ijmlt}\)**: Quantity threshold beyond which a price break becomes valid during period \(t\) for item \(i\) for \(l^{th}\) price break from \(j^{th}\) source to \(m^{th}\) destination
- **\(d_{ijmlt}\)**: Discount factor that is valid if more than \(a_{ijmlt}\) unit are purchased \(0 < d_{ijmlt} < 1\)
- **\(d_f\)**: Discounted slab
- **\(\omega\)**: Weight transported in full truckload
- **\(\phi_{ijmt}\)**: Unit purchase cost for \(i^{th}\) item in \(t^{th}\) period during \(t^{th}\) period for \(m^{th}\) destination from \(j^{th}\) source.
- **\(s\)**: cost of shipping unit weighted quantity in LTL mode
4.1.1.6 Quantity Discounts

Quantity Discounts for the ordered products are defined as:

\[
d_f = \begin{cases} 
  d_{ijmlt} & a_{ijmlt} \leq X_{ijmlt} \leq a_{ijml(t+1)t} \\
  d_{ijmlLt} & X_{ijmlt} \geq a_{ijmlLt}
\end{cases}
\]

where \( i = 1, \ldots, P; j = 1, \ldots, J; m = 1, \ldots, M; l = 1, \ldots, L; t = 1, \ldots, T \)

As discussed above, \( d_{ijmlt} \) specifies the fact that when the order size in period \( t \) is larger than \( a_{ijmlt} \), it results in discounted prices for the ordered products.

4.1.1.7 The Mathematical Formulation

The following is the formulation for above described analysis:

\[
\begin{align*}
\text{Min } C & = \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{j=1}^{J} \sum_{i=1}^{P} \left[ h_{ijmlt} l_{ijmlt} + \sum_{t=1}^{T} R_{ijmlt} d_{ijmlt} \phi_{ijmlt} X_{ijmlt} \right] \\
& + \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{j=1}^{J} \sum_{l=1}^{L} \left[ (\alpha_{jml} + \beta_{jml}) u_{ijmlt} + (\alpha_{jml} + 1) \beta_{jml} (1 - u_{ijmlt}) \right] \\
\text{subject to } & \\
\sum_{j=1}^{J} I_{ijml} = I_{ijml-1} + \sum_{j=1}^{J} X_{ijml} - D_{ijml} & \forall i = 1, \ldots, P; m = 1, \ldots, M; t = 2, \ldots, T \\
\sum_{j=1}^{J} I_{ijml} = I_{ijml} + \sum_{j=1}^{J} X_{ijml} - D_{ijml} & \forall i = 1, \ldots, P; m = 1, \ldots, M \\
\sum_{j=1}^{J} X_{ijml} & \geq \sum_{t=1}^{T} D_{ijml} \forall i = 1, \ldots, P; m = 1, \ldots, M \\
X_{ijmlt} & \geq \sum_{t=1}^{T} a_{ijmlt} R_{ijmlt} \forall i = 1, \ldots, P; m = 1, \ldots, M; t = 1, \ldots, T \\
\sum_{i=1}^{P} R_{ijmlt} & = 1 \forall i = 1, \ldots, P; j = 1, \ldots, J; m = 1, \ldots, M; t = 1, \ldots, T \\
\delta_{jmlt} & = \sum_{i=1}^{P} \left[ w_{i} X_{ijmlt} \sum_{i=1}^{L} R_{ijmlt} \right] \forall j = 1, \ldots, J; m = 1, \ldots, M; t = 1, \ldots, T \\
\delta_{jmlt} & \leq (\alpha_{jml} + \omega_{jml}) u_{jmlt} + (\alpha_{jml} + 1) \omega_{jml} (1 - u_{jmlt}) \forall j = 1, \ldots, J; m = 1, \ldots, M; t = 1, \ldots, T \\
\delta_{jmlt} & = (\alpha_{jml} + \omega_{jml}) \forall j = 1, \ldots, J; m = 1, \ldots, M; t = 1, \ldots, T
\end{align*}
\]
Procurement-Distribution Coordination in Supply Chain under Quantity Discounts & Varying Truck Load Policies

\[ X_{ijmt}, I_{ijmt}, \delta_{jmt}, y_{jmt}, \alpha_{jmt} \geq 0 \text{ and integers; } R_{ijmt}, u_{jmt} \in \{0,1\} \]
\[ i = 1,...,P; j = 1,...,J; m = 1,...,M; l = 1,...,L; t = 1,...,T \]

4.1.1.8 Analysis of Model Formulation

The objective function (4.1.1) of the optimization problem is to minimize the sum of total cost incurred in ending inventory carrying cost at the destination reflected by the first term, purchasing the goods reflected by the second term of the objective function; and the last term reflects the transportation cost at final destinations. The cost is calculated for the duration of the planning horizon.

Constraints (4.1.2 – 4.1.4) are the balancing equations, which calculate the ending inventory level during period \( t \). In eq. (4.1.2), ending inventory depends upon the inventory left in the last period, the quantity \( X_{ijmt} \) ordered in period \( t \) and at demand \( D_{imt} \). Eq. (4.1.3) calculates inventory level at the end of the first period for all the products using the inventory level at the beginning of the planning horizon, and the net change at the end of period one. Eq. (4.1.4) takes care for shortages i.e. the sum of ending inventory and optimal order quantity is more than the demand of all the periods. Eq. (4.1.5) shows that the order quantity of all products during period \( t \) exceeds the quantity break threshold. Eq. (4.1.6) restricts the activation at exactly one level, either discount or no discount situation. The integrator for procurement and distribution is eq. (4.1.7), which calculates transported quantity according to product weight. In eq. (4.1.8), the minimum weighted quantity transported is calculated and further eq. (4.1.9) measures the overhead units from truckload capacity in weights.

4.1.2 Results and Discussions

The crucial objectives of the firm includes, amount of the quantity to order and how to minimize the total cost. Here, we have aimed to answer this question with the help of case study in procurement-distribution scenario of a SC. The solution of the above formulated optimization problem is obtained by programming it in Lingo 11.0 software. The required data sets and parameters such as quantity demanded, various
Multiple Source-Multiple Destination Coordination

costs, initial inventory, weights per product, quantity thresholds and discounts tabulated in Appendix B(Table 4.1.1 – 4.1.5) (data is changed due to cutting edge competition and cannot be revealed but the model is applicable in same scenarios in big data) are fed in the lingo program to generate the solution. In particular we have considered 4 periods, 4 products, two sources, 4 destinations, 4 price breaks and attained the solution presented in Appendix B. In general we can incorporate any no. of products, periods, sources, destinations and price breaks to obtain the desired solution of the problem.

The solution obtained and presented in Appendix B(Table: 4.1.6) reflects that, during first period, for all four garments considered in our case study, ordered quantity is 400, 0, 60 and 1,510 units from WH1 to RS1 with discount of 20%, 0%, 25% and 30%, respectively on purchase cost. The ending inventories are 300 units, 760, 0 and 600 units of Jeans, Pants, Shirt and T-Shirts respectively. Weighted quantity of garments from WH1 to RS1 during first period is 1,043 kgs. Firm employs only TL mode during the same period. Rest of the procurement and distribution strategies are shown in Appendix B. After solving the problem we find that one of the possible minimum optimal total costs incurred by the company is $5,530,914.

4.2 PROCUREMENT-DISTRIBUTION COORDINATION FROM MULTI SOURCE TO MULTI DESTINATION FOR PERISHABLE PRODUCTS UNDER FUZZY ENVIRONMENT

One of the largest and the most tangible investment of any retail and manufacturing organization is, applying smart supply chain management strategies that not only help boost profit but they can mean the difference between a business thriving or barely surviving, [Simchi-Levi(2000)]. The American Production and Inventory Control Society Dictionary defines the term supply chain as “the process from the initial raw materials to the ultimate consumption of the finished product linking across supplier–user companies.” Procurement and distribution in supply chain are relatively more important decisions when the demand is uncertain and products are perishable in nature. This necessitates the high inventory level that makes the situation worst. Many business owners don’t realize the true cost of carrying excess inventory, which can be
as high as 29 percent of the inventory’s value when you include all the carrying costs (interest, storage, damage, obsolescence, etc.), [U4]. These costs come directly off the bottom-line profit. Therefore, it is required to find out the optimum levels of ordered quantity, thereby, carrying inventory, so that total procurement and distribution cost can be minimized.

Better coordination amongst the suppliers, distributors and retailers is the key to success for every supply chain. Tersine and Barman (1991) developed a lot-for-lot discount pricing policy for deteriorating items with constant demand rate. Yang and Wee (2003) developed an optimal quantity-discount pricing strategy in a collaborative system for deteriorating items with instantaneous replenishment rate. Chen and Kang (2007) thought out integrated vendor-buyer cooperative inventory models with variant permissible delay in payments. Singh and Singh (2007) discussed optimal policy for decaying items with stock dependent demand under inflation in a supply chain. Rau and Ouyang (2008) have introduced an optimal batch size for integrated production-inventory policy in a supply chain. Hwang et al. (1990) determined optimal order quantity when all units’ quantity discounts are available on purchasing price and freight cost. Tersine and Barman (1991) assumed a constant demand rate and developed a model with freight and price discounts, where freight discount structure is based on weight. Ertogral (2008) took a single stage multi incapacitated dynamic lot sizing problem (MILSP) with transportation cost and assumed finite planning horizon with dynamic demand. He considered all unit inventory management models to formulate the problem with piece wise linear transportation cost function. Mendoza and Ventura (2008) developed an unconstrained integrated inventory-transportation model to decide optimal order quantity for inventory system over a finite horizon.

In the crisp environment, all parameters in the total cost such as holding cost, set-up cost, purchasing price, rate of deterioration, demand rate, production rate etc. are known and have definite value without ambiguity. Some of the business situations fit such conditions, but in most of the situations and in the day-by-day changing market scenario the parameters and variables are highly uncertain or imprecise. For any particular problem in the crisp scenario, the aim is to maximize or minimize the objective function under the given constraint. But in many practical situations, the
decision maker may not be in the position to specify the objective or the constraints precisely, but rather specify them uncertainly or imprecisely, [Singh and Singh (2011)]. In such situations, these parameters and variables are treated as fuzzy parameters. The fuzzification grants authenticity to the model in the sense that it allows vagueness in the whole setup which brings it closer to reality. The theory of fuzzy set approach [Guiffrida(1998)] has found wide applications in operations management. As a part of operations management, inventory control and supply chain management have also seen an exhaustive applications of fuzzy sets. A brief review of supply chain models based on fuzzy sets is discussed below. Chen et al. (1996) have analyzed a fuzzy inventory model. Two fuzzy models with fuzzy parameters have been developed and derived the optimal production quantity by using graded mean integration representation method and extended Lagrangian method, [Hsieh (2002)]. The author had shown that a crisp model is a specific case of the fuzzy model. Chang et al.(2006), Gen et al. (1997), Lee and Yao (1998), Lee and Yao (1999), Mendoza and Ventura (2008) and Yang et al. (2003) have discussed different problems that consider inventory with backorder, inventory and production inventory in the fuzzy sense. An inventory model has been discussed in [Yao and Chiang (2003)] without backorder where total demand and holding costs were assumed to be fuzzy in nature and the authors had used different methods to derive total cost. Park (1987) has developed an EOQ model with uncertain inventory cost under arithmetic operations of extension principle and the author used trapezoidal fuzzy numbers to represent the inventory costs. Further, Yao and Lee (1996) have considered shortages while developing their EOQ model with fuzzy order quantity.

The next session develops policies for SC model that includes procurement, inventory holding and transportation decisions incorporating quantity and freight discount policies in order to keep the total cost to its minimum.

4.2.1 Problem Description & Formulation

This section discusses the current problem in a retail chain and formulates a mathematical model based on the description.
4.2.1.1 Problem Description

Many observers [Goldman (1974)] of retailing in developing countries, prior to the 1990s, had been predicting that there was not going to be a primary and extensive retail transformation in the near future. The limited means for urban consumers and barriers to local sourcing of products by supermarkets related to poor traditional agri-food supply chains and intermediate distribution sectors were considered nearly challenging obstacles to widespread retail modernization. The ‘supermarket revolution’ in developing countries with its ‘take-off” in the early mid-1990s flies in the face of these earlier predictions with presence of retail chains Reliance Fresh, Food bazaar, More, Spencers etc. In a current study, a well established company is surveyed for its procurement and distribution policies for three months (periods) of three retail stores (RS1, RS2, RS3) in a city. Stores procure food items like grains, grocery, dairy, poultry etc from two warehouses (WH1, WH2) of a supplier, whose carrying cost is borne by the stores. A perishable food segment of food grains viz. rice, wheat, sorghum and maize are considered in the study that requires regular inspection with inspection cost of $2 per sack while assuming the 5% of perishability in a lot with the weight per sack of grains are 6, 7, 8 and 5 kgs respectively. Company appreciates and understands the importance of business with its suppliers, therefore, rarely breaks contractual agreements, hence is offered by discounts on bulk purchase. Also, goods are transported from supplier to retail stores through various modes i.e. truckload (TL), less than truckload (LTL) and combination of both. In TL transportation, the cost is a fixed for one truck up to a given capacity. The capacity for each truck is 1,500kgs. However in some cases the weighted quantity may not be large enough to substantiate the cost associated with a TL mode. In such situation, a LTL mode is used. The cost of transporting each sack in this mode is $2 per kg. As discussed the nature of the products is perishable, predicting a concrete demand is impossible and leads to uncertainty for procurement and distribution. The cost – benefit measurements are very important in any business, the same is the case in the current study that company desires to minimize the cost incurred on procurement and distribution supply chain. Here we are examining such situations where demand is uncertain and try to minimize the vagueness of total costs using fuzzy sets and membership functions.
4.2.1.2 Assumptions

The assumptions of this research are essentially the same as in previous section except that demand and total cost are fuzzy. Also products under study are perishable in nature.

4.2.1.3 Sets

The sets are same as in previous section 4.1

4.2.1.4 Decision Variables

The decision variables are same as in Section 4.1

4.2.1.5 Parameters

The additional parameters for this section are as follows:

\[ \tilde{C} \]  
Fuzzy total cost

\[ C_0 \]  
Aspiration level of fuzzy total cost

\[ \tilde{C}_0^* \]  
Tolerance level of fuzzy total cost

\[ \tilde{D}_{limt} \]  
Fuzzy demand for product \( i \) in period \( t \) for destination \( m \)

\[ \tilde{D}_{limt}^d \]  
Defuzzified demand for product \( i \) in period \( t \) for destination \( m \)

\[ m_i \]  
Rate of inspection of the \( i^{th} \) item

\[ \eta \]  
Percentage of the defective items of the stored units

Rest of the parameters used, are same as in Section 4.1

4.2.1.6 Quantity Discounts

The quantity discounts slab is same as in Section 4.1
4.2.1.7 The Fuzzy Optimization Model Formulation

The fuzzy optimization problem with fuzzy total cost function and fuzzy demand is defined as under:

\[
\text{Min } \bar{C} = \sum_{i=1}^{T} \sum_{m=1}^{M} \sum_{j=1}^{I} \left\{ h_{ijmt} I_{ijmt} + m_{I} X_{ijmt} + \sum_{l=1}^{L} R_{ijmlt} d_{ijmlt} \bar{d}_{ijmt} X_{ijmt} \right\} \\
+ \sum_{i=1}^{T} \sum_{m=1}^{M} \sum_{j=1}^{I} \left[ (s_{y_{ijmt}} + \alpha_{jmt} \beta_{jmt}) u_{ijmt} + (\alpha_{jmt} + 1) \beta_{jmt} (1 - u_{ijmt}) \right] \\
\sum_{j=1}^{J} I_{ijmt} = \sum_{j=1}^{J} I_{ijmt} - \sum_{j=1}^{J} X_{ijmt} - \bar{D}_{ijmt} - \eta \sum_{j=1}^{J} I_{ijmt} \quad \forall \quad i = 1,...,P; m = 1,...,M; t = 2,...,T \\
\sum_{j=1}^{J} I_{ijmt} = \sum_{j=1}^{J} I_{ijmt} + \sum_{j=1}^{J} X_{ijmt} - \bar{D}_{ijmt} - \bar{I}_{ijmt} \quad \forall \quad i = 1,...,P; m = 1,...,M \\
(1 - \eta) \sum_{j=1}^{J} I_{ijmt} + \sum_{j=1}^{J} X_{ijmt} - \bar{D}_{ijmt} - \bar{I}_{ijmt} \quad \forall \quad i = 1,...,P; m = 1,...,M \\
X_{ijmt} \geq \sum_{l=1}^{L} d_{ijmlt} R_{ijmlt} \quad \forall \quad i = 1,...,P; j = 1,...,J; m = 1,...,M; t = 1,...,T \\
\sum_{l=1}^{L} R_{ijmlt} = 1 \quad \forall \quad i = 1,...,P; j = 1,...,J; m = 1,...,M; t = 1,...,T \\
\bar{d}_{ijmt} = \sum_{i=1}^{I} \left[ w_{i} x_{ijmt} \sum_{l=1}^{L} R_{ijmlt} \right] \quad \forall \quad j = 1,...,J; m = 1,...,M; t = 1,...,T \\
\bar{d}_{ijmt} \leq (y_{jmt} + \alpha_{jmt} \omega_{jmt}) u_{ijmt} + (\alpha_{jmt} + 1) \omega_{jmt} (1 - u_{ijmt}) \quad \forall \quad t = 1,...,T; j = 1,...,J; m = 1,...,M \\
\bar{d}_{ijmt} = (y_{jmt} + \alpha_{jmt} \omega_{jmt}) \quad \forall \quad j = 1,...,J; m = 1,...,M; t = 1,...,T \\
X_{ijmt}, I_{ijmt}, \bar{d}_{ijmt}, \bar{y}_{jmt}, \bar{\omega}_{jmt}, \alpha_{jmt} \geq 0 \text{ and integers; } R_{ijmlt}, u_{ijmt} \in [0,1] \\
i = 1,...,P; t = 1,...,T; l = 1,...,L; j = 1,...,J; m = 1,...,M

4.2.1.8 Analysis of Model Formulation

Equation (4.2.1), the fuzzy objective function is to minimize the cost incurred in holding ending inventory at source borne by firm, cost of purchasing the products, and cost of inspection on ordered quantity by destination m in period t reflected
(where the inspection rate can be taken same at all the destination in all period for product \( i \)) by the first term of the objective function; the transportation cost from the source to the destination, is the second term. The cost is calculated for the duration of the planning horizon. The ordering cost is a fixed cost not affected by the ordering quantities and therefore is not the part of objective function. Equation (4.2.2) – (4.2.4) are the balancing equations for sources and destinations where equation (4.2.2) finds total ending inventory at all sources of \( i^{th} \) product in \( t^{th} \) period by reducing the fuzzy demand of all the destinations and fraction of perished inventory in \( t^{th} \) period from total of ending inventory of previous period and ordered quantity at \( t^{th} \) period of all the destinations. Equation (4.2.3) finds total ending inventory at all sources of \( i^{th} \) product in first period by reducing the fuzzy demand of all the destinations and fraction of perished inventory of the same period from total of initial inventory if the planning horizon and ordered quantity at first period of all the destinations. Equation (4.2.4) shows that total fuzzy demand in all the periods from all destinations is less than or equal to total of ending inventory and ordered quantity at all the sources in all the periods i.e. shortages are not allowed. Equation (4.2.5) & (4.2.6) find out the order quantity of all products in period \( t \) which may exceed the quantity break threshold, and avails discount on ordered quantity at exactly one quantity discount level. Eq. (4.2.6) restricts the activation at exactly one level, either discount or no discount situation. The integrator for procurement and distribution is eq. (4.2.7), which calculates transported quantity according to product weight. In eq. (4.2.8), the minimum weighted quantity transported is calculated and further eq. (4.2.9) measures the overhead units from TL capacity in weights.

### 4.2.2 The Crisp Formulation

In this section, we convert fuzzy formulation to crisp formulation by applying the sequential steps of the algorithm given in chapter 2. The formulation is given as under:

Maximize \( \theta \)

subject to: \( \mu_c (X) \geq \theta \),

\[
\sum_{j=1}^{J} I_{ijmt} = \sum_{j=1}^{J} I_{ijmt-1} + \sum_{j=1}^{J} X_{ijmt} - D_{ijmt} - \eta \sum_{j=1}^{J} I_{ijmt} \\
\forall \ i = 1, \ldots, P, \ j = 1, \ldots, J, \ m = 1, \ldots, M, \ t = 2, \ldots, T
\]
\[
\sum_{j=1}^{J} l_{ijm1} = \sum_{j=1}^{J} I_{nj} + \sum_{j=1}^{J} X_{ijm1} - D_{ijm1} - \eta \sum_{j=1}^{J} l_{ijm1} \forall i = 1, \ldots, P; m = 1, \ldots, M.
\]

\[
(1-\eta) \sum_{j=1}^{J} \sum_{t=1}^{T} l_{ijmt} + \sum_{j=1}^{J} \sum_{t=1}^{T} X_{ijmt} \geq \sum_{t=1}^{T} D_{ijmt} \forall i = 1, \ldots, P; m = 1, \ldots, M
\]

\[
X \in S = \{X_{ijmt}l_{ijm1}, \delta_{ijmt}, \alpha_{ijmt}, \gamma_{ijmt} \geq 0 \text{ and integer};
\]

\[
R_{ijmt}, u_{ijmt} \in \{0,1\} / \text{satisfying eq (4.2.5) to (4.2.9)};
\]

\[
i = 1, \ldots, P; t = 1, \ldots, T; l = 1, \ldots, L; m = 1, \ldots, M; \theta \in [0,1]
\]

can be solved by the standard crisp mathematical programming algorithm.

Now main challenge is to reduce various cost components viz. purchase, transportation, inspection cost and holding cost in order to get maximum benefits. Also the relevant data is provided in Appendix B

### 4.2.3. Results and Discussions

The crucial objectives of the firm includes, amount of the quantity to order and how to minimize the total cost. Here, we have aimed to answer this question with the help of case study in procurement-distribution scenario of a Supply Chain. The required data sets and parameters such as quantity demanded, various costs, initial inventory, weights per product, quantity thresholds and discounts tabulated in Appendix B (data is changed due to cutting edge competition and cannot be revealed but the model is applicable in same scenarios in big data) are fed in the lingo program to generate the solution where the tolerance and aspiration levels taken are $30,00,000 and $20,00,000 respectively. In particular, we have considered 3 periods, 4 products, 2 sources, 4 destinations, 3 price breaks and attained the solution presented in Appendix B (Table 4.2.1 – 4.2.6). In general we can incorporate any no. of products, periods, sources, destinations and price breaks to obtain the desired solution of the problem.

The solution obtained and presented in Appendix B (Table 4.2.7) reflects that, during first period, for all four grains considered in our case study, ordered quantity of four grains from WH1 to RS1 are 200, 0, 100 and 0 sacks with discount on purchase cost of 20%, 0%, 30% and 0%, respectively. The ending inventories are 100 units, 560, 0
and 0 sacks of Rice, Sorghum, Wheat and Maize, respectively. Weighted quantity of grains from WH1 to RS1 during first period is 500 kgs. Firm employs only TL mode during the same period. Rest of the procurement and distribution strategies are shown in Appendix B. After solving the problem by means of well known software suite Lingo 13.0, we find that one of the possible minimum optimal total costs incurred by the company is $2,256,314 with 75% minimization of vagueness.