Supply chains (SC) are generally complex and are characterized by numerous activities spread over multiple functions and organizations, which pose interesting challenges for effective SC coordination. To meet these challenges, SC members must work towards a unified system and coordinate with each other. Importance of coordination in SC has escalated many folds. SC coordination requires that each stage take into account the effects of its actions on the other stages. Lack of coordination results when objectives of different stages conflict or information moving between stages is distorted. As a part of SCM literature and practice, it has long been recognised that there can be significant gains in integrating procurement and transportation decisions. Many companies rely on the third party carrier for transportation of goods from supply location to retail or consumption point. Freight rate discounts depending on shipment size has become a common practice. [Carter and Ferrin (1996)]. The freight costs are usually in the form of discounts tariffs in the quantity, which reflects economies of scale. The quantity to order at a given time must be determined by balancing two factors: 1) the cost of possessing or carrying materials 2) the cost of acquiring or ordering materials. Purchasing larger quantities may decrease the unit cost of acquisition, but this saving may not be more than offset by the cost of carrying materials in stock for a longer period of time.

The economic order quantity (EOQ) is the amount of procurement to be ordered at one time for purposes of minimising annual inventory cost. In other words, EOQ is that size of the order which gives maximum economy in purchasing any material and ultimately contributes towards maintaining the materials at the optimum level and at


the minimum cost. Baumol and Vinod (1970) first introduced freight cost into EOQ model. The model developed by them is intended to explain the choice of transport made by shippers, as well as their total demand for transportation services. The optimal choice of mode is shown to involve a trade-off among freight rates, speed, dependability (variance in speed) and enroute loss age. Swenseth and Godfrey (2002) included freight rate discounts on ordering decision, when over declaring a shipment is possible to exploit a smaller unit transportation cost. Unnikrishnan and Srinivasan (2010) addressed the impact of freight discount on the design of the network considering product recovery. They considered two types of discounting methods namely: all unit discount and incremental unit discount. The freight discount was assumed to be applicable to the stage where products are transported in large quantities. While Sawadogo and Anciaux (2011) considered, discounted growth of carriage of goods per transportation mode. They formulated a model to check the performance of an intermodal transportation system within the green SC, taking into account the economic, environmental and societal criteria in order to help decision-makers in choosing the path with the best compromised benefit/impacts in an intermodal transportation system.

Quantity also plays a vital role in emphasising EOQ. Monahan (1984) proposed the quantity discount model where the vendor can determine the discount schedule and batching policy to maximise its economic gains while adding no cost to the buyer. Banerjee (1986) and Joglekar (1988) extended Monahan’s model. Banerjee (1986) incorporated vendor’s inventory carrying cost while Joglekar (1988) allowed the frequency of buyer’s ordering to be different from that of manufacturer’s production set up. Rosenblatt and Lee (1985) discussed the quantity discount problem from the view of supplier assuming retailer always uses his EOQ. It is been said that sometimes quantity discounts can be beneficial for both supplier and retailer. Dada and Srikanth (1987) analysed the seller’s decision to offer quantity discounts to the buyer. They made the range of order sizes and prices which reduce costs for buyer and seller. Traditionally, the supplier designs a quantity discount scheme, and when buyers choose their new order quantities under the quantity discount scheme offered by the supplier, the coordinated solution is achieved. Zhou et al. (2008) considered a
simple profit-sharing mechanism that would ultimately achieve perfect channel coordination. The manufacturer is provided with a quantity discount scheme to induce the retailer to increase the order quantity to maximise the manufacturer’s profit. In literature, integrated procurement and distribution can be considered as the building block for wide SC systems. The global SC can be very complex and link-by-link understanding of joint policies can be very useful (Ben-Daya et al., 2008). Yang et al. (2010) proposed an integrated production, inventory and distribution optimisation model that considers important and interrelated decisions simultaneously. The model assumed that carriers have finite truckload and drivers for both inbound and outbound shipments. The integrated model is solved via modified Benders decomposition, which is a combination of Benders decomposition and a genetic algorithm. Hwang et al. (1990) investigated the problem of determining optimal order quantity when all units quantity discounts are available on purchasing price and freight cost. Tersine and Barman (1991) assumed a constant demand rate and developed a model with freight and price discounts, where freight discount structure is based on weight. Ertogral (2008) took a single stage multi incapacitated dynamic lot sizing problem (MILSP) with transportation cost and assumed finite planning horizon with dynamic demand. He considered all unit inventory management models to formulate the problem with piece wise linear transportation cost function. Mendoza and Ventura (2008) developed an unconstrained integrated inventory-transportation model to decide optimal order quantity for inventory system over a finite horizon. Gajpal and Abad (2010) presented the variant of vehicle routing problem that considers the optimal integration of forward flow of materials from manufacturer to customers with a backward flow of materials from customers to specialised warehouses or recycling sites. The problem is called vehicle routing problem with simultaneous pickup and delivery (VRPSPD). In VRPSPD, a customer requires a given shipment to be delivered as well as a given load to be picked up simultaneously. Abad (2006) considered the lot size problem faced by a buyer who plans product availability for a style good which has uncertain demand for the season. The buyer has only one opportunity to procure the good prior to the season and is responsible for paying for the freight. A buyer may opt for paying for the freight if he sees an opportunity for savings on freight costs. Shaw et al. (2011) determined economic ordering quantity
(EOQ) and shipping strategy of an inventory system integrated with a single-product, single-vendor, and multi-customer scenario. Depending on the demand of all customers a suitable model is illustrated to transfer equally-sized batches. Optimum values of the decision variables are determined using a direct search method.

This chapter of the thesis is divided into two sections. In the first section of this chapter we have developed mathematical models to compute optimal order for integrated procurement-transportation problem of SC with two modes of transportation namely, Truckload (TL) and Less Than Truck Load (LTL) carriers by introducing all unit and incremental quantity discount structure into analysis. Finite planning horizon and dynamic demand is assumed to make ordering decisions from single supplier to single destination of a buyer. Section 2 of the chapter incorporates quantity discounts (all unit) and freight policies for multi items that are perishable in nature, which are again supplied from single source to single destinations under uncertain environment. It is assumed that, single source or a supplier is offering various discounts on the purchased quantity and is supplying to a destination. A transporter hired by the buyer also offers different policies to choose on distribution of products, thereby buyer is enticed by dual benefit on procurement as well as distribution. A fuzzy optimization problem is converted to crisp mathematical programming problem using membership function. The problems are solved and cases are presented in both sections to validate the models.

2.1 PROCUREMENT DISTRIBUTION COORDINATION FROM SINGLE-SOURCE TO SINGLE-DESTINATION

2.1.1 Problem Description & Formulation

This section discusses the current problem in a shoe store and formulates a mathematical model based on the description. Two models are developed based on all unit and incremental discounts whose details are provided in following sub sections.

2.1.1.1 Problem Description

Gone are the days when the lack of sops by government and proper infrastructure left the footwear industry in turmoil, hardest hit among those were the players who have
pan India presence. But thanks to the liberal economic policies of the government which lead to major reforms such as permitting 100% Foreign Direct Investment through automatic route in the footwear sector and de-licensing, helped the influx of much needed investment which drastically improved the manufacturing facilities that are now in line with those available in the developed economies, hence considerably reducing the lead times.

A case in point is the processing unit of a Footwear Park being established in city’s special economic zone, typically reserved by the government to boost industrial setups. An already established footwear park in town opened a processing unit for footwear which it sells in its store under its own brand name with total sixteen styles for both men and women. Now, the biggest area of concern is the procurement of footwear, its stamping and packaging under its brand name. Because of the prevailing competition the firm wants to have, as low as possible total cost of procuring, holding and transporting the footwear from processing unit to the store. The firm already forecasted that purchasing all 16 styles of foot wears every month of year will poise the seasonal demand. Because of bulk buying supplier and transporter offer discounts, where cost of transporting (in $) products is $2/kg(\omega) and weight each truck carries is 500 kgs. (\omega). Another cost escalation factor which is already haunting is inventory carrying cost because of bulk buying, so it is also keen to minimize the unnecessary burden on its capital and ever crunching space which is limited due to floor considerations. The professional representation of data is provided in Appendix B (Table 2.1.1 - 2.1.12).

2.1.1.2 Assumptions

1) Demand is dynamic
2) Supply is instantaneous and no shortages are allowed
3) Initial inventory of each product is zero at the beginning of planning horizon
4) Two transportation policies are used, namely TL and LTL.

Figure 2.1 represents the multi item EOQ environment where per unit percentage benefit of procurement cost and per unit additional benefit from truckload/less then truckload (TL/LTL) cost jointly trade off inventory carrying charge is shown.
2.1.1.3 Sets

- Product set with cardinality \( P \) and indexed by \( i \).
- Period set with cardinality \( T \) and indexed by \( t \).
- Quantity discount break point set with cardinality \( L \) and indexed by small \( l \).

2.1.1.4 Decision Variables

\( X_{it} \)  
Amount of item \( i \) ordered in period \( t \)

\( R_{il\text{lt}} \)  
If the ordered quantity falls in \( l^{\text{th}} \) price break then the variable takes value 1 otherwise zero

\( I_{it} \)  
Inventory level at the end of period \( t \) for product \( i \)

\( IN_i \)  
Inventory level at the beginning of planning horizon for product \( i \)

\( \delta_i \)  
Total weighted quantity transported in period \( t \)

\( \alpha_t \)  
Total number of trucks in period \( t \)

\( y_t \)  
Overhead weighted quantity above truckload capacity

\( u_t \)  
Reflects usage of either both TL and LTL policies or only TL policy

\[
u_t = \begin{cases} 
1, & \text{if considering TL and LTL both} \\
0, & \text{otherwise} 
\end{cases}
\]
2.1.1.5 Parameters

\( C \) \hspace{1em} Total inventory cost
\( \beta_t \) \hspace{1em} Fixed freight cost for each truck in period \( t \)
\( D_{it} \) \hspace{1em} Demand for product \( i \) in period \( t \)
\( h_i \) \hspace{1em} Inventory holding cost for product \( i \)
\( w_i \) \hspace{1em} Per unit weight of Product \( i \)
\( a_{ilt} \) \hspace{1em} Quantity threshold beyond which a price break becomes valid in period \( t \) for product \( i \) for \( l^{th} \) price break
\( d_{ilt} \) \hspace{1em} Discount factor that is valid if more than \( a_{ilt} \) unit are purchased, \( 0 < d_{ilt} < 1 \)
\( d_f \) \hspace{1em} Discounted slab
\( \omega \) \hspace{1em} Weight transported in full truckload
\( \varphi_{it} \) \hspace{1em} Unit purchase cost for product \( i \) in period \( t \)
\( S \) \hspace{1em} Cost of shipping unit weighted quantity in LTL mode

2.1.1.6 Quantity Discounts

Quantity Discounts for the ordered products are defined as:

\[
d_f = \begin{cases} 
    d_{ilt} & \text{if } a_{ilt} \leq X_{it} \leq a_{i(t+1)l} \\
    d_{iLt} & \text{if } X_{it} \geq a_{iLt}
\end{cases}
\]

where \( i = 1, \ldots, P, l = 1, \ldots, L, t = 1, \ldots, T \)

As discussed above, \( d_{ilt} \) specifies the fact that when the order size during period \( t \) is larger than \( a_{ilt} \), it results in discounted prices for the ordered products.

2.1.1.7 The Mathematical Formulation

The model for integrated procurement-distribution all unit problem for \( i^{th} \) period, \( i^{th} \) product from single source to single destination is as follows:
Min \( C = \sum_{t=1}^{T} \left[ \sum_{i=1}^{P} \left( h_i I_{it} + \sum_{l=1}^{L} R_{ilt} d_{ilt} \phi_{ilt} X_{ilt} \right) + \sum_{i=1}^{P} (s y_i + \alpha_i \beta_i u_t + (\alpha_i + 1) \beta_i (1 - u_t)) \right] \) ... (2.1.1)

\( I_{it} = I_{it-1} + X_{it} - D_{it}, \quad \forall \; i=1,...,P, \; t=2,...,T \) ... (2.1.2)

\( I_{t|1} = I_{N|1} + X_{t|1} - D_{t|1}, \quad \forall \; i=1,...,P \) ... (2.1.3)

\[ \sum_{t=1}^{T} I_{it} + \sum_{t=1}^{T} X_{it} \geq \sum_{t=1}^{T} D_{it}, \quad \forall \; i=1,...,P \] ... (2.1.4)

\( X_{it} \geq \sum_{l=1}^{L} a_{ilt} R_{ilt} \quad \forall \; i=1,...,P, t=1,...,T \) ... (2.1.5)

\[ \sum_{l=1}^{L} R_{ilt} = 1 \quad \forall \; i=1,...,P, \; t=1,...,T \] ... (2.1.6)

\[ \delta_{t} \geq \sum_{i=1}^{P} w_i X_{it} \sum_{l=1}^{L} R_{ilt} \quad \forall \; t=1,...,T \] ... (2.1.7)

\[ \delta_{t} \leq (y_t + \alpha_t \omega) u_t + (\alpha_t + 1) \omega (1 - u_t) \quad \forall \; t=1,...,T \] ... (2.1.8)

\[ \delta_{t} = (y_t + \alpha_t \omega) \quad \forall \; t=1,...,T \] ... (2.1.9)

\( X_{it}, I_{it}, \delta_{t} \geq 0 \) and integer; \( R_{ilt}, u_t \in \{0,1\} \); \( \forall \; i=1,...,P, t=1,...,T, l=1,...,L \)

The model for integrated procurement-distribution incremental discount problem for \( i^{th} \) period and \( l^{th} \) product from single source to single destination with the changed objective function is as follows:

Min \( C = \sum_{t=1}^{T} \left[ \sum_{i=1}^{P} \left( h_i I_{it} + \sum_{l=1}^{L} R_{ilt} \left\{ \sum_{q=1}^{l-1} d_{iqt} \phi_{lt} \left( a_{iqt} - a_{iq-1t} \right) + d_{ilt} \phi_{lt} \left( X_{it} - a_{ilt} \right) \right\} \right) \right] + \sum_{t=1}^{T} \left[ (s y_t + \alpha_t \beta_t u_t + (\alpha_t + 1) \beta_t (1 - u_t)) \right] \) ... (2.1.10)

Subject to (2.1.2)- (2.1.9)
2.1.1.8 Analysis of Model Formulation

The objective function (2.1.1) and (2.1.10) of the optimization problem is to minimize the sum of total cost incurred in inventory carrying cost reflected by the first term, purchasing the goods reflected by the second term; and the transportation cost reflected by the last term at final destinations for both models. The cost is calculated for the duration of the planning horizon and the price breaks are already defined in section 2.1.1.6.

Constraints (2.1.2 –2.1.4) are the balancing equations, which calculate the ending inventory level during period $t$. In eq. (2.1.2), ending inventory depends upon the inventory left in the last period, the quantity $X_t$ ordered in period $t$ and at demand $D_t$. Eq. (2.1.3) calculates inventory level at the end of the first period for all the products using the inventory level at the beginning of the planning horizon, and the net change at the end of period one. Eq. (2.1.4) takes care for shortages i.e. the sum of ending inventory and optimal order quantity is more than the demand of all the periods. Eq. (2.1.5) shows that the order quantity of all products during period $t$ exceeds the quantity break threshold. Eq. (2.1.6) restricts the activation at exactly one level, either discount or no discount situation. The integrator for procurement and distribution is eq. (2.1.7), which calculates transported quantity according to product weight. In eq. (2.1.8), the minimum weighted quantity transported is calculated and further eq. (2.1.9) measures the overhead units above truckload capacity in weights.

2.1.2 Results and Discussions

Following points needed to be addressed by the firm for strategic decision making. How to minimise inventory carrying cost, transportation cost and purchase cost. Also which procurement and distribution policy firm should opt for. How much firm should order to fulfill demand of all the products and thereby fulfilling customer’s demand of footwear on time and availing maximum discount.

We have used well known programming tool LINGO 11.0 to solve and answer the above described problem. Our problem is integer non-linear programming problem which contains a huge data and required to be coded in a Lingo Program. A period
January is discussed for firm’s purchasing and distribution policy of all unit based model and rest may be seen in Tables 2.1.13 to 2.1.19 in Appendix B. In month of the January firm purchases 1,300 pairs of style 1 and style 9 footwear with a discount of 6% on procurement. Procurement of 1,500 pairs for style 2 and 10 is done with discount of 5%, 1,900 pairs of style 3 and 11 are to be bought with discount of 35% on purchase cost. For style 4, 5, 7, 8, 12, 13, 15 and 16, 1700 pairs are purchased with discount of 6, 5, 5, 6, 5, 6 and 5% respectively. 1900 pairs are to be bought for style 6 and 14 shoes with discount of 35% on total procurement cost. The pairs come out to be 55,800 kgs and sent in 111 full trucks and 300 kgs in partial truck by using combination of TL and LTL mode.

Obviously, procurement cost while using second policy will be more but an entrepreneur may choose any of the two policies in consideration with other parameters like holding cost and transportation costs borne by the firm to order all the 16 styles of shoes would cost $101,978,300 when firm chooses first option of procurement i.e. all unit based model as discussed in the case and $103,800,900 if it chooses the option second i.e. incremental based model. Costs at a glance are given in Table 2.1.20(Appendix B).

2.2 PROCUREMENT DISTRIBUTION COORDINATION FROM SINGLE-SOURCE TO SINGLE-DESTINATION FOR PERISHABLE PRODUCTS UNDER FUZZY ENVIRONMENT

Over the past few years prevailing economic condition have made the companies around the world more customer centric. Customer satisfaction and service have become the highest priority, Georgios and Elmar (2004). SCM has come up as a source of gaining competitive advantage in the business world. Organizations are grappling with identifying and improving the strategic issues that their SC should cater to Sahay (2004). Due to pressure from increased competition, globalization of supply and networks, corporate restructuring, introduction of new manufacturing methods, high level of services, as well as low price expectations, the importance of SC will continue to intensify, Sahay and Mohan (2004). Suppliers frequently offers discount schedules based on all units model, which divides the range of possible order quantities into intervals with progressively lower unit costs Rubin and Benton (2003).
Transporters, these days are looking for more innovative ways to attract customers by thriving to satisfy their ever increasing demand of cost effective transportation by offering freight policies such as TL and LTL. In the SCM literature, there are several articles that address about quantity discounts and freight cost.

Least has been said about perishability/deterioration of stock (which is the main factor in determining the productivity of stock) and its implication on the procurement and distribution of products. Following factor may lead to deterioration of stock viz. damage, spoilage, obsolescence, decay, decreasing usefulness etc. The first model where the factor of perishability was consiered as noteworthy is projected by Ghare and Shrader (1963), who pointed out that inventory decay could exert a significant impact on the overall inventory cost, while considerable cost saving could be achieved if the inventory analysis took the inventory decay into consideration. Lakdere(1995) finds the replenishment schedule for an inventory system with shortages, in which items deteriorate at a constant rate and demand rates are decreasing over a known and finite planning horizon. Mandala and Roy(2006) built up an inventory model for perishable items with limited storage space. They took demand rate for the items are finite; items deteriorate at constant rates and are replenished instantaneously. All the above authors studied about constant rate of deteriorating items but none of them included quantity discounts and freight policies. The few studies performed on the subject of quantity discounts and freight also reveal interesting results. Shina and Benton (2006) quoted that quantity discounts provide a practical foundation for inventory coordination in SCs. Munsona and Hub (2010) analyzed that multi-site organizations must balance conflicting forces to determine the appropriate degree of purchasing centralization for their respective supplies. The ability to garner quantity discounts represents one of the primary reasons that organizations centralize procurement. Li and Liu (2006) examined that if both the buyer and supplier can find a coordination mechanism to make joint decisions, the joint profit in this situation is more than the sum of their profits in the decentralized decision situation. They show that quantity discount policy is a way that may be implemented to achieve coordination. Darwish (2008) emphasized that, transportation costs has enhanced the need to develop models with transportation consideration.
explicitly. The uncertainties in demand due to perishable nature of products and hence uncertainty in costs doubles the problems.

In order to quantify these uncertainties defining the problem under fuzzy environment offers the opportunity to model subjective imagination of the decision maker as precisely as a decision maker will be able to describe it. Optimization under fuzzy environment is gaining even more importance in the era of globalization due to some additional sources i.e. stiff competition, shorter life cycles of products, contributing to bringing uncertainties in the problem definitions. It is a flexible approach that permits a more adequate solution of real problems in the presence of vague information. It also deals with the concept of linguistic variables by formulating vague description in natural languages using precise mathematical terms. Crisp mathematical programming to solve fuzzy optimization problems provides no mechanism to quantify these uncertainties directly. The fuzzy set concept and fuzzy optimization techniques can be used efficiently in such situations to defuzzify the fuzzy parameters, constraints and objectives and formulating an equivalent crisp problem which can further be solved using the mathematical programming techniques. Even though the fuzzy optimization procedure is highly subjective to the problem solver due to the subjectivity of the defuzzification techniques it is preferred as it provides flexibility in terms of the alternative courses of action to the decision maker. An important advantage of fuzzy systems is the fact that they allow an adequate mapping of real problems and is the first step to incorporate human knowledge into engineering systems in a systematic and efficient manner.

Fuzzy set theory was originally established by Zadeh(1973). Since then this theory has become one of the emerging areas in contemporary technologies of information processing. It has its strength in modeling, interfacing humans with computers and modeling certain uncertainties. In the two decades since its inception, the theory has matured into a wide ranging collection of concepts and techniques for dealing with complex phenomena that do not lend themselves to analysis by classical methods. It involves capturing, representing, and working with linguistic notions-objects with unclear boundaries. The resulting framework of fuzzy sets is essential to many human endeavors and permeates many studies on the role of uncertainty. Essentially such a
framework provides a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables. “Imprecision” here is meant in the sense of vagueness rather than the lack of knowledge about the value of a parameter. It provides a strict mathematical framework in which vague conceptual phenomenon can be precisely and rigorously studied. [Zimmerman(2001)].

The fuzzy set-based optimization was introduced in seminal paper on decision making in a fuzzy environment in which the concepts of fuzzy constraint, fuzzy objective and fuzzy decision were introduced. These concepts were then widely used and applied by many researchers. It emerged as a new paradigm in which linguistic uncertainty could be modeled systematically. Differently from the classical optimization methods, the main idea in fuzzy optimization is to optimize objective function and constraints, simultaneously. In order to determine the optimal point (solution point), both objective function and constraints must be characterize by membership functions and they must be linked by a linguistic conjunction: “and” (for maximization) and “or” (for minimization). In this chapter we have formulated procurement-distribution coordination problems under fuzzy environment and have discussed fuzzy optimization procedure to solve them.

2.2.1 Problem Description & Formulation

This section discusses the current problem in an established restaurant and formulates a mathematical model based on the description.

2.2.1.1 Problem Description

The most recent dietary survey of the country cropped up the evidence that many individuals had dietary practice which either leads to deficiency or excessive intake of nutrients, missing is the phenomenon of balanced diet. It was found that best way to make your dietary intake balanced in all respect is by having meals made up of whole grains, which can keep check on obesity and other diet related chronic diseases.

As the society is getting health conscious day by day, a leading restaurant in the city decided to en-cash this prevailing health phenomenon by introducing “Whole Grain” meals in their restaurant, keeping only those items made from “Whole Grain” in the
menu, especially the ones which have great nutrient value viz. Wheat, Maize and Ragi to keep the quality of the food to max they decided to open a small storage and processing unit in their restaurant only.

After lengthy negotiations with a grain mill, it was decided that mill will supply the grains i.e. wheat, ragi and maize in 20, 25 & 10 kg sacks, every quarter of the year. The processing unit of the restaurant requires large amount of grains for its progression. Since grains in bulk quantities are relatively inexpensive, it makes sense to purchase grain in extremely large quantities that are shipped by trucks or rails. It would not make a lot of sense to purchase small quantities losing the price discount for ordering bulk and paying to ship grain in smaller, high-cost transportation movements. A leading transporters are hired to transport the grains from the warehouse of grain mill to the processing the centre of the restaurant. Transporters also offered various schemes (like Truckload, less-than Truckload or a combination) to suit the needs of the customer, where weight transported in each full vehicle is 2000 kgs & cost of shipping a kg in LTL mode of transportation, is $2/kg.

Restaurant was also faced with the herculean task of minimizing the holding cost, as special environment is needed to store the grains in storage house and also minimizing inspection cost due to perishable nature of the grains which tend to decay of almost 10% of grains per sack, therefore periodic inspections are needed to keep decay of the grains to minimum. Since its a new strategy demand is imprecise. The professional representation of data is provided in Appendix (Table 2.2.1-2.2.7).

2.2.1.2 Assumptions

1) Demand is uncertain and dynamic in nature.

2) Supply is instantaneous and no shortages are allowed

3) Initial inventory of each product for the beginning of planning horizon is zero.

4) Constant rate of deterioration for perishable items, as a percentage of defective items of stored units.

5) Constant rate of inspecting the multiple perishable items.
2.2.1.3 Sets

- Product set with cardinality $P$ and indexed by $i$.
- Period set with cardinality $T$ and indexed by $t$.
- Item discount break point set with cardinality $L$ and indexed by small $l$.

2.2.1.4 Decision Variables

The decision variables are same as in Section 2.1

2.2.1.5 Parameters

The additional parameters for this section are as follows:

\[
\begin{align*}
\tilde{c} & \quad \text{Fuzzy total cost} \\
c_0 & \quad \text{Aspiration level of fuzzy total cost} \\
c_0^* & \quad \text{Tolerance level of fuzzy total cost} \\
\tilde{D}_it & \quad \text{Fuzzy demand for product } i \text{ in period } t \\
\tilde{D}_{it} & \quad \text{Defuzzfied demand for product } i \text{ in period } t \\
m_i & \quad \text{Rate of inspection of } i^{th} \text{ item} \\
\eta & \quad \text{Percentage of defective items of the stored units}
\end{align*}
\]

Rest of the parameters used are same as in Section 2.1

2.2.1.6 Quantity Discounts

The discounted slab is same as provided in Section 2.1

2.2.1.7 The Fuzzy Optimization Model Formulation

Most of our traditional tools of modeling are crisp, deterministic, and precise in character. But for many practical problems there are incompleteness and unreliability of input information. This is caused to use fuzzy optimization method with fuzzy parameters. Crisp mathematical programming approaches provide no such mechanism
to quantify these uncertainties. Fuzzy optimization is a flexible approach that permits more adequate solutions of real problems in the presence of vague information, providing the well defined mechanisms to quantify the uncertainties directly. Therefore, we formulate fuzzy optimization model for on vague aspiration levels on total cost and demand, the decision maker may decide his aspiration levels on the basis of past experience and knowledge possessed by him.

\[
\begin{align*}
\text{Min } \hat{C} &= \sum_{t=1}^{T} \sum_{i=1}^{P} \left[ h_i I_{i,t} + m_i X_{i,t} + \sum_{l=1}^{L} R_{i,l,t} d_{i,l,t} f_{i,l,t} X_{i,t} \right] + \sum_{l=1}^{T} \left( s y_l + \alpha_l \beta_l \right) u_l + (\alpha_l + 1) \beta_l (1-u_l) \quad \ldots(2.2.1)
\end{align*}
\]

\[
I_{i,t} = I_{i,t-1} + X_{i,t} - D_{i,t} - \eta I_{i,t} \quad \forall \ i = 1, \ldots, P, t = 2, \ldots, T \quad \ldots(2.2.2)
\]

\[
I_{i,t} = I_{i,t-1} + X_{i,t} - \bar{D}_{i,t} - \eta I_{i,t} \quad \forall \ i = 1, \ldots, P \quad \ldots(2.2.3)
\]

\[
(1-\eta) \sum_{i=1}^{T} I_{i,t} + \sum_{i=1}^{T} X_{i,t} \geq \sum_{i=1}^{T} \bar{D}_{i,t}, \quad \forall \ i = 1, \ldots, P \quad \ldots(2.2.4)
\]

\[
X \in S = \{X_{i,t}, I_{i,t}, \delta_t, \alpha_t, y_t \geq 0 \text{ and integer}; R_{i,l,t}, u_t \in \{0,1\} \} \quad i = 1, \ldots, P, l = 1, \ldots, L, t = 1, \ldots, T
\]

\[
X_{i,t} \geq \sum_{l=1}^{L} \alpha_{i,l} R_{i,l,t} \quad \forall \ i = 1, \ldots, P, t = 1, \ldots, T \quad \ldots(2.2.5)
\]

\[
\sum_{l=1}^{L} R_{i,l,t} = 1 \quad \forall \ i = 1, \ldots, P, t = 1, \ldots, T \quad \ldots(2.2.6)
\]

\[
\delta_t = \sum_{i=1}^{P} \left[ w_i X_{i,t} \sum_{l=1}^{L} R_{i,l,t} \right], \quad \forall \ t = 1, \ldots, T \quad \ldots(2.2.7)
\]

\[
\delta_t \leq (y_t + \alpha_t \omega) u_t + (\alpha_t + 1) \omega (1-u_t) \quad \forall \ t = 1, \ldots, T \quad \ldots(2.2.8)
\]

\[
\delta_t = (y_t + \alpha_t \omega) \quad \forall \ t = 1, \ldots, T \quad \ldots(2.2.9)
\]

\subsection*{2.2.1.8 Analysis of Model Formulation}

Equation (2.2.1), the fuzzy objective function is to minimize the cost incurred in holding ending inventory borne by firm, cost of purchasing the products, and cost of inspection on ordered quantity in period \( t \), reflected by the first term of the objective function; the transportation cost from the source to the destination, is the second term. The cost is calculated for the duration of the planning horizon. The ordering cost is a
fixed cost not affected by the ordering quantities and therefore is not the part of objective function. Equation (2.2.2) – (2.2.4) are the balancing equations where equation (2.2.2) finds total ending inventory of \(i^{th}\) product in \(t^{th}\) period by reducing the fuzzy demand and fraction of perished inventory in \(t^{th}\) period from total of ending inventory of previous period and ordered quantity during \(t^{th}\) period. Equation (2.2.3) finds total ending inventory of \(i^{th}\) product in first period by reducing the fuzzy demand and fraction of perished inventory of the same period from total of initial inventory in the planning horizon and ordered quantity at first period. Equation (2.2.4) shows that total fuzzy demand in all the periods is less than or equal to total of ending inventory and ordered quantity in all the periods i.e. shortages are not allowed. Equation (2.2.5) & (2.2.6) find out the order quantity of all products in period \(t\) which may exceed the quantity break threshold, and avails discount on ordered quantity at exactly one quantity discount level. Eq. (2.2.6) restricts the activation at exactly one level, either discount or no discount situation. The integrator for procurement and distribution is eq. (2.2.7), which calculates transported quantity according to product weight. In eq. (2.2.8), the minimum weighted quantity transported is calculated and further eq. (2.2.9) measures the overhead units from TL capacity in weights.

2.2.2 Solution Procedure

Using the basic concepts of fuzzy set theory discussed in Appendix A, we will discuss the optimization technique to solve the above stated problem. The following algorithm specifies the sequential steps to solve it.

1. Compute the crisp equivalent of the fuzzy parameters using a defuzzification function (ranking of fuzzy numbers) \(F_2(A) = (a_1 + 2a + a_u) / 4\). Same defuzzification function is to be used for each of the parameters.

2. Incorporate the objective function of the fuzzifier min (max) as a fuzzy constraint with a restriction (aspiration) level. The inequalities are defined softly if the requirement (resource) constants are defined imprecisely.

3. Define appropriate membership functions for fuzzy inequalities. The membership function for the fuzzy less than or equal to and greater than or equal to type are given as
\[
\mu_C(X) = \begin{cases} 
1 & ; C(X) \leq C_0 \\
\frac{C_0^* - C(X)}{C_0^* - C_0} & ; C_0 \leq C(X) < C_0^* \\
0 & ; C(X) > C_0^* 
\end{cases}
\]

respectively, where \( C_0 \) is the restriction and \( C_0^* \) is the tolerance levels to the fuzzy total cost. The membership functions can be a linear or piecewise linear function that is concave or quasi concave.

4. Employ the extension principle [Bector and Chandra(2005)] to identify the fuzzy decision, which results in a crisp mathematical programming problem given by

Maximize \( \theta \)

Subject to \( \mu_i(X) \geq \theta \implies i=1,2,...,n \ ; \ \theta \geq 0, \ \theta \leq 1, \ T \geq 0 \)

and can be solved by the standard crisp mathematical programming algorithms.

### 2.2.3 The Crisp Formulation

By using above algorithm, the fuzzy optimization problem converted to crisp problem and is as follows:

Maximize \( \theta \)

subject to: \( \mu_i(X) \geq \theta \)

\[
I_{it} = I_{it-1} + X_{it} - \bar{D}_{it} - \eta I_{it} \quad \forall \ i = 1,...,P; t = 2,...T
\]

\[
I_{i1} = I_{i1} + X_{i1} - \bar{D}_{i1} - \eta I_{i1} \quad \forall \ i = 1,...,P
\]

\[
(1-\eta) \sum_{t=1}^{T} I_{it} + \sum_{t=1}^{T} X_{it} - \bar{D}_{it} \quad \forall \ i = 1,...,P
\]

\[
X \in S = \{X_{it}, I_{it}, \delta, \alpha, \gamma \geq 0 \text{ and integer}; R_{il}, u_l \in \{0,1\} \}
\]

/satisfying equations (2.2.5 to 2.2.9; i = 1,...,P; t = 1,...,T; l = 1,...,L; \theta \in [0,1])

can be solved by the standard crisp mathematical programming algorithm.
2.2.4 Results and Discussions

The various cost and demand parameters $C_0$ and $\tilde{d}_t$ respectively are the ranking fuzzy numbers represented as $A = (a_L, a, a_U)$. The value of these fuzzy numbers are specified by the management based on the past experiences and/or expert opinion. The values of these fuzzy parameters are assumed and are tabulated in table 2.2.7 in Appendix B. Using the defuzzification function $F_2(A) = (a_L + 2a + a_U)/4$ we defuzzify these fuzzy numbers. Defuzzified values of these parameters are also given in table 2.2.7 in Appendix B. The aspiration level of total cost is $C_0 = $320,000 and tolerance level of cost is $C^* = $480,000 provided by the firm.

Using the values of the fuzzy parameters as given in Table 2.2.7 and substituting in the defuzzification values in above problem also by using, we obtained the solution of the case study is as follows.

Restaurant is able to estimate the ordered quantity for all the products of every period for his restaurant. For Wheat, it will order 88 sacks in first period, nothing in second period, 94 sacks in third period and 90 sacks in last period. Thereby it will have to store extra 54 sacks in first period which it will use in second period. It will receive a discount of 4% in first, third and fourth period.

Regarding Maize, it orders 96, 109, 10 and 80 sacks in four periods thereby after fulfilling demand left with only 30 sacks in period two. It will receive a discount of 15% in first and second periods, no discount in third period and 10% in last period. And for Ragi, ordersizes of sacks are 57, 67, 100 and 5 in four periods respectively, thereby getting discounts of 4% in first and second periods, 12% in third period and no discount in last period.

As ordersizes of sacks decide the weighted quantity for distribution, so weighted quantity of all the grains in every period is 3,770 kg; 2,305kg; 3,030kg and 3,050 kgs respectively. In first period TL policy will be used and 2 trucks will ship the whole quantity. In second, third and fourth periods both TL & LTL policies will be used. In each of these periods 1 full truck will go and rest of 305 kg, 1,030 kg; and 1,050 kgs will use LTL policy for distributing the grains. As far as the vagueness is concerned, restaurant able to minimize the uncertainty by more than 75% and by incurring a total cost of $359,405.