Supply chain management has been both an important and a productive aim of corporations and companies in today’s competitive environment as they are striving hard to minimize the cost component viz. procurement, production, distribution, and inventory holding cost to increase their operation profit and to sustain it for longer period of time. In many industries, components and sub components are manufactured at variety of sites and the same are sold across the cities and continents, such a scenario necessitate companies to reduce various cost by adopting comprehensive supply chain policies. Effective supply chain management solves many problems encountered in the business today, first, the vendors involved in the chain will actually have a clearer idea of what the buyer needs and can then adequately provide for these needs. Slow response times and delays in project start dates also become less frequent because the automated supply chain helps shave the time off of the order placement and fulfillment process, furthermore problem like the one(intermediate stoppage in transportation) discussed in this chapter can also be solved by applying wide-ranging supply chain policies of optimization. It's no secret that department store retailing is no longer simply operating an anchor store at the mall-you need expertise across channels. And with private-label merchandise, your supply chain gets longer and more complex every day managing the overseas purchasing process. To complicate matters, discounters now offer the brands that have been your staple offerings. Top brands-and best sellers-compete directly with their own stores and Internet presence.

Supply chain optimization requires development of models that require integrating the process of procurement, processing, storing, shipping and getting them to the


consumer. The overall objective of these models is to minimize total cost by obtaining the economic order quantity and transporting needed quantity in a timely and efficient manner. Levi and Kaminsky (2000) mentioned that, supply chain management is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses and stores, so that merchandise is produced and distributed at the right quantities, to the right locations and to the right time, in order to minimize system-wide costs while satisfying service level requirements. Modeling of two stage models taking into considerations the procurement and distribution decisions for multi products has least been studied by researchers, though subjects have been studied separately and extensively. Quantity discounts is one of the most popular mechanism of coordination in business for a long time to entice the buyers for purchasing larger quantity, [Munson and Rosenblatt (1998)]. However, unless enough care is given to the design of a discount scheme, much of the potential benefits could be lost, [Altintas et. al. (2008)]. Cheung and Lee (2002) forced shipment co-ordination in order to have full truckload (TL) shipments. Corbett and Groote (2000) considered coordinating the supply chain when the buyer has some private information. Ertogral (2008) took a single stage multi incapacitated dynamic lot sizing problem (MILSP) with transportation cost and assumed finite planning horizon with dynamic demand. He considered all unit inventory management models to formulate the problem with piece wise linear transportation cost function. Mendoza and Ventura (2008) developed an unconstrained integrated inventory-transportation model to decide optimal order quantity for inventory system over a finite horizon. All the above integrations are on single staged supply chains. Below are few studies of two stage supply chains. Subramanya and Sharma (2009) integrated two stage supply chain network of an automobile company, measured the performance parameters and established the priority decision and queuing rules for improving the utilization of resources. The study restricted to measuring operational processes in a two stage supply chain between the supplier - manufacturer - distributors. Wang et al. (2009) examined the dynamics of a two stage supply chain consisting of one retailer and one distributor with order-up-to control policy. Lee et al. (2000) examined the value of information in a two-stage supply chain under an autoregressive demand process. Rieksts and Ventura (2010) discussed two staged inventory models over an infinite planning
horizon with constant demand rate and two modes of transportation. These transportation options include truckloads and a less than truck-load carrier. An optimal algorithm is derived for a one-warehouse one-retailer system. A power-of-two heuristic algorithm is also proposed for a one-warehouse multi-retailer system. Capar et al. (2011) dealt with two stage supply chain with two distribution centers (DC) and two retailers. Each member of supply chain used a (Q,R) inventory policy and incurred standard holding, backlog, ordering and transportation costs. Each DC is able to serve retailer on a first-come-first-served basis, but the transportation cost is lower for retailers within the DC’s service area. Their goal is to provide an effective retailer ordering policy and to compare the overall supply chain performance of their policy to the current policy used by the company. Hsiao (2008) investigated the integrated stochastic inventory problem for a two-stage supply chain consisting of a single retailer and a single supplier. By using batch shipment policy, the expected total cost can be significantly reduced. Equally sized batch shipment models, controlled by both the reorder and the shipping points with sharing information, variable safety factors are constructed. The problem is solved optimally by the proposed algorithms that determine the economic lot size, the optimal batch sizes, number of batches, and safety factor. These studies barely included procurement decisions into analysis.

This chapter considers the dynamics of two stage supply chain where there is single source of supply, an intermediate stoppage and destination of consumption i.e. the integration of procurement and distribution decision making under two stage environment. In first section, our attention is focused on an integrated procurement-distribution two stage supply chain model incorporating the discounted policies on purchasing goods and transportation network. A model is formulated which explains the flow of ordered quantity from single source to multiple destinations with one intermediate stoppage point. In the model buyer at destination with various retail stores avails quantity discounts on bulk order and freight discounts on bulk transported quantity. Quantity discounts are provided by the supplier, in which supplier has fixed the quantity level beyond which discount would be given, in particular, on the basis of all unit discount model. The mode of transporting the goods
from source to destinations takes place in two stages, where in the first stage, goods are moved using cargo. The unloading point of goods is the intermediate stoppage. At the stoppage, unloading of goods and their further processing takes a specified time for which the holding cost is free. Since cargo points require space to unload goods of number of cargos, the holding cost at stoppage increases with very high rates after a preset time. Keeping inventory for long time may not be beneficial for the buyer but sometimes the inventory has to be kept because of some undue causes like transportation facility is not available to move goods from cargo point to destination. As the buyer has received discounts from supplier on the ordered quantity mentioned above, Cargo Company is also giving discounts on weighted quantity from supply point to intermediate stoppage.

Movement of goods from intermediate stoppage to destination is the second stage of model which is completed through modes of transportation, which are categorized as truckload (TL) and less-than-truckload (LTL) transportation. In Truckload transportation, the cost is a fixed of one truck up to a given capacity. In this mode company may use less than the capacity available but cost per truck will not be deducted. However in some cases the weighted quantity may not be large enough to substantiate the cost associated with a TL mode. In such situation, a LTL mode may be used. LTL may be defined as a shipment of weighted quantity which does not fill a truck. In such case transportation cost is taken on the bases of per unit weight. The model shows some initial and ending inventory at source to fulfill the uncertainties, as shortages are not allowed at any cost. Also, products ordered in any period will reach to a retailer at the second stage of next period. Different discount policies are offered to procure and transport goods from the one stage to other stage when it is assumed that inventory carrying charge at the stoppage is very high after a pre-specified time. Model will benefit organizations in a long run by helping them determining optimal quantity to be ordered which not only reduces the cost of procurement and transportation costs.

Section II describes the model where there is uncertainty in demand and cost, when procurement and distribution decisions of supply chain need to be taken. The model incorporates a single supplier transporting its products to multiple destinations of a retailer. This process becomes tedious, when items are moving with stoppage as on
stoppage point inventory carrying cost would also be incurred due to perishable nature of products. Different discount policies are offered to procure and transport goods from the one stage to other stage. By using the fuzzy set theory, optimum decisions are taken. In both sections, case studies are presented to validate the procedure.

5.1 TWO STAGE MODEL FROM SINGLE SOURCE TO MULTIPLE DESTINATIONS

5.1.1 Problem Description & Formulation

This section discusses the current problem in a retail chain of stationery items and formulates a mathematical model based on the description.

5.1.1.1 Problem Description

India, slowly and steadily becoming the hub for global BPO services. Due to this rise, India already got a new nick name “Call center of the world”. Many non-metro cities across India witnessing rapid growth in IT sector because of booming BPOs and Call centers with the availability of talented English speaking graduates. One such city is Jaipur, because of the beaming BPOs in its proximity with state capital, Delhi.

As the IT related services are increasing at the express speed, so is the need for various computer peripherals, especially storage devices such as CDs, DVDs and Pen drives. Showrooms cum retail stores in Jaipur selling various computer peripherals and hardware is already on rise. One such leading stationary selling store in city decided to open stores selling various computer peripherals. They got the good deal for the peripherals other than storage devices for the bulk purchase, in their own city but for the storage devices such as CDs, DVDs and Pen drives they decided to look for a supplier from other cities as they were not getting the best deal in Jaipur.

They contracted one of the leading companies in Bangalore which specializes in manufacturing of storage devices for the supply of devices for their shops in Jaipur thrice a year. The deal was struck and it was decided that each item will be supplied per carton from the warehouse in Bangalore. Supplies will have to be flown from Bangalore airport to a storage house at the Jaipur airport from where it will be transported to the retail shops.
Keeping supplies for the first day at the storage house at Jaipur airport is free of charge but subsequent stay costs hefty charges. So to keep the transportation cost at bare minimum one of the most efficient transporters in Jaipur was chosen, who will supply the products from the Bangalore airport to Jaipur airport then to the retail shops.

Transporter offers various discounts depending upon the mode of transport chosen where, weight transported in each full vehicle is 1000 kgs (ω) and cost of shipping one unit in LTL mode of transportation, $2 per kgs (s). Quantity discounts are also offered by the supplier in order to encourage large orders from the retailers. The relevant data is provided in Appendix B (Table 5.1.1 – 5.1.10).

The formulation and solution of the above enlightened model has been discussed in the following sections.

5.1.1.2 Assumptions

The assumptions of this research are essentially the same as those of EOQ model except for the transportation cost. The section considers a two stage system from single source to multiple destinations with finite planning horizon. The demand is dynamic in nature. Shortages are not allowed. Supply is immediate for both modes of transportation available, namely TL and LTL. Initial inventory of each product is zero at the beginning of the planning horizon and the holding cost is independent of the purchase price and any capital invested in transportation.

5.1.1.3 Sets

- Product set with cardinality \( P \) and indexed by \( i \).
- Period set with cardinality \( T \) and indexed by \( t \).
- Item discount break point set with cardinality \( L \) and indexed by small \( l \).
- Freight discount break point set with cardinality \( K \) and indexed by small \( k \).
- Waiting time set at intermediate stoppage with cardinality \( \Gamma \) and indexed by \( \tau \).
- Destination set with cardinality \( M \) and indexed by \( m \).
5.2.1.4 Decision Variables

\( X_{it} \)  
Amount of item \( i \) ordered in period \( t \)

\( R_{i,l} \)  
If the ordered quantity falls in \( l^{th} \) price break then the variable takes value 1 otherwise zero

\( I_{it} \)  
Inventory level at the end of period \( t \) for product \( i \)

\( L_{At} \)  
Total weighted quantity transported to intermediate stoppage at stage 1 in period \( t \).

\( L_{2mt} \)  
Total weighted quantity transported to \( m^{th} \) destination at stage 2 in period \( t \).

\( a_{mt} \)  
Total number of trucks in period \( t \) transported to destination \( m \)

\( y_{mt} \)  
Amount in excess of truckload capacity (in weights) to \( m^{th} \) destination in \( i^{th} \) period.

\( v_{\tau t} \)  
Time period for which quantity is stored at intermediate stoppage

\[
v_{\tau t} = \begin{cases} 
1 & \text{if } L_{At} \text{ waits at halt for period } t \\
0 & \text{otherwise}
\end{cases}
\]

\( Z_{kt} \)  
If the weighted quantity transported falls in \( k^{th} \) price break then the variable takes value 1 otherwise zero

\[
Z_{kt} = \begin{cases} 
1 & \text{if } L_{i} \text{ falls in } k \text{ pricebreak} \\
0 & \text{otherwise}
\end{cases}
\]

\( u_{mt} \)  
Reflects usage of either both TL and LTL policies or only TL policy

\[
u_{mt} = \begin{cases} 
1, & \text{if considering TL & LTL both policies or only LTL} \\
0, & \text{if considering only TL policy}
\end{cases}
\]

5.1.1.5 Parameters

\( C \)  
Total inventory cost

\( \beta_{mt} \)  
Fixed freight cost for each truck load

\( D_{imt} \)  
Demand for item \( i \) in period \( t \) for destination \( m \)

\( h_{i} \)  
Inventory holding cost per unit of item \( i \) per period
5.1.1.6 Quantity and Freight Discounts

The quantity and freight discounts applicable in first stage of procurement and
distribution are defined as:

**Quantity Discounts:**

\[
d_f = \begin{cases} 
    d_{ilt} & a_{ilt} \leq X_{it} \leq a_{ilt+1}t \\
    d_{ilt} & X_{it} \geq a_{ilt} 
\end{cases} 
\]

**Freight Discounts:**

\[
d_f = \begin{cases} 
    f_{kt} & b_{kt} \leq L_{it} \leq b_{k+1}t \\
    f_{kt} & L_{it} \geq b_{k}t 
\end{cases} 
\]
5.1.1.7 The Mathematical Formulation

The following is the formulation for above described analysis:

\[
\text{Min } C = \sum_{t=1}^{T} \left[ \sum_{l=1}^{L} \left( h_{lt} + \sum_{i=1}^{P} \frac{R_{ili} d_{ili} \phi_{ili}}{X_{it}} \right) \right] + \sum_{k=1}^{K} Z_{kt} f_{kt} c_{kt} + \sum_{t=1}^{T} L_{itt} \gamma_{rt} t \quad \ldots(5.1.1)
\]

\[
l_{it} = l_{i(t-1)} + X_{it} - \sum_{m=1}^{M} D_{imt} \quad \forall i = 1 \ldots P, t = 2 \ldots T \quad \ldots(5.1.2)
\]

\[
l_{il} = IN_{i} + X_{il} - \sum_{m=1}^{M} D_{iml} \quad \forall i = 1 \ldots P \quad \ldots(5.1.3)
\]

\[
\sum_{i=1}^{T} l_{it} + \sum_{i=1}^{T} X_{it} \geq \sum_{i=1}^{T} \sum_{m=1}^{M} D_{imt} \quad \forall i = 1 \ldots P \quad \ldots(5.1.4)
\]

\[
X_{it} \geq \sum_{l=1}^{L} a_{ilt} R_{ilt} \quad \forall i = 1 \ldots P, t = 1 \ldots T \quad \ldots(5.1.5)
\]

\[
\sum_{l=1}^{L} R_{ilt} = 1 \quad \forall i = 1 \ldots P, t = 1 \ldots T \quad \ldots(5.1.6)
\]

\[
L_{it} = \sum_{l=1}^{L} \left[ w_{i} X_{it} + \sum_{l=1}^{L} R_{ilt} \right] \quad \forall t = 1 \ldots T \quad \ldots(5.1.7)
\]

\[
L_{it} \geq \sum_{k=1}^{K} b_{kt} Z_{kt} \quad \forall t = 1 \ldots T \quad \ldots(5.1.8)
\]

\[
\sum_{k=1}^{K} Z_{kt} = 1 \quad \forall t = 1 \ldots T \quad \ldots(5.1.9)
\]

\[
\sum_{t=1}^{T} \gamma_{rt} = 1 \quad \forall t = 1 \ldots T \quad \ldots(5.1.10)
\]

\[
L_{1t} = \sum_{m=1}^{M} L_{2m(t+1)} \quad \forall t = 1 \ldots T \quad \ldots(5.1.11)
\]

\[
L_{2mt} \leq (\gamma_{mt} + \alpha_{mt}) u_{mt} + (\alpha_{mt} + 1) \omega (1 - u_{mt}) \quad \forall m = 1 \ldots M, t = 2 \ldots T + 1 \quad \ldots(5.1.12)
\]

\[
L_{2mt} = 0 \quad \forall m = 1 \ldots M, t = 2 \ldots T + 1 \quad \ldots(5.1.13)
\]

\[
X_{i1}, L_{it}, L_{2mt}, \alpha_{mt}, \gamma_{mt}, y \geq 0 \text{ and integer; } R_{ilt}, u_{mt}, \gamma_{rt} \in \{0, 1\}
\]

\[
i = 1 \ldots P; t = 1 \ldots T; l = 1 \ldots L; m = 1 \ldots M
\]
5.1.1.8 Analysis of Model Formulation

Equation (5.1.1) is the objective function to minimize the cost incurred in holding inventory, cost of purchasing the products, during period t reflected by the first term of the objective function; the transportation cost from the source to intermediate stoppage point and holding cost at stoppage point is reflected by the second term and third term respectively. Distribution cost for destination m by using TL, LTL and combination of two modes is presented by the third term of the function. The cost is calculated for the duration of the planning horizon. The ordering cost is a fixed cost not affected by the ordering quantities and therefore is not the part of objective function. Equations (5.1.2) – (5.1.4) are the balancing equations, where equation (5.1.2) finds total ending inventory of \( i^{th} \) product during \( t^{th} \) period is found by reducing the demand of all the destinations during \( t^{th} \) period from the sum of total of ending inventory of previous period and ordered quantity at \( t^{th} \) period. Equation (5.1.3) finds total ending inventory of \( i^{th} \) product during first period is found by reducing the demand of all the destinations of the same period from the sum of total of initial inventory of the planning horizon and ordered quantity at first period. Equation (5.1.4) shows that total demand in all the periods from all destinations is less than or equal to total ending inventory and ordered quantity during all the periods i.e. shortages are not allowed. Equation (5.1.5) and (5.1.6) evaluates the order quantity of all products during period t which may exceed the quantity break threshold, and avails discount on ordered quantity at exactly one quantity discount level. Equation (5.1.7) is the integrator for procurement equations (5.1.2 - 5.1.6) and transportation equations (5.1.8 - 5.1.13), where (5.1.7) calculates weighted quantity to be transported according to weights per product. Equation (5.1.8) & (5.1.9) find out the weighted transport quantity of all products in period t which may exceed the freight break threshold, and avails discount on transportation quantity at exactly one freight discount break. Equation (5.1.10) calculates the time of halt at intermediate stoppage. Equation (5.1.11) shows the total weighted quantity transported in stage 1 of period t is equal to the total weighted quantity transported in stage 2 of period (\( t + 1 \)). In equation (5.1.12), the minimum weighted quantity transported is calculated and further Equation (5.1.13) measures the overhead units from truckload capacity in weights.
5.1.2 Results and Discussions

The vital objectives, the firms are concerned about are how much to order and how to minimize the total cost. Here, we have tried to answer these questions with the help of a case in procurement-distribution scenario of a two stage supply chain. Ordered quantity for CDs in period 1 is 464 packs with discount of 15% on purchase cost and remaining ending inventory would be nothing after fulfilling the demand. Distribution in first stage from supplier to intermediate stoppage is 19,662 kgs with 10% discount on cargo. At intermediate stoppage Storage Devices will be halted for two days and then at second stage distributed through trucks at shops. The quantity will reach at shop 1 at second stage of second period with load of 17,662 kgs by using both TL & LTL policy, in 17 trucks and 662 extra kgs. Rest of the procurement and distribution policies for Storage Devices is given in Appendix B (Table 5.1.11 – 5.1.14). Total cost incurred by store is $5,323,678.

5.2 TWO STAGE COORDINATION FROM SINGLE SOURCE TO MULTIPLE DESTINATIONS FOR PERISHABLE PRODUCTS UNDER FUZZY ENVIRONMENT

Today’s global supply chains are complex systems of organizations, people, processes, technology, and information that move products or services from concept to finished goods and from suppliers to customers. To gain the competitive edge supply chain should be flexible and agile which in turn help organizations streamline operations, increase delivery reliability, and maximize profits. Procurement and distribution in supply chain are relatively more important decisions when the demand is uncertain and products are perishable in nature. This necessitates the high inventory level that makes the situation worst. Therefore, it is required to find out the optimum levels of ordered quantity, so that total procurement and distribution cost can be minimized.

Modeling of two stage models taking into considerations the procurement and distribution decisions for perishable products under fuzzy environment has least been studied by researchers, though subjects have been studied separately and extensively among which few are as follows. Regarding the two stage supply chains, Kaminsky
and Simchi-Levi (2003) developed a two stage model of a manufacturing supply chain. This two stage production-transportation model featured capacitated production in two stages, and a fixed cost for transporting the product between the stages. They show that their model reduces to a related model, with one capacitated production stage with linear production cost, and transportation between two inventory locations with non-linear transportation cost. However, Hyun-Soo and Kaminsky (2005) considered a model of a two-stage push-pull production-distribution supply chain. In their model, orders arrive at the final stage according to a Poisson process. Two separate operations, which take place at different places with exponential service times, were located to convert the raw materials into finished goods. When the first operation is completed the intermediate inventory is held at the first stage and then transported to the second stage where the items are produced to order. Kalaiaarasi and Ritha (2011) studied two stage supply chain with fuzzy parameters. The optimal policy developed by them is determined by using Lagrangian Conditions after defuzzification of the cost function with the graded mean integration method. Another model proposed by Sinha and Sarmah (2006) studied a two-stage supply chain coordination under uncertain demand and cost information.

A wide range of literature is available on supply chain coordination based on quantity discounts and transportation policies. A detailed review on quantity discounts is presented by Goyal and Gupta (1989), Benton and Park (1996) and Sarmah et al. (2006). An integrated supply chain model is studied by Hwang et al. for determining optimal order quantity when all units’ quantity discounts are available on purchasing price and freight cost. Tersine and Barman (1991) assumed a constant demand rate and developed a model with freight and price discounts, where freight discount structure is based on weight. Ertogral (2008) took a single stage multi incapacitated dynamic lot sizing problem (MILSP) with transportation cost and assumed finite planning horizon with dynamic demand. He considered all unit inventory management models to formulate the problem with piece wise linear transportation cost function. Mendoza and Ventura (2008) developed an unconstrained integrated inventory-transportation model to decide optimal order quantity for inventory system
over a finite horizon. There is hardly any study about perishable products in two stage procurement-distribution supply chain under uncertainty.

5.2.1 Problem Description & Formulation

This section discusses the current problem in a departmental chain and formulates a mathematical model based on the description.

5.2.1.1 Problem Description

Recently, departmental stores have become the most important social facilities of the urban tier cities. These stores take care of all the needs of consumers who are dependent on them for fulfilling their daily chores. Leading the way in multi brand retail segment, a retail outlet has multiple outlets in a city to cater to ever growing consumer demand. The store deals in different categories like consumable, durable, usable and perishable items. Consumable products of the store are kept such that they strive to quench the people’s appetite by providing them quick bites like cream/non-cream biscuits, cakes, chips, chocolate etc. Products such as toiletries, cosmetics, perfumes etc are kept in usable products segment. Most of the electrical and electronic items in the store falls under the durable category, like washing machine, TV, geysers, ACs and kitchen appliances. Poultry, fruits, vegetables and food grains are under perishable product segment.

Besides many operational and day to day problems, departmental store has to deal with one of most important and strategic issue ie. optimization of the procurement and distribution cost that includes purchase cost, storage cost, inspection cost and transportation cost of fruits & vegetables and its uncertain demand. Procured vegetables and fruits are first coming from train to the railway yard of city from where it is sent to various branches of the store through trucks. Also, the products can be kept at yard for first period with no charges but further storage will cost them hefty amounts unless the trucks pick them for distribution to outlets. Vegetables and fruits are subject to inspection at the local warehouses due to pilferage which is almost 10% per product. Supplier provides discounts on bulk quantity purchased and transporter offers freight discounts and TL/LTL policies on transportation which varies on the weighted quantity ordered, where each truck can carry 2,000 kgs with cost of carrying per kg of product is $2.0. Now
company’s main challenge is to reduce various cost components viz. purchase, transportation, inspection cost and holding cost and minimize the uncertainty in demand in order to get maximum benefits. As company has some past experience in similar business it has some idea on total cost. Below you can find data pertaining to order quantity and the various discounts options given to the departmental store by the suppliers and transporter. Please note that due to company’s privacy policies (competitors may not sense the exact figures) we have taken dummy/part of the data which will depict the company’s scenario as close as possible which otherwise will not reflect the exact state. It is to be noted that outlet wishes to have the aspiration level on total cost is $C_0 = $2,000,000 and tolerance level of cost is $C^* = $3,000,000. The relevant data pertaining to case is presented in Appendix B (Table 5.2.1-5.2.11)

5.2.1.2 Assumptions

The assumptions of this research are essentially the same as those of EOQ model except for the transportation cost. The section considers a single stage system with finite planning horizon. The demand is dynamic and fuzzy in nature. Shortages are not allowed. Lead times are assumed to be zero for both modes of transportation available, namely TL and LTL i.e. supply is immediate. Initial inventory of each product is zero at the beginning of the planning horizon and the holding cost is independent of the purchase price and any capital invested in transportation.

5.2.1.3 Sets

The sets are same as in previous section 5.1

5.2.1.4 Decision Variables

The decision variables are same as in Section 3.1

5.2.1.5 Parameters

The additional parameters for this section are as follows:

$\tilde{C}$  Fuzzy total cost

$C_0$  Aspiration level of fuzzy total cost
$C_0^*$ Tolerance level of fuzzy total cost

$\tilde{D}_{imt}$ Fuzzy demand for product $i$ in period $t$ for destination $m$

$\tilde{D}_{imt}$ Defuzzfied demand for product $i$ in period $t$ for destination $m$

$m_i$ Rate of inspection of the $i^{th}$ item

$\eta$ Percentage of the defective items of the stored units

Rest of the parameters used, are same as in Section 5.1

### 5.2.1.6 Quantity & Freight Discounts

The discount slabs are same as in previous section 5.1

#### 5.2.1.7 The Fuzzy Optimization Model Formulation

The fuzzy mathematical problem is described as:

$$
\begin{align*}
\text{Min } \tilde{C} &= \sum_{t=1}^{T} \left[ \sum_{i=1}^{P} \left( h_i I_{it} + m_i X_{it} + \sum_{l=1}^{L} R_{ilt} h_i X_{it} \right) + \sum_{k=1}^{K} Z_{kt} f_{kt} \gamma_{it} + \sum_{t=1}^{T} I_{it} \phi_{yt} \right] \\
&+ \sum_{i=2}^{T} \sum_{m=1}^{M} \left[ (s_{y mt} + a_{mt} b_{mt}) u_{mt} + (a_{mt} + 1) b_{mt} (1 - u_{mt}) \right] \\
I_{it} &= I_{it-1} + X_{it} - \sum_{m=1}^{M} \tilde{D}_{imt} \cdot \eta \quad \forall i = 1...P \quad , \quad t = 2...T \\
I_{i1} &= I_{i1-1} + X_{i1} - \sum_{m=1}^{M} \tilde{D}_{im1} \cdot \eta \quad \forall i = 1...P \\
(I \cdot \eta) \sum_{t=1}^{T} I_{i t} + \sum_{t=1}^{T} X_{i t} \geq \sum_{t=1}^{T} \sum_{m=1}^{M} \tilde{D}_{im t} \quad \forall i = 1...P \\
X_{i t} \geq \sum_{l=1}^{L} a_{ilt} R_{ilt} \quad \forall i = 1...P \quad t = 1...T \\
\sum_{t=1}^{T} R_{ilt} = 1 \quad \forall i = 1...P \quad t = 1...T \\
L_{it} &= \sum_{i=1}^{P} \left[ w_i X_{it} + \sum_{l=1}^{L} R_{ilt} \right] \quad \forall t = 1...T \\
L_{it} \geq \sum_{k=1}^{K} b_{kt} Z_{kt} \quad \forall i = 1...T
\end{align*}
$$
\[
\sum_{k=1}^{K} Z_{kt} = 1 \quad \forall \ t = 1...T \quad \ldots(5.2.9)
\]
\[
\sum_{r=1}^{\Gamma} v_{rt} = 1 \quad \forall \ t = 1...T \quad \ldots(5.2.10)
\]
\[
L_{mt} = \sum_{m=1}^{M} L_{2m(t+1)} \forall \ t = 1...T \quad \ldots(5.2.10)
\]
\[
L_{2mt} \leq (y_{mt} + \alpha_{mt} \omega)u_{mt} + (\alpha_{mt} + 1)\omega(1-u_{mt}) \quad \forall \ m = 1...M; t = 2,...,T+1 \quad \ldots(5.2.11)
\]
\[
L_{2mt} = (y_{mt} + \alpha_{mt} \omega) \quad \forall \ m = 1...M; t = 2...T+1 \quad \ldots(5.2.12)
\]

5.2.1.8 Analysis of Model Formulation

Equation (5.2.1) is the fuzzy objective function to minimize the cost incurred in holding inventory, cost of inspection on ordered quantity and cost of purchasing the products, during period \( t \) reflected by the first term of the objective function; the transportation cost from the source to intermediate stoppage point and holding cost at stoppage point is reflected by the second term and third term respectively. Distribution cost for destination \( m \) by using TL, LTL and combination of two modes is presented by the third term of the function. The cost is calculated for the duration of the planning horizon. The ordering cost is a fixed cost not affected by the ordering quantities and therefore is not the part of objective function. Equations (5.2.2) – (5.2.4) are the balancing equations, where equation (5.2.2) finds total ending inventory of \( i^{th} \) product during \( i^{th} \) period is found by reducing the fuzzy demand of all the destinations and fraction of deteriorated ending inventory during \( i^{th} \) period from the sum of total of ending inventory of previous period and ordered quantity at \( i^{th} \) period. Equation (5.2.3) finds total ending inventory of \( j^{th} \) product during first period is found by reducing the fuzzy demand of all the destinations and fraction of deteriorated ending inventory of the same period from the sum of total of initial inventory of the planning horizon and ordered quantity at first period. Equation (5.2.4) shows that total fuzzy demand in all the periods from all destinations is less than or equal to total ending inventory and ordered quantity during all the periods i.e. shortages are not allowed. Equation (5.2.5) and (5.2.6) evaluates the order quantity of all products during period \( t \) which may exceed the quantity break threshold, and avails
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discount on ordered quantity at exactly one quantity discount level. Equation (5.2.7) is
the integrator for procurement equations (5.2.2 - 5.2.6) and transportation equations
(5.2.8-5.2.13), where (5.2.7) calculates weighted quantity to be transported according
to weights per product. Equation (5.2.8) & (5.2.9) find out the weighted transport
quantity of all products in period t which may exceed the freight break threshold, and
avails discount on transportation quantity at exactly one freight discount break.
Equation (5.2.10) calculates the time of halt at intermediate stoppage. Equation
(5.2.11) shows the total weighted quantity transported in stage 1 of period t is equal to
the total weighted quantity transported in stage 2 of period \( t + 1 \). In equation (5.2.12),
the minimum weighted quantity transported is calculated and further Equation
(5.2.13) measures the overhead units from truckload capacity in weights.

### 5.2.2 The Crisp Formulation

Here we have converted fuzzy formulation to crisp formulation

Formulation is given as under:

Maximize \( \theta \)

subject to: \( \mu_c(X) \geq \theta \),

\[
I_{it} = I_{it-1} + X_{it} - \sum_{m=1}^{M} D_{imt} \cdot \eta \cdot I_{it},
\]

\[
I_{i1} = IN_i + X_{i1} - \sum_{m=1}^{M} \tilde{D}_{i1m} \cdot \eta \cdot I_{i1},
\]

\[
(I \cdot \eta) \sum_{t=1}^{T} I_{it} + \sum_{t=1}^{T} X_{it} \geq \sum_{t=1}^{T} \sum_{m=1}^{M} \tilde{D}_{imt} \]

\( X \in S = \{X_{it}, I_{it}, L_{it}, L_{2mt}, a_{mt}, y_{mt} \geq 0 \text{ and integer}; R_{it}, z_{kt}, u_{mt}, v_{kt} \in \{0,1\} \} \)

/satisfying eq (5.2.5) to (5.2.13); \( i = 1,...,P; t = 1,...,T; l = 1,...,L; m = 1,...,M, k = 1,...,K, \tau = 1,...,\Gamma \} ; \theta \in [0,1] \)

can be solved by the standard crisp mathematical programming algorithm.

Now main challenge is to reduce various cost components viz. purchase, transportation,
inspection cost and holding cost in order to get maximum benefits. The various demand
parameters \( \tilde{D}_{lmt} \) are the ranking numbers represented as \( A = (a^1, a^2, a^3) \). The value of these fuzzy numbers are specified by the management based on the past experiences and/or expert opinion that are tabulated in Table: 5.2.11. Using the defuzzification function \( F_2(A) = (a^1 + 2a^2 + a^3)/4 \) we defuzzify these fuzzy numbers.

### 5.2.3 Results and Discussions

The vital objectives, the firms are concerned about are how much to order and how to minimize the total cost. Here, we have tried to answer these questions with the help of case in procurement-distribution scenario of a two stage supply chain in uncertain environment for perishable products. Ordered Quantity for Carrots in period 1 is 387 packs with discount of 5% on purchase cost and remaining ending inventory would be nothing after fulfilling the demand. Distribution in first stage from supplier to intermediate stoppage is 6,842 kgs with 5% discount on vegetables. At intermediate stoppage vegetables will be halted for two days and then at second stage distributed through trucks at different stores of the city. The quantity will reach at store 1 at second stage of second period with load of 6,003 kgs by using both TL & LTL policy in 3 trucks and 3 extra kgs. Rest of the procurement and distribution policies for vegetables is given in Appendix B (Table 5.2.12 – 5.2.15). Total cost incurred by firm is $2,324,712 with removal of 81.59% vagueness.