Ph.D Thesis of Navneet Singh Rana

Suggestions for Revision with Replies

(i) In Chapter 1, my only criticism concerns the statement on Page 11 that “the non-existence of perfect codes have been proved in a series of papers by van Eupen (1995), ...”. In fact, five of the six papers listed have nothing to do with perfect codes, but are concerned with optimal linear codes, a topic not discussed in this introductory chapter.

*In Chapter 1, changes as per the suggestions of the examiner have been made and a paragraph on optimal codes has been added.*

(ii) On page 33, a simple example of a (1+2,1) optimal ternary code is given. It is then stated at the bottom of Page 33 that “the parity check matrices for (1+5, 3), (1+14, 11), ... codes can similarly be constructed ...”. I tried to construct the (1+5, 3) code for myself but succeeded only in showing that it does not exist! Indeed, I found later in the thesis (theorem on p. 72), the author’s own result showing that this code does not exist”! So the above statement either needs correcting and justifying, or should be withdrawn.

An optional suggestion: The penultimate example on Page 34, with second block of length 10, is of interest. It would be interesting to know how this was found (trial and error?) and whether this is the maximum possible length of the second sub-block of a (1,2) ternary code with n − k = 4 [an upper bound is 14 by Equation (2)]. Is it feasible to find this maximum length by exhaustive search, looking at all the inequivalent ways of building up the matrix, column by column? It actually looks as though this might be too big to do by hand, but should be amenable to a computer search.

*In Chapter 2 (p. 35), the necessary corrections have been made as suggested by the examiner.*

Further (13+10, 19) non-binary ternary code with second sub-block of length 10 was constructed by Trial and Error method. Also we have been able to constructed (10+11, 17)-code where the length of the second sub-block is 11, the technique remain the same.
(iii) At the top of p. 35, it is stated that “Based on this study, we can say that, for each \( n_1 = 1, 4, 7, \ldots \) there exists an infinite class of non-binary (1,2)-optimal codes over GF(3).” Is the author really saying that for each of \( n_1 = 1, 4, \ldots \) there is an infinite class of codes? I don’t see any evidence in the thesis for such a claim, and would be very surprised if it were true.

Further, on p. 35, it is stated that “a large number of cases for (2, 2), (2, 3), (3, 3), (3, 4) and (4, 4) non-binary optimal codes have been studied as shown in the following table ...” and that “it is interesting to note that non-binary optimal codes over GF(3) do not exist for any other values of \( b_1 \) and \( b_2 \).” It is not clear what the author means by “have been studied”. In fact, it follows instantly from Equation (2) on P. 21 that there cannot possibly be any \((b_1, b_2)\)-optimal code over GF(q) with \( b_1 \) and \( b_2 \) both greater than 1, as taking Equation (2) modulo q would give the immediate contradiction that \( 1 \equiv 0 \pmod{q} \). So no extensive study of many separate cases is required.

Changes as per the suggestions of the examiner have been made

(iv) Section 2.3 looks at (1, 2)-optimal codes over GF(5) and GF(7). For GF(5) codes, the author looks at the case of \( n_1 = 86 \) (it is not clear why this particular value should be considered). For \( n - k = 4 \), a suitable check matrix is given to show the existence of an \((86+15, 97)\) code. It is then stated that “the matrices for the remaining codes [i.e. \((n_2, k) = (140, 221) \) and \((765, 845)\)] can similarly be constructed by a procedure to be given in Section 2.4 of this chapter”. But section 2.4 gives a procedure which works only for \( n_2 \leq 4 \). Here, for the case of \( n_2 = 140 \), say, this looks to me to be a difficult problem. So, again, this statement needs to be properly clarified and/or justified.

Exactly the same comments apply to the treatment for \( q = 7 \) given on pages 37 to 39.

In section 2.3 (chapter 2), we have made changes as per the suggestions of the examiner and also justify the considerations of these particular values of \( n_1 \) as 86 and 218 for \((87+15, 97)\) and \((218 + 27, 241)\) non-binary codes over GF(5) and GF(7) respectively.

(v) On p. 46, Theorem 2, is incorrectly stated. The word “if” should be replaced by “only if”. The author’s proof is then correct, though
somewhat laboured, as the result follows instantly just by taking Equation (2) modulo q to get $n_1 \equiv 1 \pmod{q}$.

On p. 48 (Chapter 2), the word “if” has been replaced by “only if” in the statement of theorem 2.1 and the proof of Theorem 2.2 has been shortened as suggested by the examiner.

(vi) Chapter 3 generalizes the treatment to $(b_1, b_2, ..., b_m)$-codes for $m \geq 3$. A necessary condition for the existence of such codes is generalized in a straightforward way. A sufficient condition is also given. This is based on a condition which ensures the availability of each new column of the check matrix after all non-eligible potential new columns have been excluded (and assuming a worst-case scenario that all these non-eligible columns are distinct). I found the proof a little difficult to follow, but I think I now understand it and believe that it is probably correct (though I think (3.22) on p. 55 has a factor $(q - 1)$ missing).

In Chapter 3 (p. 55), a factor $(q - 1)$ was missing in eq. (3.17) due to typographical mistake and we have added the factor $(q - 1)$ in the said equation.

(vii) Chapter 4 considers the non-existence of certain codes. The theorem on p.72 shows that (1,2)-optimal codes do not exist for the case $n - k = 3$ and $n_2 = 5$, but the proof needs greater rigour. As the author points out, the key step is to show that the second sub-block cannot be constructed. The author then considers

\[
\begin{align*}
100 \\
010 \\
001 
\end{align*}
\]

as the first three columns of this sub-block and correctly shows that a fourth column cannot be added. However, the justification for choosing the first three columns in this way is inadequate. The author simply says that this is “the most trivial way in which the first three columns of 2nd sub-block can be constructed”. But the point here is that we must show that a fourth column cannot be added whatever valid choice of first three columns is chosen. So it is necessary to say that the first three columns can be chosen in the above way without loss of generality. To do this properly requires one to use the fact that parity-check matrices for the
same code can be obtained by using elementary row operations. The required properties of the code ensure that the 2\textsuperscript{nd} column must be independent of the first and then that the third must be independent of the first two. Hence, wlog (via row operations), we can choose each of the first three columns in turn to be as above. I have rather laboured this point because it is an error which appears to be more seriously made in Section 4.2, as follows.

*In Theorem 4.1 on p. 72 (Chapter 4), the changes suggested by the examiner have been incorporated and we have justified the selection of first three columns in the second sub-block by wlog arguments as suggested by the examiner.*

(viii) Let us look at the proof of the result on p. 75 in Section 4.2, The author says “Suppose we take H as ...”. Here the first 7 columns have been fixed. How can this supposition be justified? Just because no 8\textsuperscript{th} column can be added to this particular choice of first seven, it doesn’t mean that an 8\textsuperscript{th} column cannot be added some other possible choice of first seven. To do this proof properly, it is necessary first to justify the choice of the first four columns by “wlog” arguments and then consider all possible eligible choices for the 5\textsuperscript{th} column, then for the 6\textsuperscript{th} and so on. The result may well still be true, but it requires a somewhat longer proof than that given in the thesis.

*The proof of the result in section 4.2 (p. 76) has been changed as suggested by the examiner. Truly, the proof is now longer as observed by the examiner.*