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*** ***
SOME FINITE AND INFINITE SUMMATION FORMULAE FOR
THE H-FUNCTION OF SEVERAL COMPLEX VARIABLES

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In the present paper we establish some finite and infinite series summation
formulæ for the H-function of several complex variables defined by
Srivastava and Panda as a multiple Mellin-Barnes contour integral.

1. INTRODUCTION AND NOTATIONS

The H-function of several complex variables, which was introduced and
studied systematically in a series of recent papers by Srivastava and Panda (1975,
1976, 1978, 1979), is an extension of the G-function of several complex variables.
This H-function of several variables includes Fox's H-functions of one and two
variables, Meijer's G-functions of one and two variables, the generalized Lauricella
function of Srivastava and Daoust (1969, p. 454), Appell functions, the Whittaker
functions, etc. Therefore, the results established in this paper are of a general
character and hence encompass several cases of interest.

The object of this paper is to establish some finite and infinite summation
formulæ for the H-function of several complex variables and recurrence relations
based upon them. These formulæ will yield a number of new and known results
including the results of Sharma (1972, 1973).

The H-function of several complex variables defined by Srivastava and Panda
(1975, 1976) and represented in the following manner (Srivastava et al. 1982, p. 251):

\[
H[Z_1, \ldots, Z_r] \equiv H_{p_1, \ldots, p_r; q_1, \ldots, q_r}^{m_1, \ldots, m_r; n_1, \ldots, n_r}
\]

\[
\times \prod_{i=1}^{r} \left( \frac{1}{(2\pi i)^{\xi_i}} \right) \int_{a_1}^{b_1} \cdots \int_{a_r}^{b_r} f_1(\xi_1) \cdots f_r(\xi_r) \psi(\xi_1, \ldots, \xi_r) Z_1^{\xi_1} \cdots Z_r^{\xi_r} d\xi_1 \cdots d\xi_r, \quad (1.1)
\]
where

\[
\omega = \sqrt{-1}
\]

\[
\phi_l (\xi_l) = \frac{\prod_{j=1}^{\nu_l} \Gamma \left( d^{(l)}_j - \delta^{(l)}_j \xi_l \right) \prod_{k=1}^{\nu_l} \Gamma \left( 1 - c^{(l)}_j + \gamma^{(l)}_j \xi_l \right)}{\prod_{j=m+1}^{\rho_l} \Gamma \left( 1 - d^{(l)}_j + \delta^{(l)}_j \xi_l \right) \prod_{k=m+1}^{\rho_l} \Gamma \left( c^{(l)}_j - \gamma^{(l)}_j \xi_l \right)} \quad \ldots(1.2)
\]

\[
\psi (\xi_1, \ldots, \xi_r) = \frac{\prod_{j=1}^{\rho} \Gamma \left( 1 - a_j + \sum_{i=1}^{r} \alpha^{(l)}_j \xi_i \right)}{\prod_{j=n+1}^{\rho} \Gamma \left( a_j - \sum_{i=1}^{r} \alpha^{(l)}_j \xi_i \right) \prod_{i=1}^{q} \Gamma \left( 1 - b_j + \sum_{i=1}^{r} \beta^{(l)}_j \xi_i \right)} \quad \ldots(1.3)
\]

and the integral (1.1) converges absolutely if

\[
\Omega \equiv \sum_{i=1}^{n} \alpha^{(l)}_i - \sum_{i=n+1}^{\rho} \alpha^{(l)}_i + \sum_{i=1}^{\varphi_l} \gamma^{(l)}_i - \sum_{i=n+1}^{\varphi_l} \gamma^{(l)}_i - \sum_{j=1}^{\varphi_l} \beta^{(l)}_j + \sum_{j=n+1}^{\varphi_l} \delta^{(l)}_j - \sum_{j=m+1}^{\varphi_l} \delta^{(l)}_j > 0,
\]

\[
i \in \{ 1, \ldots, r \}, \quad | \arg (z_l) | < \frac{1}{2} \Omega \quad \pi.
\]

The multiplication formula for the gamma function is

\[
\Gamma \left( mz \right) = (2\pi)^{1/2(1-m)} m^{mz - 1/2} \prod_{i=0}^{m-1} \Gamma \left( z + \frac{i}{m} \right) \quad \ldots(1.4)
\]

where \( m \) is a positive integer.

From (1.4), if \( N \) is a positive integer, the formulae

\[
\prod_{i=0}^{m-1} \Gamma \left( \frac{\alpha + N + i}{m} \right) = m^{-N} \left( \frac{\alpha}{m} \right) \prod_{i=0}^{m-1} \Gamma \left( \frac{\alpha + i}{m} \right) \quad \ldots(1.5)
\]

\[
\prod_{i=0}^{m-1} \Gamma \left( \frac{\alpha - N + i}{m} \right) = \left( \frac{-m}{1 + \alpha} \right) \prod_{i=0}^{m-1} \Gamma \left( \frac{\alpha + i}{m} \right) \quad \ldots(1.6)
\]

can easily be derived.

2. INFINITE SUMMATION FORMULAE

The infinite summation formulae to be established are

\[
\sum_{N=0}^{\infty} \frac{(-1)^N \left( \frac{1}{2} + \alpha \right) N!}{N! \left( \frac{1}{2} + \alpha \right) ^N} \left( \sum_{k=1}^{m_1} \sum_{l=1}^{n_1} \cdots \sum_{r_1}^{s_1} \cdots \sum_{k_r}^{m_r} \sum_{l_r}^{n_r} \cdots \sum_{r_r}^{s_r} \right)
\]

\[
(equation \ continued \ on \ p. \ 411)
\]
$H$-FUNCTION OF SEVERAL COMPLEX VARIABLES

\[
\frac{Z_1}{Z_r} \left[ \begin{array}{c}
(a_j, \alpha_j', \ldots, \alpha_j^{(r)})_{1,p} : (c_j', \gamma_j')_{1,q_1} \\
\vdots \\
(b_j, \beta_j', \ldots, \beta_j^{(r)})_{1,q} : (d_j', \delta_j')_{1,q_1}
\end{array} \right] \\
\frac{m^{n+1/2}}{2^{\alpha-\beta-\gamma} \Gamma(\alpha+1)} \Gamma\left(\frac{1}{2}\right) \Gamma\left(\alpha+\beta+\frac{1}{2}\right) \Gamma\left(\beta+\frac{1}{2}\right) \\
H^{\alpha+n_1, \ldots, \alpha_n, \gamma, \beta}_{\alpha, \beta, \gamma, \beta} \\
\frac{2^{2\alpha h} Z_1}{Z_r} \left[ \begin{array}{c}
(a_j, \alpha_j', \ldots, \alpha_j^{(r)})_{1,p} : (c_j', \gamma_j')_{1,q_1} \\
\vdots \\
(b_j, \beta_j', \ldots, \beta_j^{(r)})_{1,q} : (d_j', \delta_j')_{1,q_1}
\end{array} \right]
\]

$(\Delta (m, 1 + \alpha - \beta - N), h), (\Delta (m, 1 + \alpha - \gamma + N), h), (\Delta (m, 1 - \beta - N), h), (\Delta (m, 1 - \gamma - N), h); \ldots$

$(i)$

\[
\sum_{N=0}^{\infty} \frac{(\alpha)^N (\beta)^N}{N!} \Gamma\left(\alpha+\beta+\frac{1}{2}\right) \Gamma\left(\alpha+\beta+\frac{1}{2}\right) \Gamma\left(\alpha+\beta+\frac{1}{2}\right) \Gamma\left(\alpha+\beta+\frac{1}{2}\right) H^{\alpha+n_1, \ldots, \alpha_n, \gamma, \beta}_{\alpha, \beta, \gamma, \beta} \\
\frac{Z_1}{Z_r} \left[ \begin{array}{c}
(a_j, \alpha_j', \ldots, \alpha_j^{(r)})_{1,p} : (c_j', \gamma_j')_{1,q_1} \\
\vdots \\
(b_j, \beta_j', \ldots, \beta_j^{(r)})_{1,q} : (d_j', \delta_j')_{1,q_1}
\end{array} \right]
\]

$(\Delta (m, 1 + \beta), h), (\Delta (m, 1 - \gamma), h), (\Delta (m, 1 + \gamma - N), h), (\Delta (m, 1 - \beta - N), h); \ldots ; (c_j^{(r)}, \gamma_j^{(r)})_{1,p_1}, \ldots ; (d_j^{(r)}, \delta_j^{(r)})_{1,q_1}$

Re $\left(\alpha - 2\beta - 2\gamma\right) > -2;$ ...$(2.1)$

$(ii)$

\[
\sum_{N=0}^{\infty} \frac{(\alpha)^N (\beta)^N}{N!} \Gamma\left(\alpha+\beta+\frac{1}{2}\right) \Gamma\left(\alpha+\beta+\frac{1}{2}\right) \Gamma\left(\alpha+\beta+\frac{1}{2}\right) \Gamma\left(\alpha+\beta+\frac{1}{2}\right) H^{\alpha+n_1, \ldots, \alpha_n, \gamma, \beta}_{\alpha, \beta, \gamma, \beta} \\
\frac{Z_1}{Z_r} \left[ \begin{array}{c}
(a_j, \alpha_j', \ldots, \alpha_j^{(r)})_{1,p} : (c_j', \gamma_j')_{1,q_1} \\
\vdots \\
(b_j, \beta_j', \ldots, \beta_j^{(r)})_{1,q} : (d_j', \delta_j')_{1,q_1}
\end{array} \right]
\]

$(\Delta (m, 1 + \alpha - \beta - N), h); \ldots ; (c_j^{(r)}, \gamma_j^{(r)})_{1,p_1}, \ldots ; (d_j^{(r)}, \delta_j^{(r)})_{1,q_1}$

\[
\frac{\Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\alpha+\frac{1}{2}\right) \Gamma\left(\beta+\frac{1}{2}\right)} H^{\alpha+n_1, \ldots, \alpha_n, \gamma, \beta}_{\alpha, \beta, \gamma, \beta} \\
\frac{Z_1}{Z_r} \left[ \begin{array}{c}
(a_j, \alpha_j', \ldots, \alpha_j^{(r)})_{1,p} : (c_j', \gamma_j')_{1,q_1} \\
\vdots \\
(b_j, \beta_j', \ldots, \beta_j^{(r)})_{1,q} : (d_j', \delta_j')_{1,q_1}
\end{array} \right]
\]

$(\Delta (m, 1 + \beta), h), (\Delta (m, 1 - \gamma), h), (\Delta (m, 1 + \gamma - N), h), (\Delta (m, 1 - \beta - N), h); \ldots ; (c_j^{(r)}, \gamma_j^{(r)})_{1,p_1}$

$\ldots ; (d_j^{(r)}, \delta_j^{(r)})_{1,q_1}$

(equation continued on p. 412)
\[(\Delta (m, \frac{1}{2} + \gamma - \alpha), h), (\Delta (m, \frac{1}{2} + \gamma - \beta), h); \ldots; (c_j^{(r)}, \gamma_j^{(r)})_{1,q_1}^{(r)}\]
\[(\Delta (m, \gamma - \alpha - \beta + \frac{1}{2}), h), (d_j^{'}, \delta_j^{'})_{1,q_1}^{(r)}; \ldots; (d_j^{(r)}, \delta_j^{(r)})_{1,q_r}^{(r)}\]
\[\ldots (2.2)\]

**Proof:** Expressing the $H$-function of several complex variables on the left-hand side of (2.1) by Mellin-Barnes contour integral (1.1), changing the order of summation and integration as permissible by absolute convergence for $\text{Re} (\alpha - 2\beta - 2\gamma) < -2$ and the above stated conditions, finally using (1.5) and (1.6), and simplifying the expression with the help of result (MacRobert 1966, p. 368), the right-hand side of (2.1) obtained.

Similarly, eqn. (2.2) can be obtained by using the result given by MacRobert (1958) in place of MacRobert (1966, p. 368).

3. **Finite Summation Formulae**

The finite summation formulae to be established are

\[(\sum_{n=0}^{N} \binom{N}{u}^{m_2} \frac{m_{2u}}{(1+k)_u (1-k-N)_u} H_{p, q : p_1 + m_1, q_1 + m_1; \ldots; p_r, q_r}^{0; n; m_1, n_1; \ldots; m_r, n_r} \times \left[ a_i, a_i', \ldots; a_i^{(r)} \right]_{1,p}^{(r)} \times \left[ b_j, b_j', \ldots; b_j^{(r)} \right]_{1,q}^{(r)}\]

\[(\Delta (m, \gamma - \alpha - \beta + \frac{1}{2}), h), (d_j^{'}, \delta_j^{'})_{1,q_1}^{(r)}; \ldots; (d_j^{(r)}, \delta_j^{(r)})_{1,q_r}^{(r)}\]

\[= (-1)^N m^{2N} \frac{\Gamma (k + 1) \Gamma (k)}{\Gamma (k + N) \Gamma (k + 1 + N)} H_{p, q : p_1 + 2m_1, q_1 + 2m_1; \ldots; p_r, q_r}^{0; n; m_1, n_1; \ldots; m_r, n_r} \times \left[ a_i, a_i', \ldots; a_i^{(r)} \right]_{1,p}^{(r)} \times \left[ b_j, b_j', \ldots; b_j^{(r)} \right]_{1,q}^{(r)}\]

\[(c_j^{'}, \gamma_j)_{1,p_1}^{(r)}, (\Delta (m, \alpha + k), h), (\Delta (m, \alpha - k - N), h); \ldots; (c_j^{(r)}, \gamma_j^{(r)})_{1,p_r}^{(r)}\]

\[(\Delta (m, \alpha + k + N), h), (\Delta (m, \alpha - k), h), (d_j^{'}, \delta_j^{'})_{1,q_1}^{(r)}; \ldots; (d_j^{(r)}, \delta_j^{(r)})_{1,q_r}^{(r)}\]

\[\ldots (3.1)\]
\[ \sum_{u=0}^{N} \binom{N}{u} (k)_{u} (-k + N)_{u} m^{-3u} H_{p+q}^{0, n; m_1 + m_2; n_1, \ldots, m_r, n_r} (\Lambda (m, \alpha + u), h), \]

\[ \times \left[ \begin{array}{c} Z_1(a_j; \alpha_j, \gamma_j)_{1+p_1} \\ Z_2(b_j; \beta_j, \gamma_j)_{1+q_1} \end{array} \right], \]

\[ (\Delta (m, \alpha + u), h); \ldots; (c_j^{(r)}, \gamma_j^{(r)})_{1+p_r}, \]

\[ (d_j^{(r)}, \delta_j^{(r)})_{1+q_r} \]

\[ = H_{p+q}^{0, n; m_1 + m_2; n_1, \ldots, m_r, n_r} \left[ \begin{array}{c} Z_1(a_j; \alpha_j, \gamma_j)_{1+p_1} \\ Z_2(b_j; \beta_j, \gamma_j)_{1+q_1} \end{array} \right], \]

\[ (\Delta (m, \alpha + k), h), (\Delta (m + N), h), \]

\[ (d_j^{(r)}, \delta_j^{(r)})_{1+q_1}, (\Delta (m, \alpha + k), h), \]

\[ (\Delta (m, \alpha - k - N), h); (\Delta (m, \alpha - k + N), h); \ldots; (c_j^{(r)}, \gamma_j^{(r)})_{1+p_r}, \]

\[ (\Delta (m, \alpha - N), h); (\Delta (m, \alpha - k - N), h); \ldots; (d_j^{(r)}, \delta_j^{(r)})_{1+q_r} \] \quad \ldots \quad (3.2)

\[ \sum_{u=0}^{N} (-1)^{u} \binom{N}{u} \left( \frac{1}{2} \alpha + 1 \right) \left( \frac{1}{2} \alpha - 1 \right) \frac{(\gamma)_{u}}{(\alpha - \delta + 1)_{u} (\alpha - \epsilon + 1)_{u}} \]

\[ \binom{N}{u}, \binom{(\delta)_{u}}{(\alpha - N + 1)_{u}} \]

\[ H_{p+q}^{0, n; m_1 + m_2; n_1, \ldots, m_r, n_r} \left[ \begin{array}{c} Z_1(a_j; \alpha_j, \gamma_j)_{1+p_1} \\ Z_2(b_j; \beta_j, \gamma_j)_{1+q_1} \end{array} \right], \]

\[ (\Delta (m, 1 + \alpha - \gamma + u), h), (\Delta (m, 1 - \gamma - u), h); \ldots; (c_j^{(r)}, \gamma_j^{(r)})_{1+p_r}, \]

\[ (\Delta (m, \beta - \alpha - u), h), (d_j^{(r)}, \delta_j^{(r)})_{1+q_1}, \ldots; (d_j^{(r)}, \delta_j^{(r)})_{1+q_r} \] \quad \ldots \quad (3.3)

\[ = \frac{(\alpha + 1)^{N}}{(\alpha - \delta + 1)^{N}} \frac{(\alpha - \beta - \gamma + 1)^{N}}{(\alpha - \beta - \gamma - \delta + 1)^{N}} \cdot \]

\[ H_{p+q}^{0, n; m_1 + m_2; n_1, \ldots, m_r, n_r} \left[ \begin{array}{c} Z_1(a_j; \alpha_j, \gamma_j)_{1+p_1} \\ Z_2(b_j; \beta_j, \gamma_j)_{1+q_1} \end{array} \right], \]

\[ (\Delta (m, \beta), h), \]

\[ (\text{equation continued on p. 414}) \]
\((\Delta (m, 1 + \alpha - \gamma + N), h), (\Delta (m, 1 - \gamma), h), (\Delta (m, 1 + \alpha - \delta), h),
(\Delta (m, \beta - \alpha - N), h), (\Delta (m, \beta + \delta - \alpha), h),
(\Delta (m, 1 + \beta + \delta - \alpha - N), h); \ldots;
(\Delta (m, \alpha - \gamma - \delta + 1 + N), h), (d'_j, \delta'_j)_{1 \leq q_j}; \ldots;
(c'_j, \gamma'_j)_{1 \leq r_j}
\)

\[
(c'_j, \gamma'_j)_{1 \leq r_j} \\
(d'_j, \delta'_j)_{1 \leq q_j}
\]

...(3.3)

provided \(m\) and \(N\) are positive integers, \(h > 0\).

**Proof:** Expressing the \(H\)-function on the left-hand side of (3.1) by Mellin-Barnes contour integral (1.1), changing the order of summation and integration, which is justified due to the absolute convergence involved in the process and finally using (1.5), (1.6) and simplifying with the help of Saalschütz's theorem (MacRobert 1966, p. 360) and (1.4), the right-hand side of (3.1) is obtained.

Similarly, we can obtain the formulae (3.2) and (3.3) by using the results

\[
_{3}F_2 \left[ \begin{array}{c}
\alpha, \beta, \gamma \\
\rho, \sigma
\end{array} ; 1 \right] = \frac{\Gamma (\rho) \Gamma (\alpha - \sigma + 1) \Gamma (\beta - \sigma + 1) \Gamma (\gamma - \sigma + 1)}{\Gamma (1 - \sigma) \Gamma (\rho - \alpha) \Gamma (\rho - \beta) \Gamma (\rho - \gamma)}
\]

and [MacRobert 1966, p. 171 (34)] respectively.

Setting \(N = 1\), (3.1), (3.2) and (3.3) yields

(i) \(\frac{k (k + 1)}{m^2} H \left[ Z_1, \ldots, Z_r \right] + H \left[ \ldots, \frac{m + m_1 q_1}{Z_1}, \ldots; \ldots, \frac{m + m_1 q_1}{Z_r}, \ldots \right]
\]
\[
\times \left[ \begin{array}{c}
Z_1 \\
\vdots \\
Z_r
\end{array} \right] = \left[ \begin{array}{c}
\ldots, (c'_j, \gamma'_j)_{1 \leq r_j} \\
\ldots, (\Delta (m, 1 + \alpha + k), h),
\end{array} \right]
\]
\[
(\Delta (m, 1 + \alpha + k), h), (\Delta (m, \alpha - k), h); \ldots; \\
(\Delta (m, \alpha + k), h), (d'_j, \delta'_j)_{1 \leq q_j}; \ldots;
\]

(ii) \(H \left[ Z_1, \ldots, Z_r \right] + \frac{k (1 - k)}{m^2} H \left[ \ldots, \frac{m + m_1 q_1}{Z_1}, \ldots; \ldots, \frac{m + m_1 q_1}{Z_r}, \ldots \right]
\]

\[
\left( \begin{array}{c}
Z_1 \\
\vdots \\
Z_r
\end{array} \right] = \left[ \begin{array}{c}
\ldots, (c'_j, \gamma'_j)_{1 \leq r_j}, (\Delta (m, \alpha), h); \ldots; \\
\ldots, (\Delta (m, 1 + \alpha), h), (d'_j, \delta'_j)_{1 \leq q_j}; \ldots;
\end{array} \right]
\]

...(3.4)

(equation continued on p. 415)
\[ H \cdot \ldots \cdot \frac{Z_{1}}{Z_{r}} \cdot \ldots \cdot \left[ \begin{array}{c}
\vdots \\
\ldots
\end{array} \right] \cdot \left[ \begin{array}{c}
\ldots
\ldots
\end{array} \right] \cdot \left[ \begin{array}{c}
\ldots
\ldots
\end{array} \right] = H \cdot \ldots \cdot \frac{Z_{1}}{Z_{r}} \cdot \ldots \cdot \left[ \begin{array}{c}
\vdots \\
\ldots
\end{array} \right] \cdot \left[ \begin{array}{c}
\ldots
\ldots
\end{array} \right] \cdot \left[ \begin{array}{c}
\ldots
\ldots
\end{array} \right] = \ldots (3.5)

\]

\[
\begin{align*}
H \cdot \ldots \cdot \frac{Z_{1}}{Z_{r}} \cdot \ldots \cdot \left[ \begin{array}{c}
\vdots \\
\ldots
\end{array} \right] \cdot \left[ \begin{array}{c}
\ldots
\ldots
\end{array} \right] = & \frac{2\delta \varepsilon \left( \frac{1}{2} \alpha + 1 \right)}{(\alpha + 2) (\alpha - \delta + 1) (\alpha - \epsilon + 1)} \\
& \ldots (3.6)
\end{align*}
\]

where

\[
2\alpha = \beta + \gamma + \delta + \epsilon.
\]

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DOUBLE SUMMATION FORMULAE FOR THE $H$-FUNCTION
OF SEVERAL COMPLEX VARIABLES

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ABSTRACT

In the present paper, an attempt has been made to derive unified double summation formulae for the $H$-function of several complex variables introduced by H. M. Srivastava and R. Panda. Since the $H$-function of several complex variables includes a large number of special functions of one and more variables as its particular cases, the results established here serve as key formulae giving us a large number of new and interesting results by specialising the parameters involved.

1. Introduction and Notations

For the $H$-function of several complex variables, which was introduced by Srivastava and Panda ([9], [10]), is an extension of the $G$-function of several complex variables. This $H$-function of several variables includes Fox's $H$- and Meijer's $G$-functions of one and two variables, the generalized Lauricella function of Srivastava and Daoust [7], Appell functions, the Whittaker functions, etc. Therefore, the results established in this paper are of a general character and hence encompass several cases of interest.

The object of this paper is to establish four unified double summation formulae for the $H$-function of several complex variables. These formulae will yield a number of new and known results including the results of Gupta and Goyal ([4], [5]).

The $H$-function of several complex variables defined by Srivastava and Panda ([9], [10]) and represented in the manner already detailed by Srivastava, Gupta and Goyal ([8], p. 251).

2. Double Summation Formulae

The double summation formulae to be established are
\[ (i) \sum_{M,N=0}^{\infty} \frac{1}{M! N!} H_0^0, n+1 : m_1+1, n_1+1; m_2+1, n_2+1; \ldots; m_r, n_r, \]
\[ p+1, q+3 : p_1+1, q_1+1; p_2+1, q_2+1; \ldots; p_r, q_r \]
\[ \begin{bmatrix} Z_1 \\ Z_r \end{bmatrix} \begin{array} \{ \begin{array} \{ 1-C-M-N; \gamma, \delta, 1, \ldots, i, \\
1-d-e-M-N; \alpha, \beta, 1, \ldots, 1, \end{array} \} \end{array} \]

\[ 1-C-M; \gamma, \delta, 1, \ldots, 1), (1-C-N; \gamma, \delta, 1, \ldots, 1): \]
\[ 1-d-M, d) ; (1-e-N, \beta), \ldots ; \ldots \]
\[ (a+M, r), \ldots; (b+N, o), \ldots; \ldots \]

\[ = H_0^0, n+3 : m_1+1, n_1; m_3+1, n_3; \ldots; m_r, n_r, \]
\[ p+3, q+3 : p_1, q_1+1; p_2, q_2+1; \ldots; p_r, q_r \]
\[ \begin{bmatrix} Z_1 \\ Z_r \end{bmatrix} \begin{array} \{ \begin{array} \{ 1-a-M-N; \alpha, \beta, 1, \ldots, 1, \\
1-d-e+a+b; \alpha+p, \beta+\sigma, 1, \ldots, 1, \end{array} \} \end{array} \]

\[ 1-e+a; \rho, \beta, 1, \ldots, 1, \]
\[ 1-c+a; \gamma, \rho+\delta, 1, \ldots, 1, \]
\[ 1-C+a+b; \gamma+\rho, \delta+\sigma, 1, \ldots, 1, \]
\[ 1-C+b; \gamma, \delta+\sigma, 1, \ldots, 1, \]: \]
\[ (a, \rho) ; (b, \sigma), \ldots; \ldots ; \ldots ; \ldots \]

\[ \text{Re} (d+e-C) > 0, \text{Re} (C-a-d) > 0, \text{Re} (C-b-e) > 0; \]

\[ (ii) \sum_{M,N=0}^{\infty} \frac{1}{M! N!} H_0^0, n+2 : m_1+2, n_1; m_2+2, n_2; \ldots; m_r, n_r, \]
\[ p+2, q+4 : p_2, q_3+2; p_3, q_3+2; \ldots; p_r, q_r \]
\[ \begin{bmatrix} Z_1 \\ Z_r \end{bmatrix} \begin{array} \{ \begin{array} \{ 1-a-M-N; \alpha, \beta, 1, \ldots, 1, \\
1-a/2-M-N; \alpha/2, \beta/2, 1, \ldots, 1, \end{array} \} \end{array} \]

\[ (-a/2-M-N; \alpha/2, \beta/2, 1, \ldots, 1), \]
\[ (-a+b-M-N; \alpha+p, \beta, 1, \ldots, 1), \]
\[ (-a+C-M-N; \alpha, \beta+\sigma, 1, \ldots, 1), (-a+d+e-M-N; \alpha+\gamma, \beta+\delta, 1, \ldots, 1), \]
\[ \ldots \]

\[ b+M+N, \rho), (d+M, \gamma), \ldots; (C+M+N, o), (e+N, \delta) ; \ldots; \ldots \]

\[ = \frac{1}{2} H_0^0, n+1 : m_1+2, n_1; m_2+2, n_2; \ldots; \]
\[ p+1, q+3 : p_1, q_1+2; p_2, q_2+2; \ldots; \ldots \]
\[
\begin{align*}
\left[ Z_1 \right] &= (-a + b + c + d + e; \alpha + \gamma + \rho, \beta + \delta + \sigma, 1, \ldots, 1) \\
\left[ Z_r \right] &= (-a + b + c; \alpha + \rho, \beta + \sigma, 1, \ldots, 1), \\
\text{...} &
\end{align*}
\]

\[(a + b + d + e; x + \gamma + \rho, \beta + \delta, 1, \ldots, 1), (-a + c + d + e; \alpha + \gamma, \beta + \delta + \sigma, 1, \ldots, 1): \]

\[
\begin{align*}
\text{...} &
\end{align*}
\]

\[
(b, \rho), (d, \gamma); (c, \sigma), (\rho, \delta), \ldots; \ldots \\
\text{... (2.2)}
\]

\[
\text{Re } (b + c + d + e - a) < 1;
\]

(iii) \[
\sum_{M=0}^{R} \sum_{N=0}^{S} \frac{(-R)_M}{M!} \frac{(-S)_N}{N!} \frac{(b)_N}{H^{0, n+1 : m_1+1, n_1; \ldots; m_r, n_r}} \frac{(b)_N}{H^{0, q+2 : p_1+1, q_1+2; \ldots; p_r, q_r}}
\]

\[
\begin{align*}
\left[ Z_1 \right] &= (1 - C - M - N; \alpha, \beta, 1, \ldots, 1), \\
\left[ Z_r \right] &= (1 - C - M; \alpha, \beta, 1, \ldots, 1), \\
\text{...} &
\end{align*}
\]

\[
\left[ Z_1 \right] = (1 - C - M - N; \alpha, \beta, 1, \ldots, 1); \left[ Z_r \right] = (1 - C - M; \alpha, \beta, 1, \ldots, 1), \\
\text{... (2.3)}
\]

\[
(1 - C - N; \alpha, \beta, 1, \ldots, 1): (1 - S + b + M, \gamma), \ldots; \ldots; (1 - C - S; \alpha, \beta, 1, \ldots, 1)
\]

\[
\left[ Z_1 \right] = (1 - C - M - N; \alpha, \beta, 1, \ldots, 1), \\
\left[ Z_r \right] = (1 - C - M; \alpha, \beta, 1, \ldots, 1), \\
\text{... (2.3)}
\]

\[
\left[ Z_1 \right] = (1 - C - M - N; \alpha, \beta, 1, \ldots, 1); \left[ Z_r \right] = (1 - C - M; \alpha, \beta, 1, \ldots, 1), \\
\text{... (2.3)}
\]

\[
\text{e is not an integer:}
\]

(iv) \[
\sum_{M=0}^{R} \sum_{N=0}^{S} \frac{(-R)_M}{M!} \frac{(-S)_N}{N!} \frac{(b)_M}{H^{0, n+1 : m_1+1, n_1; \ldots; m_r, n_r}} \frac{(b)_M}{H^{0, q+2 : p_1+1, q_1+2; \ldots; p_r, q_r}}
\]

\[
\begin{align*}
\left[ Z_1 \right] &= (1 - C - d - M - N; \alpha, \beta, 1, \ldots, 1); \\
\left[ Z_r \right] &= (1 - C - d - M - N; \alpha, \beta, 1, \ldots, 1), \\
\left[ Z_1 \right] &= (1 - C - M, \alpha), \ldots; (1 + b - C - S + N, \alpha); \\
\left[ Z_r \right] &= (1 - d - N, \beta), \ldots; (1 + b - d - R + M, \beta); \ldots; \ldots
\end{align*}
\]
\[ (-1)^{R+S} \begin{bmatrix} 0, n+1 : m_1, n_1+1 ; m_2, n_2+1 ; \ldots ; \\ p+1, q+2 : p_1+1, q_1 ; p_2+2, q_2 ; \ldots ; \\ Z_1, \ldots, (1-C-d+b-R-S; \alpha, \beta, 1, \ldots, 1), \\ Z_2, \ldots, (1-C-d-R-S; \alpha, \beta, 1, \ldots, 1), \\ \vdots \end{bmatrix} \]

(1-C-d+b; \alpha, \beta, 1, \ldots, 1); (1-C-S, \alpha), \ldots, (1-C+b, \alpha); (1-d-R, \beta), \ldots, (1-d+b, \beta); \ldots; \ldots (2.4)

provide that the double series involved in all the above formulae (2.1) to (2.4) is absolutely convergent, \( \alpha, \beta, \gamma, \delta, \rho, \sigma, \geq, 0 \) (not all zero simultaneously).

**Proof.** Expressing the \( H \)-function of several complex variables involved in the left-hand side of (2.1) in terms of its Mellin-Barnes integral with the help of [8, p. 251 (C.1)], changing the order of integration and summation therein which is easily seen to be permissible under the conditions mentioned with (2.1) and finally using the results of Carlitz [3, p. 231].

Proofs of (2.2) to (2.4) can be developed on lines similar to those given above by using the results of Sharma [6, p. 187] and Carlitz [2, p. 138 and 1, p. 416, eq (9) in place of that of Carlitz [3, p. 231].

**Acknowledgements**

The authors are grateful to Prof. H. M. Srivastava, University of Victoria, Victoria, B. C., Canada, for making several suggestions.

**REFERENCES**


Received: February 2, 1983
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PRIORITY

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The National Academy of Sciences, India, Allahabad

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Date: 29 October, 1983.

Certified that Prof./Dr./Shri/Shmt. Kumari V S SHUKLA of
Dept. Math. Govt. P. V. M. Univ. Sugar, Panipat,
attended the 53rd Annual Session of the National Academy of Sciences,
India held at National Institute of Oceanography, Goa. He/She attended
the Session on October 27, 28 and 29, 1983. He/She also presented
research paper/s at the Session.

Chief Editor

Physical Sciences Section

Biological Sciences Section

53rd Annual Session
National Academy of Sciences, India
National Institute of Oceanography.

---

Dear Sir,
Your paper
"Fuzzy Sets: A Computational Approach"

is accepted for presentation in the Mathematics Section of the Indian Science Congress
from Jan 3-8, 1985.

Yours sincerely
T.C. Khare
156. Blue Shifted Iodine Band in Molecular Complexes of Iodine with Halobenzenes

V. P. Shedbalkar and S. N. Bhat, Department of Chemistry, North-Eastern Hill University, Shillong.

The blue shifted bands of iodine in its complexes with weak donors, such as benzene, chlorobenzene, bromobenzene, iodobenzene, o-dichlorobenzene and p-chloronitrobenzene in cyclohexane have been determined by a recently proposed method of maintaining a constant activity of iodine between two tetramethylammonium polyiodide solids. The results show that band positions, molar extinction coefficients, oscillator strengths and transition dipole moments depend on the interaction between iodine and the donors.


S. M. Rafique, Department of Physics, Marwari College, Bhagalpur University, N. R. Mitra, Department of Physics, T. N. B. College, Bhagalpur University, Bhagalpur, and P. L. Srivastava, Department of Physics, Bhagalpur University, Bhagalpur.

The choice of a set of eigenvalues and eigenfunctions in Harrison’s First Principle approach has been analysed. It is observed in case of Al that choosing the eigenvalues and eigenfunctions for the ionic core electrons gives the best agreement with the form factor inferred from experimental observations and also with the liquid electrical resistivity. Effect of core band exchange parameter on the above mentioned property has also been considered.

158. Double Summation Formulas for H-function of Several Complex Variables.

Chandra K. Sharma and Vijay S. Shukla, Department of Mathematics, Govt. P.V.M. (University of Saugar), Parasta.

In the present paper, an attempt has been made to derive unified double summation formula for H-function of several complex variables introduced by Srivastava and Panda (1976, 1979). Since the H-function of several complex variables includes a large number of special functions of one and more variables as its particular cases. The results established here serve as key formulas giving us a large number of new and interesting results by specialising the parameters involved. Some interesting particular cases have also been given.
June 13, 1985

Dr. C.K. Sharma
and Dr. V.S. Shukla
Department of Mathematics
Government P.V. Mahavidyalaya
Parasia - 480441, M.P.
India

Dear Drs. Sharma and Shukla:

I am pleased to inform you that the revised version of your joint paper "Exponential Fourier series for the H-function of several complex variables" has been accepted for publication in Pure and Applied Math. Sciences.

Your typescript is being forwarded to the Editor-in-Chief, Dr. P.L. Maggu, who will entertain all future correspondence in this connection.

Sincerely yours,

H.M. Srivastava
Professor, Department of Mathematics
and
Regional Editor, PAMS

HMS/bp
cc: Dr. P.L. Maggu
Editor-in-Chief, PAMS
PURE AND APPLIED MATHEMATIKA SCIENCES
(An International Journal of the Mathematika Sciences Society of India)

To
Dr. C.K. Sharma and V.J. Shukla
Department of Mathematics
Government Ranch Valley Mahavidyalaya
PANJAB - 480 441, H.P., India

Dear Drs. Sharma and Shukla:

I am pleased to inform you that your paper entitled "Exponential Fourier Series for the H-function of several complex variables" (No. 1527-(1)) has been found suitable for publication in our Journal: PURE AND APPLIED MATHEMATIKA SCIENCES.

This paper is likely to appear in Vol. XXIII, or XXIV, No. 1-2 issue of the Journal: PURE APPL. MATH. SCI.

Yours sincerely,

[Signature]

(F.L. MAGGU)
Editor-in-Chief, PAMS

Enclosed: Publication charge bill for payment at the earliest.
Professor N. Sankaran
Editor, Mathematics Student

No. 3249/559-MS/(A)

Dept. of Mathematics
Panjab University
Chandigarh-160 014.

March 26, 1984.

Dr. C.K. Sharma
Mathematics Department
Govt. P V. Mahavidiyalaya,
Parasia (Dist. Chhindwara, M.P. 480 441).

Dear Dr. Sharma,

We are happy to accept your paper entitled, "On the Multiple Hypergeometric Functions and H-Function of several complex variables" (written in collaboration with V.S. Shukla) for publication in the Mathematics Student.

There is a heavy backlog of accepted material with us (because of non-cooperation from the printers) in view of financial difficulties. It will take three to four years before your paper is printed.

I hope you will bear with us.

With best regards,

Yours sincerely,

(N. Sankaran)
150. Synthesis and Antiviral Activity of Biphenylene Tellurium Dichloride and Its Reaction Products.

T. N. SRIVASTAVA AND SHASHI MEHROTRA, Chemistry Department, University of Lucknow, Lucknow.

Biphenylene tellurium (IV) dichloride \((\text{C}_6\text{H}_5)_2\text{TeCl}_2\) has been prepared by fusion of Biphenyl (1.25 moles) with \(\text{TeCl}_4\) (1.00 mole) in the solid state at 140°C - 160°C. Its interaction with various silver salts \(\text{Ag}^+\text{X}^-\) (\(\text{X}^-=\) nitrate, acetate, oxalate, perchlorate) yields new products \((\text{C}_6\text{H}_5)_4\text{Te}(\text{X})_2\) (where \(n=1\) or 2) by displacement reaction. Complexes of the formulae \([\text{(C}_6\text{H}_5)_3\text{Te][M}X_4]\) (where \(\text{M} = \text{Hg, Ni and X=CN and NCS}\) have been isolated by the interaction of \((\text{C}_6\text{H}_5)_4\text{TeCl}_2\) with \(\text{K}_4[\text{MX}_4]\). It reacts with various donor bases to yield molecular adducts of the type \((\text{C}_6\text{H}_5)_3\text{TeCl}_2\cdot n\text{L}\) (where \(n=1\) or 2 and \(\text{L}\) = various donor bases) with penta and hexa co-ordinated tellurium atom. Its chlorination with \(\text{Na}_2\text{S}_2\text{H}_4\text{O}\) yields Biphenynelentetelluride (II) which undergoes oxidative addition reactions with halogens, interhalogens and pseudohalogenes. It also acts as a Lewis base and forms Donor-acceptor type complexes with strong acceptors such as \(\text{BF}_3\) and \(\text{HgCl}_2\). The newly synthesised compounds have been characterized and their structures established through elemental analyses, molar conductance, IR and PMR spectroscopic data. Some of the complexes show appreciable antiviral activity.

151. Study of feed impedances and driving-point currents of Archimedean spiral array.

V. RAMACHANDRA AND K. K. DEY, Electronics and Radio Physics Section, Department of Physics, Banaras Hindu University, Varanasi.

A theoretical analysis based on induced EMF theory for the feed impedances of half-wave dipole radiators arranged along an Archimedean spiral form has been made. The effects of various parameters of the Archimedian Spiral Array (viz., number of dipole radiators, number of spiral turns and size of the spiral) on the feed impedances has been discussed. The analysis is carried out for constant phase distribution by assuming thin linear, center-driven, vertically placed and non-staggered dipole radiators. The feed point resistances and reactances of single and multiple turns are plotted and compared. The driving-point currents at the feed-points are also presented in a tabular form.

152. Integrals Involving Product of Orthogonal Polynomials and the H-Functions of Several Complex Variables.

CHANDRA K. SHARMA AND VIJAY S. SUKLA, Department of Mathematics, Govt. P.V.M. (University of Saugar), Parasia.

In this paper, we evaluate six integrals involving the product of orthogonal polynomials and the H-functions of several complex variables introduced by H.M. Srivastava and R. Panda (1976). The integrals are quite general in nature and from them a large number of new results can be obtained simply by specializing the parameters of the multivariable H-function.