CHAPTER 1

A BRIEF SURVEY OF SEPARATION AXIOMS

The concept of a general topological space is too general for the purpose of many types of special studies that can be taken up naturally on them. Separation axioms are one of the types of conditions that are imposed to make them more amenable and natural for a wide variety of studies. For instance, completely regular spaces form a natural framework for the study of "Uniform or proximal continuity" and $R_0$-spaces for the study of nearness structure etc.

Dr. K.K. Dube in Chapter I of his thesis [11] has presented an excellent survey of the situation regarding the separation axioms up to 1973 and has indicated an excellent bibliography which shows that the interest of Topologists continues to exist not only in the search of new axioms but also in deeper study of the implications of the known axioms.

Similarly a survey in connection with subbase approach to the study of existing separation axioms and the search of new ones has been presented in the thesis of Dr. R. Dutta [14]

Since we started our research work to the study of
some aspects of the theory of separation axioms,
a brief survey of some recent work in this direction
and not covered by the theses mentioned above
is presented below.

We cannot claim that the survey is exhaustive
and it is possible that some interesting results are not
able to find mention in it. In some cases, original
papers were not available and we had to rely on summaries
as appearing in the Mathematical reviews.

We have divided it into three parts. The first part
indicates some new separation axioms defined recently and
some interesting related results. The first section
of the second part presents recent attempts at
developing a unified approach to the study of separation
axioms \([3]\) and which can be regarded as an interesting
though partially answer to the question raised recently
by H.J. Thron, "What is a separation axiom?"

The second section of the second part deals
briefly with the approach of Dowker and Stauts of
defining separation axioms for frames \([15]\) which when
applied to the topological space situation give rise to
the usual standard separation axioms.
The third part of this survey article is devoted to the theory of some localized versions of some standard separation axioms by Dube and Miara in [12]. The idea of localization was studied first by J. Chew [9] in the context of paracompact and fully normal spaces.

Part I

A. Axioms weaker than $T_2$

A.1 $R_0$ and $R_1$ spaces

A topological space $(X, T)$ is said to be $R_0$ if one of the following equivalent conditions is satisfied. (i) If $U \in T$ and $x \in U$ then $\{x\} \subset U$ or $\{\{x\} : x \in X\}$ is a decomposition of $X$. The $R_0$ axiom was introduced by N. A. Shanin [31]. Later A. S. Davis [16] investigated the $R_0$-axiom and introduced the $R_1$-axiom as follows:

A topological space $(X, T)$ is $R_1$ iff for $x, y \in X$ there exist disjoint open sets...
$U$ and $V$ such that $\overline{U} \subseteq U$ and $\overline{V} \subseteq V$.

$R_0$-spaces have been studied by several Topologists in the different contexts e.g. proximity, uniformity and nearness types of the structures. K. Morita [31] studied it in terms of family of coverings whereas Murdeshwar and Naimpally in terms of quasi-uniformity in [32], [33] and [43]. Further in [44]Naimpally indicated the role played by $R_0$-spaces in the theory of comparison of topologies. Interestingly, this axiom was found compatible with Lodoce proximity ([45] Theo. 19,19)

K.K. Debnath and B.M. Misra [34] have studied these spaces in the bitopological context and some of their basic properties.

The $R_1$-axiom was discovered by A.C. Davis [16] and was found to be independent of $T_0$ and $T_j$-axioms. It is related to the $T_2$-axiom through the relationship $T_2 = R_1 + T_0$. This space was found to be well behaved as it is preserved hereditarily and topologically. Murdeshwar and Naimpally in [35] and [36] studied its properties in the context of para compact spaces and quasi-uniformities. G.D. Richardson [49] proves that in a locally compact $R_1$-space, the closed compact neighbourhoods of each point form a neighbourhood base.
Recently, Charles Dorsett [17] introduced the concept of \( T_0 \)-identification of any topological space \((X, \mathcal{T})\) as the space \((X_0, S_0)\) where \(X_0\) is the quotient set and \(S_0\) is the quotient topology obtained by defining the relation in \((X, \mathcal{T})\) as \(x y \iff \overline{x} = \overline{y}\). He has used \(T_0\)-identification spaces to characterize \(R_c\) and \(R_2\)-axioms. In an other paper [18] he has applied the same idea of \(T_0\)-identification spaces to generalize known results for hyperspaces of \(T_2\) and \(T_2\)-spaces to hyperspaces of \(R_c\) and \(R_2\)-spaces.

(A.2) KC and US spaces

A. Willansky [30] defined KC and US axioms as follows:

A topological space is called a KC-space if every compact set is closed and a US space if every convergent sequence has exactly one limit point to which it converges.

We list some of the results obtained by Willansky.

1) \(T_2 \Rightarrow KC \Rightarrow US \Rightarrow T_2\) but not conversely even if the space is compact.
(ii) For first countable spaces $T_2 \equiv KC \equiv US$

(iii) For locally compact spaces $T_2 \equiv KC$

(iv) Let $X$ be a $KC$-space then $X^+$ (one point compactification of $X$) is $KC$ iff $X$ is a $K$-space.

Garcia Maynez in [21] continued the study of these axioms and found the following results:

(v) If $X \times Y$ is a $KC$ space then $X \times Y$ is a $K$-space. Besides, he has shown the following properties to be equivalent for a topological space $X$.

(a) $X$ is a locally compact $T_2$-space.

(b) For every $KC$, $K$ spaces $Y$, the product $X \times Y$ is a $KC$, $K$ space.

(c) $X \times X$ is $KC$, $K$-space

M. Takashi [36] studied $KC$-spaces under the name of Weak Hausdorff spaces.

A.3 $T_{ES}$ & $T_{EF}$ spaces

KC Morita [37] introduced the separation axioms weaker than $T_1$ - spaces as follows:

A topological space $(X, \tau)$ is a $T_{ES}$-space if each singleton subset is either open or

K spaces have been studied in ([28] page 230)
closed and the topological space \((X, \mathcal{T})\) is
\(T_{EF}\) space if each finite subset is either open
or closed.

MC Sherry has studied their properties and
relationship with the well known separation axioms
due to Aull and Thron \([2]\).

MC Cartan \([38]\) continued the study of \(T_{ES}\)
and \(T_{EF}\) spaces and has characterized minimal
\(T_{ES}\) spaces and minimal \(T_{EF}\) spaces.

A4. Weakly Hausdorff spaces

A topological space is called weakly Hausdorff
if there exists a net \(S\) with \(\lim S = x\) and
\(\lim S = y\) then \(\{x\} \neq \{y\}\). This definition, it is
learnt is due to N. Levine. William Dunham \([39]\)
has shown that this separation axiom is less severe
than \(T_2\) separation and neither implies the other.
Further, he has shown that some of the good features
of Hausdorff spaces and regular spaces are preserved
in weakly Hausdorff spaces. For example, in weakly
Hausdorff spaces, the closures of compact subsets
are compact and the property of being homeomorphic to
a subspace of a compact weakly Hausdorff space
characterizes complete regularity.
N. Levine in [22] defined a subset $A$ to be semi-open if there is an open set $U$ such that $U \cap A = \emptyset$. This concept of semi-open set enabled S. Maheshwari and R. Prasad [33] to define semi $T_0$, semi $T_1$, and semi $T_2$-axioms on the basis of the definitions of the corresponding $T$-type axioms replacing open sets by semi-open sets. In their paper they have established various equivalent formulations and independence of these new axioms.

Further work in this direction was undertaken by N. Takashi [57] who investigated some relations between semi $T_2$ and semi-continuous functions and also proved that the product of any family of semi $T_2$-spaces is also a semi $T_2$-space.

Deploying the same concept of semi-open sets due to N. Levine, Charles Worsatt [19] introduced the concept of semi $R_0$ and semi $R_1$ spaces. In addition, he has established the idea of semi-convergence of a net and has characterized semi $R_0$, semi $R_1$, and semi $T_2$-space through it.
B) Minimal Hausdorff spaces

A topological space \((X, \mathcal{F})\) is said to be minimal Hausdorff if \(\mathcal{F}\) is a Hausdorff topology and there exists no Hausdorff topology on \(X\) strictly weaker than \(\mathcal{F}\). A well-known topological fact is that the topology of a compact Hausdorff space is minimal i.e., it is not strictly finer than any other Hausdorff topology. Katetov [23] was the first to characterize minimal Hausdorff spaces and proved that a Urysohn space is minimal Hausdorff iff it is compact.

B. Banaschewski [6] investigated minimal Hausdorff spaces and proved that the concepts of minimal semi-regular Hausdorff and minimal Hausdorff spaces are equivalent. In another paper [7] Banaschewski proved that a Hausdorff space can be densely embedded in a minimal Hausdorff space iff it is semi-regular.

In [4] N.P. Berri continued the study of such spaces and found an interesting characterization as follows:

A necessary and sufficient condition that a Hausdorff space \((X, \mathcal{F})\) be minimal Hausdorff is that \(\mathcal{F}\) satisfies the following conditions:

1. Every open filter base has an adherent point
2. Semi-regular. A set \(U \subseteq X\) is regular open iff \(U\) meets \(\overline{U}\).

The topology \(\mathcal{T}_S\) generated by the regular open sets is called the semi-regularization of \(\mathcal{I}\) and is coarser than \(\mathcal{I}\). A space \((X, \mathcal{I})\) is semi-regular iff \(\mathcal{I} = \mathcal{T}_S\).
(ii) If an open filter base has a unique adherent point then it converges to that point.

For the purpose of completeness, we list some of the results obtained by Berri [4]:

(iii) If a subspace of a minimal Hausdorff space is both open and closed, then it is minimal Hausdorff.

(iv) Let \( \{ (X_\alpha, J_\alpha) \} \) be a family of non-empty Hausdorff spaces. If the product \( X = \prod X_\alpha \) is minimal Hausdorff, then each factor space is minimal Hausdorff.

Recently, L. Herrington and P.E. Long [23] established a characterization of minimal Hausdorff spaces through functions having strongly closed graphs. Any map \( f: X \to Y \) is said to have strongly closed graph if for \( y = f(x) \) there exist open neighbourhoods \( U \) of \( x \) and \( V \) of \( y \) such that \( (x, y) \in G_f \) where \( G_f \) stands for the graph of \( f \).

For the purpose, they took the class \( S \) of all Hausdorff completely normal and fully normal spaces and proved the following theorem characterizing the minimal Hausdorff spaces:

A topological space \( X \) is minimal Hausdorff iff for each topological space \( X \) belonging to \( S \), each function \( f: X \to Y \) with strongly closed graph, is continuous.

In the similar way, the Joseph, James [34] gave another characterization of minimal Hausdorff spaces as
Let $S$ be the class of topological spaces containing all Hausdorff one point extensions of discrete spaces. Then any Hausdorff topological space $Y$ is minimal Hausdorff iff for each $X \in S$, and bijection $f: X \to Y$ with strongly closed graph, $f$ is weakly continuous.

C. An axiom stronger than $T_2$

A. Hajnal and I. Juhász [23] introduced the notion of strongly Hausdorff spaces as follows:

A $T_2$ space $R$ is called strongly Hausdorff (or briefly $S.H.$) iff the following condition holds:

$$\forall^\forall \exists \forall$$

For any infinite subset $X \subset R$ there exists a sequence $\{U_i\}_{i=1}^{\infty}$ of open subsets of $R$ such that $U_i \cap X \neq \emptyset$ and $U_i \cap U_j = \emptyset$ for $i, j \neq 1, 2, \ldots, i \neq j$

They have compared spaces having this property with $T_2$ spaces on the one hand and Urysohn spaces on the other where by Urysohn space is meant a space in which two distinct points possess disjoint closed neighbourhoods.

Hajnal and Juhász [23] obtained the following chain of implications where arrows cannot be reversed

$$T_3 \Rightarrow U \Rightarrow S.H. \Rightarrow T_2$$

Any map $f: X \to Y$ is said to be weakly continuous if for each point $x \in X$ and an open set $V$ containing $f(x)$ there is an open set $U$ containing $x$ such that $f(U) \subseteq \overline{V}$.
J.R. Porter \( (46) \) continued the study of such strongly Hausdorff spaces and obtained the following theorems:

(1) A strongly Hausdorff space can be embedded as a closed nowhere dense subset of a minimal strongly Hausdorff space.

(2) A strongly Hausdorff space can be densely embedded in a strongly Hausdorff closed space.

D. An axiom independent of \( T_2 \)

A topological space \( X \) is said to be \( T_2 \)-iff it is \( T_1 \) and satisfies the property that if \( x \) and \( y \) are points of \( A \) (derived set of \( A \)), there exist a subset \( B \) of \( A \) such that \( x \in B \) and \( y \notin B \).

This space was found to be independent of various known separation axioms including \( T_2 \). The relation between \( T_2 \) and \( T_2 \) is given by a theorem of C.H. All in \( [1] \) which states that a \( T_2 \)-space is \( T_2 \)-iff every finer topology is \( T_2 \). All has also studied in \( [1] \) minimal \( T_2 \)-spaces.

Continuing the study of \( T_2 \)-spaces, K. Czetsar in \( [10] \) proved an interesting product theorem that a \( T_2 \)-space \( X \) is \( T_2 \)-iff \( X \times X \) is \( T_2 \).

D. A space is strongly Hausdorff closed iff it is strongly Hausdorff and \( R \)-closed (Recall that \( X \) is said to be \( R \)-closed if \( X \) is a closed subspace in every \( R \)-space in which it is embedded.) The term Hausdorff closed is usually shortened to \( R \)-closed.
3.1 Minimal and strongly minimal regular spaces

In a survey article [4] on minimal topological spaces, M.F. Berri, J.R. Porter and R.H. Stephenson have given the systematic development of the theory of minimal topological spaces till 1966. This section will include only some of the recent interesting developments in this direction.

Given a set $X$ let $P(X)$ denotes the set of all topologies on the set $X$ partially ordered by inclusion. A topological space $(X, \mathcal{T})$ is said to be minimal provided that $\mathcal{T}$ is a minimal element in $P(X)$.

M.P. Berri and R.H. Sorgenfrey [5] introduced the concept of minimal regular spaces by saying that regular topological space $(X, \mathcal{T})$ will be called minimal regular if there exists no regular topology on $X$ which is strictly weaker than $\mathcal{T}$.

The concept of strongly minimal regular space was later introduced by R.H. Stephenson [32]. A regular $T_\Sigma$ space $X$ is called strongly minimal if there is a base $\mathcal{U}$ for the topology on $X$ such that for each $V$ in $\mathcal{U}$, $X - V$ is an $R$-closed subspace of $X$. It may be mentioned that a regular $T_\Sigma$ space is said to be $R$-closed iff there is no regular $T_\Sigma$-space in which it can be embedded as a non-closed subspace. Further, he observed that every strongly minimal regular space is minimal regular.
The converse of this result was settled by an example given by D.H. Petry [43]. He showed that minimal regular space need not be strongly minimal. Further Stephenson in the paper referred to above proved that the product of a strongly minimal regular space and a compact $T_\omega$-space is always strongly minimal regular.

M.K. Singal and Asha Mathur [38] have characterised minimal almost regular spaces in terms of filters and covers. Earlier the notion of almost regular space had also been introduced by M.K. Singal and S.P. Arya [34]

\section*{4.2 s-regular spaces}

The concept of semi-open sets of Levine [29] led S.K. Maheshwari and R. Prasad to introduce in [46] a new regularity axiom called as s-regular and defined as follows:

A topological space $X$ is said to be s-regular iff for each closed set $F$ and a point $x \notin F$, $F$ and $x$ are contained in disjoint semi-open sets.

In their paper they have obtained some equivalent reformulations and showed that s-regularity is inherited by open subsets (considered as subspaces) in a s-regular spaces.

T. Neiri [42] continued the study of s-regular spaces and obtained some simple properties of such spaces.
P.1 k-normal spaces

E.V. Ščepkin [38] introduced a new notion of k-normality as follows:

A regular space X is k-normal iff two disjoint closures of open sets can be separated by disjoint open sets.

Her main results are the following:

(i) A regular space is k-normal iff every real-valued bounded continuous function defined on the closures of an open set can be continuously extended over X.

(ii) If either X is k-normal and locally connected or the cartesian product XX is k-normal then every real-valued continuous function defined on closure of open sets can be continuously extended over X.

P.2 Continuously perfectly normal (CPN), continuously normal (CN) and continuously completely regular spaces (CCR)

Let X be any topological space and \( \mathcal{F} \) be the space of closed subsets of X with finite topology generated by the family \( \{ RU : U \text{ is a finite collection of open subsets of } X \} \) where each \( RU = \{ F \in \mathcal{F} : F \subset U \} \) and F intersects each member of \( U \). Let \( CX \) denote \( \mathcal{F}X \).

Finite topology is often called Vietoris topology or the exponential topology. Good study of these topologies can be found in [27] and [41].
the space of continuous non-negative real valued functions defined on $X$ with compact open topology.

P. Zenor \cite{62} showed that a $T_2$-space $X$ is metrizable iff there is a continuous function $\lambda : \mathcal{F}X \rightarrow CX$ such that

(a) If $H \in \mathcal{F}X$, then $H = \{ x : \lambda (H)(x) = 0 \}$ and

(b) If $K$ is a finite subset of $X$ and if $x \in K$, then

\[
(\lambda (\{x\})(y) > (\lambda (K))(y)) \text{ for all } y \in X
\]

Zenor's attempt to check whether (b) part of this theorem could be removed, led to the notions of CPH, CN and CCR axioms in \cite{63}.

Take $\mathcal{F}X$ as the space $\{(H, K) \in \mathcal{F}X \times \mathcal{F}X : H \cap K = \emptyset \}$ and $UX$ as the space $\{(x, K) \in X \times \mathcal{F}X : x \notin K \}$.

Then,

1. A function $\phi : X \times \mathcal{F}X \rightarrow [0, 1]$ is called perfectly normality operator (PM-operator) if for each $H \in \mathcal{F}X$ it is true that $H = \{ x : \phi(x, H) = 0 \}$.

2. A function $\phi : X \times UX \rightarrow [0, 1]$ is a normality operator (N-operator) if it is true that

\[
(H, K) \in \mathcal{F}X \quad \text{then} \quad H \subseteq \{ x \in X : \phi(x, (H, K)) = 1 \}
\]

and $K \subseteq \{ x \in X : \phi(x, (H, K)) = 0 \}$.

3. A function $\phi : X \times UX \rightarrow [0, 1]$ is a complete regularity operator (CR-operator) if for each

\[
(x, H) \in UX, \quad \phi (x, (x, H)) = 0
\]

and $H \subseteq \{ y \in X : \phi(y, (x, H)) = 1 \}$.
We can now introduce the definitions of above mentioned spaces as follows:

A topological space $X$ is said to be CPN or CN or CCR spaces iff $X$ admits respectively a continuous perfect normality or a continuous normality or a continuous complete regularity operator.

P. Zenor [63] found the following chain of implications:

$X$ is CPN $\Rightarrow$ CN $\Rightarrow$ CCR.

In reverse situations, he showed by an example that CCR space need not be CN but the possibilities in the other reverse situation remain unanswered up to now to the best of my knowledge.

F.3 Monotonically normal spaces

A $T_L$ -space $X$ is said to be monotonically normal if corresponding to each pair $(A, B)$ of disjoint closed subsets of $X$ there is an open subset $G(A, B)$ such that

(i) $A \subset G(A, B) \subset G(A, B) \subset XB$

(ii) $G(A, B) \subset G(A, B)$ whenever $A \subset A_L$ and $B \subset B_L$

As an equivalent reformulation, Borges [3] observed that the space $X$ remains monotonically normal even if the above conditions (i) and (ii) are true for each pair of sets $(A, B)$ such that $A \cap \overline{B} = \phi = \overline{A} \cap B$
In an attempt to characterize monotonically normal spaces, Borges [3] has established the following interesting results:

X is monotonically normal iff for each open set $U \subseteq X$ and $x \in U$ there exists an open neighbourhood $U_x$ of $x$ such that $U_x \cap V_y \neq \emptyset$ implies that $x \in V$ or $y \in U$. 
Part II

1. A unified approach to the study of separation axioms

Several separation axioms, defined in terms of continuous functions were examined by Van Est and Freudenthal [63]. Since then a number of new separation properties were defined by Aull and Thron [5] and others which made Prof. Thron raise the question "what is a separation axiom?"

T. Crone [3] gave a unified approach for introducing several standard separation axioms. All the S-type separation axioms, axioms of Aull and Thron [5] and of Robinson and Wu [65] fall within the scope of this definition.

We mention here briefly his method for defining separation axioms.

Let \( A \) and \( B \) be the classes of subsets of topological spaces and \( \mathcal{X} \) a class of topological spaces with a distinguished pair of subsets \( A_x \) and \( B_x \) of each member \( x \in \mathcal{X} \). \( T(A, B; \mathcal{X}) \) is defined to be the class of topological spaces \( Y \) such that for every disjoint pair of non-empty subsets \( A \) and \( B \) of \( Y \) with \( A \in \mathcal{A} \) and \( B \in \mathcal{B} \) there is a continuous map \( f : Y \to X \) with \( f(A) = A_x \) and \( f(B) = B_x \) for some \( x \in \mathcal{X} \).
A separation axiom is then defined to be a class $T (A,B,X)$ or the intersection of such classes. By judiciously choosing $A,B$ and $X$, it is shown that the standard axioms $T_0, T_1, T_2$, regularity, complete regularity, and normality fit into his definition as well as other separation properties such as those of S. M. Robinson and Y. C. Wu [50] and of Aull and Thron [2]

However, this approach does not cover all the separation axioms. For instance, Cramer [3] has shown that strong $T_0$-axiom can not be so represented.

2. Theory of Frames

A frame is a complete lattice satisfying the infinite distributive law $a \land \lor \{b_i\} = \lor \{a \land b_i\}$ If $(X, \tau)$ is a topological space with topology $\tau$, then $\tau$ ordered by inclusion is an example of a frame. A frame map $f: L \rightarrow M$ is a function from a frame $L$ to a frame $M$ which commutes with finite meets and arbitrary joins.

In [15] Dowker and Strauss show how some of the separations can be stated in terms of frames in such a way that they reduce to the usual separation axioms when applied to the topological space situation.

For example, corresponding to normality axiom Dowker and Strauss [15] defined separation axiom $S_4$ for the frame $L$ as follows:
If \(a, b \in L\) and \(a \vee b = 1\), then there exist \(c, d\) in \(L\) such that \(a \wedge d = 1\), \(c \vee b = 1\) and \(c \wedge d = 0\).

Similarly, corresponding to \(T_2\)-axiom, the frame axiom \(T'_2\) is as follows:
If \(a \leq b\) there exist an \(c\) such that \(c \vee b \neq 1\) and \(c \vee d = 1\).

Corresponding to \(T_2\) is the frame axiom \(S_2\) as follows:
If \(a\) is an element of a frame \(L\) such that \(a \neq 1\) there is a quotient map \(f: L \rightarrow N\) with \(f(a) = 0\), \(f(1) \neq 0\) such that if \(a \vee b = 1\) with \(b \neq 1\), then there are \(c, d\) in \(L\) such that \(c \wedge d = 0\), \(f(d) = 1\) and \(c \neq b\).
1. Some localised separation axioms

The idea of localization used by Jane Chow in the context of paracompact and fully normal spaces led K.K. Dube and S.K. Misra to introduce localised formulations of the standard separation axioms $T_o$, $T_1$, and $T_2$.

(i) In topological space $(X, \mathcal{T})$, a point $x$ in $X$ is said to be a $T_o$-distinct point if for any $y \in X$ with $y \neq x$ there exists $U \in \mathcal{T}$ such that either $x \notin U$, $y \notin U$ or $y \in U$, $x \notin U$. The notions of $T_1$ and $T_2$-distinct points are defined similarly.

(ii) In a topological space $(X, \mathcal{T})$, a set $A \subseteq X$ is said to be $T_2$-distinct point set if each $x \in A$ is a $T_2$-distinct point for $1 \neq 2$ and 2.

They have discussed results leading to some variants of regularity and normality properties. For example, if $(X, \mathcal{T})$ is paracompact (with no separation axioms assumed) and if $x$ is a $T_2$-distinct point not belonging to the closed set $F$, then $x$ and $F$ have disjoint neighbourhoods.

A similar result holds for two disjoint closed sets, one of which consists of $T_2$-distinct points.
Furthermore, they have characterized $\mathcal{R}_D$-spaces as follows:

A topological space $(X, \mathcal{T})$ is an $\mathcal{R}_D$-space if for each $T_0$-distinct point $x \in X$, $\overline{\{x\}} \setminus \bigcup \{ Y : x \in \overline{Y} \} = \{x\}$.

Earlier $\mathcal{R}_D$-spaces were introduced and studied by the authors in [13]. In continuation to the study of $\mathcal{R}_D$-axioms, they have established the equivalence of the following statements in a topological space $(X, \mathcal{T})$:

(i) $(X, \mathcal{T})$ is an $\mathcal{R}_D$-space.

(ii) For any $T_0$-distinct point set $A$, the derived set $A'$ is closed.

(iii) For any $T_0$-distinct point $x$, there exist open set $G$ and closed set $F$ such that $\{x\} = F \cap G$.

They have also discussed some results on $T_2$-distinct points which show that such points are worthy of study. For example,

(iv) In a topological space $(X, \mathcal{T})$ if an almost compact set and a $T_2$-distinct point set form a partition of $X$, then the almost compact set is closed.

(v) If $f$ and $g$ are continuous functions from any topological space $X$ to a topological space $Y$ and if for each $x \in X$, $f(x) \neq g(x)$ implies $f(x)$ or $g(x)$ is a $T_2$-distinct point in $Y$, then the set $A = \{x : f(x) = g(x)\}$ is closed in $X$. 
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