CHAPTER-VII

ANALYSIS OF A TWO-UNIT COLD STANDBY SYSTEM WITH THREE STAGES OF OPERATION AND PRIORITY FOR OPERATION AT SECOND STAGE
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Analysis of a Two-Unit Cold Standby System with Three Stages of Operation and Priority for Operation at Second Stage

In the previous chapter, a two-unit cold standby system having three operational stages of the units i.e. proper installation or burn in period, useful life period and wear-out period and FCFS pattern for repair has been analysed. In the system the failure rate of the unit operating during its second stage, i.e. during useful life period, is lower in comparison to the failure rate at the other two operational stages. It, therefore, seems logically better to give priority to a unit operating in second stage over the unit working in any other stage.

Thus, the present chapter deals with a two-unit cold standby sophisticated system wherein each unit have three operational stages, i.e. burn-in period (Stage-I), useful life period (Stage-II) and wear out period (Stage-III) with priority for operation is given to a unit operating at its second stage. Other assumptions are same as taken in the previous chapter.

The system is analysed by using the semi-Markov processes and regenerative point technique and the following measures of the system effectiveness are obtained

- Mean Time to System Failure
- Steady-State Availability
- Busy period of the repairman
- Expected number of repairs
- Expected number of replacements
- Expected number of visits of the repairman

The expected profit incurred to the system is obtained and various conclusions regarding the system have been drawn on the basis of graphical study.

Notations and States of the System

\( \lambda_1/\lambda_2/\lambda_3 \) : failure rate of the unit during Stage-I/Stage-II/ Stage-III
Transition Probabilities and Mean Sojourn Times

A state transition diagram in fig. 7.1 shows the various states of transition of the system. The epochs of entry into states 0, 1, 3, 4, 6, 7, 8, 9, 11 and 14 are regenerative points and thus these are regenerative states. The states 2, 5, 10, 12, 13 and 15 are failed states and these are non-regenerative states. The states 1, 4, 6, 9, 11 and 14 are down states. The transition probabilities are given by

\begin{align*}
q_{01}(t) &= \lambda_1 e^{-(\lambda_1 + \eta_1)t}; & q_{03}(t) &= \eta_1 e^{-(\lambda_1 + \eta_1)t}; \\
q_{10}(t) &= e^{-\lambda_1(t)} g_1(t); & q_{11}^{(2)}(t) &= [\lambda_1 e^{-\lambda_1 t} \otimes 1] g_1(t); \\
q_{12}(t) &= \lambda_1 e^{-\lambda_1 t} \overline{G_1(t)}; & q_{34} &= \lambda_2 e^{-(\lambda_2 + \eta_2)t};
\end{align*}
\[ q_{38} = \eta_2 e^{-(\lambda_2 + \eta_2) t}; \]
\[ q_{45} (t) = \lambda_1 e^{-\lambda_1 t} G_2 (t); \]
\[ q_{63} (t) = e^{-\lambda_3 t} g_1 (t); \]
\[ q_{76} (t) = g_1 (t); \]
\[ q_{9, 11} (t) = \lambda_1 e^{-\lambda_1 t} G_3 (t); \]
\[ q_{11, 8} (t) = e^{-\lambda_3 t} g_1 (t); \]
\[ q_{11, 12} (t) = p \lambda_3 e^{-\lambda_3 t} G_1 (t); \]
\[ q_{11, 14} (t) = [q \lambda_3 e^{-q \lambda_3 t} \otimes 1] g_1 (t); \]
\[ q_{14, 11} (t) = [\lambda_1 e^{-\lambda_1 t} \otimes 1] h (t); \]
\[ q_{14, 13} (t) = \lambda_1 e^{-\lambda_1 t} H(t); \]

The non-zero elements \( p_{ij} \) obtained as \( p_{ij} = \lim_{s \to 0} q_{ij} \) and are given by

\[ p_{01} = \frac{\lambda_1}{D_0}; \]
\[ p_{03} = \frac{\eta_1}{D_0}; \]
\[ p_{10} = g_1 (\lambda_1); \]
\[ p_{1, 1} = \frac{\lambda_1}{D_1}; \]
\[ p_{12} = 1 - g_1 (\lambda_1); \]
\[ p_{34} = \frac{\lambda_2}{D_1}; \]
\[ p_{38} = \frac{\eta_2}{D_1}; \]
\[ p_{43} = g_2 (\lambda_1); \]
\[ p_{45} = p_{46} = 1 - g_2 (\lambda_1); \]
\[ p_{63} = g_1 (\lambda_2); \]
\[ p_{67} = 1 - g_1 (\lambda_2); \]
\[ p_{76} = 1; \]
\[ p_{89} = p; \]
\[ p_{8, 14} = q; \]
\[ p_{98} = g_3 (\lambda_1); \]
\[ p_{9, 10} = p_{9, 11} = 1 - g_3 (\lambda_1); \]
\[ p_{11, 8} = g_1 (\lambda_3); \]
\[ p_{11, 9} = p_{11, 12} = p [1 - g_1 (\lambda_3)]; \]
\[ p_{11, 13} = p_{11, 14} = q [1 - g_1 (\lambda_3)]; \]
\[ p_{14, 0} = h (\lambda_4); \]
\[ p_{14, 15} = 1 - h (\lambda_4); \]

where

\[ D_0 = \lambda_1 + \eta_1 \] and \( D_1 = \lambda_2 + \eta_2 \)
By these transition probabilities, it can be verified that
\[ p_{01} + p_{03} = p_{10} + p_{11} = p_{10} + p_{12} = p_{34} + p_{38} = p_{43} + p_{45} = p_{43} + p_{46} = p_{46} \]
\[ = p_{63} + p_{67} = p_{76} + p_{89} + p_{9,14} = p_{9,10} + p_{9,11} = p_{11,8} + p_{11,12} + p_{11,13} \]
\[ = p_{11,8} + p_{11,9} + p_{11,14} = p_{14,0} + p_{14,1} = p_{14,0} + p_{14,1} = 1 \]

The mean sojourn times (\( \mu_i \)) in state ‘i’ are:
\[ \mu_0 = \frac{1}{D_0} ; \quad \mu_1 = \frac{1}{\lambda_1} [1 - g_1^*(\lambda_1)] ; \quad \mu_3 = \frac{1}{D_1} ; \]
\[ \mu_4 = \frac{1}{\lambda_1} (1 - g_2^*(\lambda_1)) ; \quad \mu_6 = \frac{1}{\lambda_2} [1 - g_1^*(\lambda_2)] ; \quad \mu_8 = \frac{1}{\lambda_3} ; \]
\[ \mu_0 = \frac{1}{\lambda_1} [1 - g_3^* (\lambda_1)]; \quad \mu_{11} = \frac{1}{\lambda_3} [1 - g_1^* (\lambda_3)]; \quad \mu_{14} = \frac{1}{\lambda_1} [1 - h^* (\lambda_1)]; \]

The unconditional mean time taken by the system to transit for any state, ‘j’ when it is counted from epoch of entrance into state ‘i’ is mathematically stated as:

\[ m_{ij} = \int_0^\infty t \ q_{ij}(t) dt = -q_{ij}^* (0) \]

Thus,

\[ m_{01} + m_{03} = \mu_0; \quad m_{10} + m_{11} = m_{11,8} + m_{11,9} + m_{11,14} = \int_0^\infty G_1(t) dt = k_1 \text{ (say)}; \]

\[ m_{10} + m_{12} = \mu_1; \quad m_{34} + m_{38} = \mu_3; \quad m_{43} + m_{45} = \mu_4; \]

\[ m_{43} + m_{45} = \int_0^\infty G_2(t) dt = k_2 \text{ (say)}; \quad m_{63} + m_{67} = \mu_6; \]

\[ m_{76} = \mu_7; \quad m_{89} + m_{8,14} = \mu_8; \quad m_{98} + m_{9,10} = \mu_9; \]

\[ m_{98} + m_{9,11} = \int_0^\infty G_3(t) dt = k_3 \text{ (say)}; \quad m_{11,8} + m_{11,12} + m_{11,13} = \mu_{11}; \]

\[ m_{14,0} + m_{14,15} = \int_0^\infty H(t) dt = k_4 \text{ (say)}; \quad m_{14,0} + m_{14,15} = \mu_{14}; \]

**Mean Time to System Failure**

Using the probabilistic arguments for regenerative process we obtain the following recursive relations for \( \phi_i(t) \):

\[ \phi_0(t) = Q_{01}(t) \ \& \ \phi_1(t) + Q_{03}(t) \ \& \ \phi_3(t); \quad \phi_1(t) = Q_{10}(t) \ \& \ \phi_0(t) + Q_{12}(t); \]

\[ \phi_3(t) = Q_{3,4}(t) \ \& \ \phi_4(t) + Q_{38}(t) \ \& \ \phi_8(t); \quad \phi_4(t) = Q_{43}(t) \ \& \ \phi_3(t) + \phi_{45}(t); \]

\[ \phi_8(t) = Q_{89}(t) \ \& \ \phi_9(t) + Q_{8,14}(t) \ \& \ \phi_{14}(t); \quad \phi_9(t) = Q_{9,8}(t) \ \& \ \phi_8(t) + Q_{9,10}(t); \]

\[ \phi_{14}(t) = Q_{14,0}(t) \ \& \ \phi_0(t) + Q_{14,15}(t); \]

Taking the L.S.T. of the above relations and then solving them for \( \phi_0^{**}(s) \), we get

\[ \phi_0^{**}(s) = \frac{N_0(s)}{D_2(s)} \]
where

\[ N_0(s) = Q_{01}^{**}(s)Q_{12}^{**}(s)[1 - Q_{89}^{**}(s)Q_{98}^{**}(s)][1 - Q_{34}^{**}(s)Q_{43}^{**}(s)] + Q_{03}^{**}(s) \]

\[ Q_{34}^{**}(s)Q_{45}^{**}(s)[1 - Q_{89}^{**}(s)Q_{98}^{**}(s)] + Q_{03}^{**}(s)Q_{38}^{**}(s)[Q_{89}^{**}(s)Q_{8,10}^{**}(s) \]

\[ + Q_{8,14}^{**}(s)Q_{14,15}^{**}(s)] \]

\[ D_2(s) = [1 - Q_{01}^{**}(s)Q_{10}^{**}(s)][1 - Q_{89}^{**}(s)Q_{98}^{**}(s)][1 - Q_{34}^{**}(s)Q_{13}^{**}(s)] - Q_{03}^{**}(s) \]

\[ Q_{38}^{**}(s)Q_{8,14}^{**}(s)Q_{14,0}^{**}(s) \]

In the steady-state, mean time to system failure is given by:

\[ T_0 = \lim_{s \to 0} \frac{[1 - \phi_0^{**}(s)]}{s} = \frac{N_0}{D_2} \]

where

\[ N_0 = [1 - p_{89} p_{98}] [\mu_0 - \mu_0 p_{34} p_{43} + \mu_1 p_{01} - \mu_1 p_{01} p_{34} p_{43} + \mu_3 p_{03} + \mu_4 p_{03} p_{34}] + \]

\[ \mu_8 p_{03} p_{38} + \mu_9 p_{03} p_{89} + \mu_{14} p_{03} p_{38} + \mu_{8,14} \]

\[ D_2 = [1 - p_{01} p_{10}] [1 - p_{34} p_{43}] [1 - p_{89} p_{98}] - p_{03} p_{38} p_{8,14} p_{14,0} \]

**Availability Analysis**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( A_i(t) \):

\[ A_0(t) = M_0(t) + q_{01}(t) \odot A_1(t) + q_{03}(t) \odot A_3(t) \]

\[ A_1(t) = q_{10}(t) \odot A_0(t) + q_{11}^{(2)}(t) \odot A_1(t) \]

\[ A_3(t) = M_3(t) + q_{34}(t) \odot A_4(t) + q_{38}(t) \odot A_8(t) \]

\[ A_4(t) = q_{43}(t) \odot A_3(t) + q_{46}^{(15)}(t) \odot A_6(t) \]

\[ A_6(t) = q_{63}(t) \odot A_3(t) + q_{67}(t) \odot A_7(t) \]

\[ A_7(t) = q_{76}(t) \odot A_6(t) \]

\[ A_8(t) = M_8(t) + q_{89}(t) \odot A_9(t) + Q_{8,14}(t) \odot A_{14}(t) \]

\[ A_9(t) = q_{98}(t) \odot A_8(t) + q_{9,11}^{(10)}(t) \odot A_{11}(t) \]
\[ A_{11} (t) = q_{11,8}(t) \odot A_8 (t) + q_{11,9}^{(12)}(t) \odot A_9 (t) + q_{11,14}^{(13)}(t) \odot A_{14} (t) \]

\[ A_{14} (t) = q_{14,0}(t) \odot A_0 (t) + q_{14,1}^{(15)}(t) \odot A_1 (t) \]

where

\[ M_0 (t) = e^{-(\lambda_1 + \eta_1) t}; \quad M_3 (t) = e^{-(\lambda_2 + \eta_2) t}; \quad M_8 (t) = e^{-\lambda_3 t}; \]

Taking the L.T. of the above relations and then solving them for \( A_0^*(s) \), we get

\[ A_0^*(s) = \frac{N_1(s)}{D_3(s)} \]

where

\[ N_1(s) = [1 - q_{11}^{(2)*}(s)][1 - q_{9,11}^{(10)*}(s)q_{11,9}^{(12)*}(s) - q_{89}^*(s)q_{98}^*(s) - q_{89}^*(s)q_{9,11}^{(10)*}(s)q_{11,9}^{(12)*}(s)] q_{11,8}^*(s) M_0^*(s) [1 - q_{34}^*(s)q_{43}^*(s)][1 - q_{67}^*(s)q_{76}^*(s)] - M_0^*(s) q_{34}^*(s)q_{46}^*(s) q_{63}^*(s) + M_3^*(s)q_{03}^*(s) - M_8^*(s)q_{03}^*(s)q_{38}^*(s) (1 - q_{11}^{(2)*}(s))[1 - q_{67}^*(s)q_{76}^*(s)][1 - q_{9,11}^{(10)*}(s)q_{11,9}^{(12)*}(s)] \]

\[ D_3(s) = [1 - q_{11}^{(2)*}(s) - q_{01}^*(s)q_{10}^*(s)][1 - q_{89}^*(s)q_{98}^*(s) - q_{9,11}^{(10)*}(s)q_{11,9}^{(12)*}(s)] - q_{89}^*(s)q_{9,11}^{(10)*}(s)q_{11,8}^*(s) [1 - q_{34}^*(s)q_{43}^*(s)] (1 - q_{67}^*(s)q_{76}^*(s)) - q_{34}^*(s)q_{46}^*(s) q_{63}^*(s) + q_{03}^*(s)q_{38}^*(s) [1 - q_{11}^{(2)*}(s)] q_{14,0}^*(s) + q_{10}^*(s)q_{14,1}^{(15)*}(s) [1 - q_{67}^*(s)q_{76}^*(s)] q_{8,14}^*(s)q_{9,11}^{(10)*}(s)q_{11,9}^{(12)*}(s) - q_{89}^*(s)q_{9,11}^{(10)*}(s)q_{11,14}^*(s) - q_{8,14}^*(s) \]

In the steady-state, the availability of the system is given by:

\[ A_0 = \lim_{s \to 0} [s A_0^*(s)] = \frac{N_1}{D_3} \]
where

\[
N_1 = \left[ \mu_0 p_{38} p_{63} + \mu_3 p_{03} \right] P_{10} \left[ 1 - p_{89} p_{98} - p_{89} p^{(10)}_{9,11} p_{11,8} - p^{(10)}_{9,11} p^{(12)}_{11,9} \right] - \mu_8 p_{03} p_{10} p_{38} p_{63} \left[ 1 - p^{(10)}_{9,11} p^{(12)}_{11,9} \right]
\]

\[
D_3 = \left[ k_1 p_{38} p_{63} + \left( \mu_6 + \mu_7 p_{43} \right) p_{10} \left( 1 - p_{34} p_{43} \right) + k_2 p_{03} p_{10} p_{63} \right] \left[ 1 - p_{89} p_{98} - p^{(10)}_{9,11} p^{(12)}_{11,9} - p^{(10)}_{9,11} p^{(13)}_{11,14} - p_{10} \right] \left[ 1 - p^{(10)}_{9,11} p^{(12)}_{11,9} - p^{(10)}_{9,11} p^{(13)}_{11,14} - p_{10} \right] \left[ 1 - p^{(10)}_{9,11} p^{(12)}_{11,9} - p^{(10)}_{9,11} p^{(13)}_{11,14} - p_{10} \right] \left[ 1 - p^{(10)}_{9,11} p^{(12)}_{11,9} - p^{(10)}_{9,11} p^{(13)}_{11,14} - p_{10} \right]
\]

**Expected Number of Repairs**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( R_i(t) \):

\[
R_0(t) = Q_{01}(t) \otimes R_1(t) + Q_{03}(t) \otimes R_3(t)
\]

\[
R_1(t) = Q_{10}(t) \otimes [1 + R_0(t)] + Q^{(2)}_{11}(t) \otimes [1 + R_1(t)]
\]

\[
R_3(t) = Q_{34}(t) \otimes R_4(t) + Q_{38}(t) \otimes R_8(t)
\]

\[
R_4(t) = Q_{43}(t) \otimes [1 + R_3(t)] + Q^{(5)}_{46}(t) \otimes [1 + R_6(t)]
\]

\[
R_6(t) = Q_{63}(t) \otimes [1 + R_5(t)] + Q_{67}(t) \otimes [1 + R_7(t)]
\]

\[
R_7(t) = Q_{76}(t) \otimes [1 + R_6(t)]
\]

\[
R_8(t) = Q_{89}(t) \otimes R_9(t) + Q_{8,14}(t) \otimes R_{14}(t)
\]

\[
R_9(t) = Q_{98}(t) \otimes [1 + R_8(t)] + Q^{(10)}_{9,11}(t) \otimes [1 + R_{11}(t)]
\]

\[
R_{11}(t) = Q_{11,8}(t) \otimes [1 + R_8(t)] + Q^{(12)}_{11,9}(t) \otimes [1 + R_9(t)] + Q^{(13)}_{11,14}(t) \otimes R_{14}(t)
\]

\[
R_{14}(t) = Q_{14,0}(t) \otimes R_0(t) + Q^{(15)}_{14,1}(t) \otimes R_1(t)
\]

Taking the L.S.T. of the above relations and then solving them for \( R_0^{**}(s) \), we get

\[
R_0^{**}(s) = \frac{N_3(s)}{D_3(s)}
\]
where

\[ N_3(s) = [Q^*_0(s)Q^*_4(s)(1 - Q^{2*}_{11}(s))Q^*_3(s) + Q^*_6(s) - Q^*_0(s)Q^*_4(s)Q^*_3(s) - Q^{10*}_{11}(s)](1 - Q^{12*}_{11}(s)) \\
- Q^{8*}_{89}(s)Q^{9*}_{98}(s) - Q^{8*}_{89}(s)Q^{9*}_{8,11}(s)Q^{10*}_{9,11}(s) + Q^*_0(s)[Q^*_8(s) \\
+ Q^{10*}_{11}(s)][1 - Q^{10*}_{34}(s)Q^{13*}_{46}(s)Q^{14*}_{63}(s) - Q^{8*}_{11}(s)Q^{10*}_{76}(s)][1 - \\
Q^{10*}_{9,11}(s)Q^{12*}_{11,9}(s) - Q^{8*}_{89}(s)Q^{9*}_{98}(s) - Q^{8*}_{89}(s)Q^{11*}_{11,8}(s)Q^{10*}_{9,11}(s)] \\
+ Q^*_3(s)Q^{8*}_{38}(s)Q^{15*}_{14,1}(s)[1 - Q^*_1(s)]Q^{10*}_{11}(s) - Q^{8*}_{11}(s)Q^{10*}_{11,9}(s)] \\
[ Q^{8,14}_8(s) + Q^{8*}_{89}(s)Q^{10*}_{9,11}(s)Q^{13*}_{11,14}(s) - Q^{8*}_{89}(s)Q^{12*}_{9,11}(s)] \\
+ Q^*_3(s)Q^*_4(s)Q^{15*}_{14,1}(s)(1 - Q^{12*}_{11}(s))][Q^*_3(s) + Q^{12*}_{11}(s) + Q^*_9(s) \\
Q^{10*}_{76}(s)[1 - Q^{10*}_{9,11}(s)Q^{12*}_{11,9}(s) - Q^{8*}_{89}(s)Q^{9*}_{98}(s) - Q^{8*}_{89}(s)Q^{11*}_{11,8}(s) \\
Q^{10*}_{9,11}(s)] + Q^*_3(s)Q^*_3(s)Q^*_8(s)[1 - Q^{12*}_{11}(s)][1 - Q^*_76(s) \\
Q^{10*}_{9,11}(s) + Q^{10*}_{9,11}(s)Q^{10*}_{11,8}(s) + Q^{10*}_{9,11}(s) \\
Q^{12*}_{11,9}(s)] \\
\]

and \( D_3(s) \) is already specified.

In the steady-state, the total fraction of time for which the system is under repair is given by:

\[ R_0 = \lim_{s \to 0} sR_0''(s) = \frac{N_3}{D_3} \]

where

\[ N_3 = p_{03} [p_{10} p_{34} + p_{01} p_{38}] [p_{88} p_{8,14} + p^{(10)}_{9,11} p^{(13)}_{11,14}] + p_{03} p_{38} p_{63} p^{(15)}_{14,1} [p_{8,14} p_{98} + \\
p^{(10)}_{9,11} p^{(13)}_{11,14} + p_{8,14} p^{(10)}_{9,11} p^{(13)}_{11,14}] + p_{03} p_{10} p_{34} p^{(15)}_{46} p_{63} [1 - p_{89} p_{98} - p^{(10)}_{9,11} p^{(13)}_{11,14} \\
- p_{91} p^{(12)}_{11,9}] + \text{other terms} \]

and \( D_3 \) is already specified.
Expected Number of Replacements

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for $R_{p_i}(t)$:

\[
\begin{align*}
R_{p_0}(t) &= Q_{01}(t) \cdot R_{p_1}(t) + Q_{03}(t) \cdot R_{p_3}(t) \\
R_{p_1}(t) &= Q_{10}(t) \cdot R_{p_0}(t) + Q_{11}(t) \cdot R_{p_1}(t) \\
R_{p_3}(t) &= Q_{34}(t) \cdot R_{p_4}(t) + Q_{38}(t) \cdot R_{p_8}(t) \\
R_{p_4}(t) &= Q_{43}(t) \cdot R_{p_3}(t) + Q_{46}(t) \cdot R_{p_6}(t) \\
R_{p_6}(t) &= Q_{63}(t) \cdot R_{p_3}(t) + Q_{67}(t) \cdot R_{p_7}(t) \\
R_{p_7}(t) &= Q_{76}(t) \cdot R_{p_6}(t) \\
R_{p_8}(t) &= Q_{89}(t) \cdot R_{p_9}(t) + Q_{8,14}(t) \cdot R_{p_{14}}(t) \\
R_{p_9}(t) &= Q_{98}(t) \cdot R_{p_8}(t) + Q_{9,11}(t) \cdot R_{p_{11}}(t) \\
R_{p_{11}}(t) &= Q_{11,8}(t) \cdot R_{p_8}(t) + Q_{11,9}(t) \cdot R_{p_9}(t) + Q_{11,14}(t) \cdot [1 + R_{p_{14}}(t)] \\
R_{p_{14}}(t) &= Q_{14,6}(t) \cdot R_{p_6}(t) + Q_{14,11}(t) \cdot [1 + R_{p_1}(t)]
\end{align*}
\]

Taking the L.S.T. of the above relations and then solving them for $R_{p_0}^{**}(s)$, we get

\[
R_{p_0}^{**}(s) = \frac{N_4(s)}{D_3(s)}
\]

where

\[
N_4(s) = Q_{03}^{**}(s)Q_{38}^{**}(s)[1 - Q_{11}^{**}(s)][1 - Q_{67}^{**}(s)Q_{76}^{**}(s)][Q_{89}^{**}(s)Q_{9,11}^{**}(s)]
\]

\[
Q_{11,14}^{**}(s) + Q_{89}^{**}(s)Q_{9,11}^{**}(s)Q_{11,14}^{**}(s)[Q_{14,0}^{**}(s) + Q_{14,1}^{**}(s) + Q_{8,14}^{**}(s)]
\]

\[
[Q_{14,9}^{**}(s) + Q_{14,11}^{**}(s)][1 - Q_{9,11}^{**}(s)Q_{11,9}^{**}(s)]
\]

and $D_3(s)$ is already specified.

In the steady-state, the total fraction of time for which the system is under replacement is given by:

\[
R_{p_0} = \lim_{s \to 0} [s R_{p_0}^{**}(s)] = \frac{N_4}{D_3}
\]

where
\[ N_4 = p_{03} p_{10} p_{38} p_{63} [p_{8,14} + p_{89} p_{9,11}^{(10)} - p_{89} p_{9,11}^{(10)} p_{11,8} - p_{9,11}^{(10)} p_{11,9}^{(12)} + p_{89} p_{9,11}^{(10)} p_{11,14}^{(13)}] \]

and D_3 is already specified.

**Expected Number of Visits by the Repairman**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( V_i(t) \):

\[
\begin{align*}
V_0(t) &= Q_{01}(t) \mathcal{S} [1 + V_1(t)] + Q_{03}(t) \mathcal{S} V_3(t) \\
V_1(t) &= Q_{10}(t) \mathcal{S} V_0(t) + Q_{11}^{(2)}(t) \mathcal{S} V_1(t) \\
V_3(t) &= Q_{34}(t) \mathcal{S} [1 + V_4(t)] + Q_{38}(t) \mathcal{S} V_8(t) \\
V_4(t) &= Q_{43}(t) \mathcal{S} V_3(t) + Q_{46}^{(5)}(t) \mathcal{S} V_6(t) \\
V_6(t) &= Q_{63}(t) \mathcal{S} V_3(t) + Q_{67}(t) \mathcal{S} V_7(t) \\
V_7(t) &= Q_{76}(t) \mathcal{S} V_6(t) \\
V_8(t) &= Q_{89}(t) \mathcal{S} [1 + V_9(t)] + Q_{8,14}(t) \mathcal{S} [1 + V_{14}(t)] \\
V_9(t) &= Q_{98}(t) \mathcal{S} V_8(t) + Q_{9,11}^{(10)}(t) \mathcal{S} V_{11}(t) \\
V_{11}(t) &= Q_{11,8}(t) \mathcal{S} V_8(t) + Q_{11,9}^{(12)}(t) \mathcal{S} V_9(t) + Q_{11,14}^{(13)}(t) \mathcal{S} V_{14}(t) \\
V_{14}(t) &= Q_{14,0}(t) \mathcal{S} V_0(t) + Q_{14,1}^{(15)}(t) \mathcal{S} V_1(t)
\end{align*}
\]

Taking the L.S.T. of the above relations and then solving them for \( V_0^{**}(s) \), we get

\[
V_0^{**}(s) = \frac{N_5(s)}{D_3(s)}
\]

where

\[
N_5(s) = [1 - Q_{11}^{(2)*}(s)][Q_{01}^{**}(s) + Q_{03}^{**}(s)Q_{34}^{**}(s) - Q_{01}^{**}(s)Q_{34}^{**}(s)Q_{43}^{**}(s)]
\]
\[
[1 - Q_{67}^{**}(s)Q_{76}^{**}(s)][1 - Q_{9,11}^{(10)*}(s)Q_{11,9}^{(12)*}(s) - Q_{89}^{**}(s)Q_{9,8}^{**}(s)]
\]
\[
+ Q_{9,11}^{(10)*}(s)Q_{11,8}^{**}(s) - (1 - Q_{11}^{(2)*}(s))Q_{01}^{**}(s)Q_{34}^{**}(s)Q_{46}^{**}(s)
\]
\[
Q_{63}^{**}(s)[1 - Q_{9,11}^{(10)*}(s)Q_{11,9}^{(12)*}(s) - Q_{89}^{**}(s)Q_{9,8}^{**}(s) + Q_{9,11}^{(10)*}(s)Q_{11,8}^{**}(s)]
\]
\[
- (1 - Q_{11}^{(2)*}(s))(Q_{03}^{**}(s)Q_{38}^{**}(s) + Q_{8,14}^{**}(s))(1 - Q_{63}^{**}(s))
\]
\[
Q_{76}^{**}(s)[1 - Q_{9,11}^{(10)*}(s)Q_{11,9}^{(12)*}(s)]
\]

and \( D_3(s) \) is already specified.
In the steady-state, the expected number of visits of the repairman per unit time is given by:

\[ V_0 = \lim_{s \to 0} [s V_0^\ast(s)] = \frac{N_5}{D_3} \]

where

\[ N_5 = \frac{p_{10}p_{63}}{p_{01}p_{38} + p_{03}p_{34}} \left[ \frac{p_{6,14}p_{98} + p_{9,10}p_{11,8}}{p_{9,10}p_{11,14}} + p_{98} \right] \]

and \( D_3 \) is already specified.

**Profit Analysis of the System**

Expected Profit (P) of the system is given by:

\[ P = C_0A_0 - C_1R_0 - C_2R_0 - C_3V_0 \]

where

- \( C_0 = \) revenue per unit up time
- \( C_1 = \) cost per unit repair
- \( C_2 = \) cost per unit replacement
- \( C_3 = \) cost per visit of the repairman.

**Graphical Interpretations and Conclusions**

Graphical analysis of the system has been carried out at various stages of operational life of the unit for the following particular case:

\[ g_1(\cdot) = \beta_1 e^{-\beta_1 t}; \quad g_2(\cdot) = \beta_2 e^{-\beta_2 t}; \quad g_3(\cdot) = \beta_3 e^{-\beta_3 t}; \]

\[ h(\cdot) = \gamma e^{-\gamma t}; \]

Therefore we have

\[ p_{01} = \frac{\lambda_1}{\lambda_1 + \eta_1}; \quad p_{03} = \frac{\eta_1}{\lambda_1 + \eta_1}; \quad p_{10} = \frac{\beta_1}{\lambda_1 + \beta_1}; \]

\[ p_{11}^{(2)} = p_{12} = \frac{\lambda_1}{\lambda_1 + \beta_1}; \quad p_{34} = \frac{\lambda_2}{\lambda_2 + \eta_2}; \quad p_{38} = \frac{\eta_2}{\lambda_2 + \eta_2}; \]
\[ p_{43} = \frac{\beta_2}{\lambda_1 + \beta_2}; \quad p_{45} = p_{46}^{(5)} = \frac{\lambda_1}{\lambda_1 + \beta_2}; \quad p_{63} = \frac{\beta_1}{\lambda_2 + \beta_1}; \]

\[ p_{67} = \frac{\lambda_2}{\lambda_2 + \beta_1}; \]

\[ p_{8,14} = q; \]

\[ p_{98} = \frac{\beta_3}{\lambda_1 + \beta_3}; \quad p_{9,10} = p_{9,10}^{(10)} = \frac{\lambda_1}{\lambda_1 + \beta_3}; \]

\[ p_{11,8} = \frac{\beta_1}{\lambda_3 + \beta_1}; \quad p_{11,13}^{(13)} = \frac{q\lambda_3}{\lambda_3 + \beta_1}; \quad p_{11,14} = \frac{p\lambda_3}{\lambda_3 + \beta_1}; \]

\[ p_{14,0} = \frac{\gamma}{\gamma + \lambda_1}; \quad p_{14,1} = p_{14,15} = \frac{\lambda_1}{\gamma + \lambda_1}; \]

\[ \mu_0 = \frac{1}{\lambda_1 + \eta_1}; \quad \mu_4 = \frac{1}{\lambda_1 + \beta_2}; \]

\[ \mu_1 = \frac{1}{\lambda_1 + \beta_1}; \quad \mu_6 = \frac{1}{\lambda_2 + \beta_1}; \quad \mu_8 = \frac{1}{\lambda_3}; \]

\[ \mu_3 = \frac{1}{\lambda_2 + \eta_2}; \quad \mu_{11} = \frac{1}{\lambda_3 + \beta_1}; \quad \mu_{14} = \frac{1}{\gamma + \lambda_1}; \]

Fig. 7.2 depicts the behaviour of MTSF with respect to failure rate (\( \lambda_1 \)) of the unit during Stage-I of the unit for different values of improvement rate (\( \eta_1 \)).

**MTSF VERSUS FAILURE RATE (\( \lambda_1 \)) FOR DIFFERENT VALUES OF IMPROVEMENT RATE (\( \eta_1 \))**

![MTSF VERSUS FAILURE RATE (\( \lambda_1 \)) FOR DIFFERENT VALUES OF IMPROVEMENT RATE (\( \eta_1 \))]
It is clear from the graph that MTSF decreases with the increase in the values of \( \lambda_1 \) and has higher values for higher values of \( \eta_1 \).

**Fig. 7.3** reveals the behaviour of MTSF with respect to failure rate (\( \lambda_2 \)) during Stage-II of the unit for different values of deterioration rate of (\( \eta_2 \)).

![MTSF VERSUS FAILURE RATE (\( \lambda_2 \)) FOR DIFFERENT VALUES OF DETERIORATION RATE (\( \eta_2 \))](image)

It can be concluded from the graph that MTSF decreases with the increase in the values of \( \lambda_2 \) and has lower values for higher values of \( \eta_2 \).

**Fig. 7.4** represents the behaviour of MTSF with respect to failure rate (\( \lambda_3 \)) during Stage-III of the unit for different values of repair rate (\( \beta_3 \)) during Stage-III.

![MTSF VERSUS FAILURE RATE (\( \lambda_3 \)) FOR DIFFERENT VALUES OF REPAIR RATE (\( \beta_3 \))]
It can be seen from the graph that MTSF decreases with the increase in the values of $\lambda_3$ and has higher values for higher values of $\beta_3$.

**Fig. 7.5** shows the behaviour of profit with respect to failure rate ($\lambda_1$) during the Stage-I of the unit for different values of repair rate ($\beta_1$) during Stage-I.

It is interpreted as follows:

(i) The profit decreases with the increase in the values of $\lambda_1$ and has higher values for higher values of $\beta_1$

(ii) For $\beta_1 = 4$, the profit is positive or zero or negative according as $\lambda_1 < $ or $= $ or $> 0.945126$ and hence $\lambda_1$ should be less than 0.945126 in this case.

(iii) For $\beta_1 = 6$, the profit is positive or zero or negative according as $\lambda_1 < $ or $= $ or $> 0.970473$ and hence $\lambda_1$ should be less than 0.970473 in this case.

(iv) For $\beta_1 = 8$, the profit is positive or zero or negative according as $\lambda_1 < $ or $= $ or $> 0.984864$ and hence $\lambda_1$ should be less than 0.984864 in this case.

![Graph showing profit versus failure rate for different values of repair rate](image)

**Fig. 7.5**

**Fig. 7.6** depicts the behaviour of profit of the system with respect to failure rate ($\lambda_2$) of the unit during Stage-II for different values of deterioration rate ($\eta_2$).
Following conclusions can be drawn from the graph:

(i) The profit decreases with the increase in the values of $\lambda_2$ and has lower values for higher values of $\eta_2$.

(ii) For $\eta_2 = 0.7$, the profit is positive or zero or negative according as $\lambda_2 < \text{or} = \text{or} > 0.782673$ and hence $\lambda_2$ should be less than 0.782673 in this case.

(iii) For $\eta_2 = 0.8$, the profit is positive or zero or negative according as $\lambda_2 < \text{or} = \text{or} > 0.521813$ and hence $\lambda_2$ should be less than 0.521813 in this case.

(iv) For $\eta_2 = 0.9$, the profit is positive or zero or negative according as $\lambda_2 < \text{or} = \text{or} > 0.260953$ and hence $\lambda_2$ should be less than 0.260953 in this case.

Fig. 7.7 depicts the behaviour of profit with respect to revenue per unit up time ($C_0$) for different values of failure rate ($\lambda_2$) during Stage-II of the unit.
From the graph following conclusions can be drawn.

(i) The profit increases with the increase in the values of $C_0$ and has lower values for higher values of $\lambda_2$.

(ii) For $\lambda_2 = 0.2$, the profit is positive or zero or negative according as $C_0 >$ or $= \text{or} < 716.8505$ and hence in this case revenue per unit up time of the system should be greater than 716.8505.

(iii) For $\lambda_2 = 0.3$, the profit is positive or zero or negative according as $C_0 >$ or $= \text{or} < 757.105$ and hence in this case revenue per unit up time should be greater than 757.105.

(iv) For $\lambda_2 = 0.4$, the profit is positive or zero or negative according as $C_0 >$ or $= \text{or} < 795.5624$ and hence in this case revenue per unit up time should be greater than 795.5624.