CHAPTER-V

ANALYSIS OF A WARRANTED SYSTEM WITH THREE STAGES OF OPERATION AND TWO TYPES OF SERVICE FACILITY
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Analysis of a Warranted System with Three Stages of Operation and Two Types of Service Facility

In the preceding chapters, single unit sophisticated systems under warranty with three operational stages considering the possibility of occurrence of different types of faults in the unit have been examined. In these chapters, it has been assumed that the service(s), i.e. maintenance/repair/replacement of the system at each of the three operational stages is provided by a single service facility, viz. service provider/service engineer. In practice, when the system is under warranty for a specified time, during the warranty period the system provider is contacted to undertake service(s) of the system, free of charges. However, on expiry of the warranty period i.e. at Stage III, the service provider/service engineer may not be immediately available whenever required or may be costlier than some other repairmen available in the open market. Then a repairman who is immediately available on payment basis is called for the service of the system by the system user. Moreover, in most of the situations, inspection is carried out on occurrence of a fault to detect whether the fault in the component(s) of the system is repairable or irreparable and the system is handled accordingly. This has also been observed while collecting information/data on failures and repairs of UPS system.

Keeping above practical aspects in mind, we, in the present chapter, deals with investigation of the single unit warranted sophisticated systems having three operational stages considering two types of service facilities, viz. service engineer during warranty period (i.e. Stage-I and Stage-II) and a repairman immediately available on payment basis (called available repairman) after the expiry of the warranty period (i.e Stage-III). That is, at Stage-III of the system, the system user gets removed fault(s) of the system through the available repairman. The available repairman takes negligible time to reach the system. Here also two reliability models for the system have been studied. In model-I, on-line repair on occurrence of a minor fault and replacement of the unit on occurrence of a major fault are taken into consideration. In model-II, it is assumed that on failure of the system due to occurrence
of a major fault the service engineer/available repairman first inspects the system to detect whether the fault(s) occurred in the component(s) is repairable/irrepairable and then accordingly, repair/replacement of the component(s) or replacement/re-installation of the complete unit is done. Thus, repairs as well as replacements of the components on occurrence of major faults are also considered in this model. Other assumptions are same as taken in the previous chapter.

The system is analysed using Markov processes and regenerative point technique and the following measures of the system effectiveness are obtained at three stages of its operation:

- Mean installation time
- Mean time to system failure
- Steady-state availability
- Expected busy period of the service engineer/available repairman
  (Repair time for the unit only)
- Expected busy period of the service engineer/available repairman
  (Replacement time for the unit only)
- Expected busy period of the service engineer/available repairman
  (Inspection time for the unit only)
- Expected busy period of the service engineer/available repairman
  (Maintenance time for components only)
- Expected busy period of the service engineer/available repairman
  (Repair time for the components only)
- Expected busy period of the service engineer/available repairman
  (Replacement time for the components only)

The expected profit for the system user as well as system provider are obtained and various conclusions are drawn on the basis of graphical study
Notations and States of the System

\( \lambda_1 / \lambda_2 / \lambda_3 \) : failure rate of the unit during Stage-I/Stage-II/State-III

\( \eta_1 / \eta_2 \) : Improvement rate / deterioration rate of the unit

\( p_1 / p_2 / p_3 \) : probability of occurrence of minor fault in the unit during Stage-I/Stage-II/State-III

\( q_1 / q_2 / q_3 \) : probability of occurrence of major fault(s) in the unit during Stage-I/Stage-II/State-III

\( a_1 / a_2 / a_3 \) : probability of occurrence of repairable fault in the component during Stage-I/Stage-II/State-III

\( b_1 / b_2 / b_3 \) : probability of occurrence of irreparable fault in the component during Stage-I/Stage-II/State-III

\( c_1 / c_2 / c_3 \) : probability of occurrence of irreparable fault in the unit during Stage-I/Stage-II/State-III

\( g(\cdot), G(\cdot) \) : p.d.f. and c.d.f. of time to maintenance/minor repair of the unit

\( i_1(\cdot), I_1(\cdot) / i_2(\cdot), I_2(\cdot) \) : p.d.f. and c.d.f. of time to inspect the unit by service engineer/available repairman

\( g_1(\cdot), G_1(\cdot) / g_2(\cdot), G_2(\cdot) / g_3(\cdot), G_3(\cdot) \) : p.d.f. and c.d.f. of time to repair the components during Stage-I/Stage-II/Stage-III

\( h_1(\cdot), H_1(\cdot) / h_2(\cdot), H_2(\cdot) / h_3(\cdot), H_3(\cdot) \) : p.d.f. and c.d.f. of time to replace the components during Stage-I/Stage-II/Stage-III

\( h(\cdot), H(\cdot) \) : p.d.f. and c.d.f. of time to replace and re-install the unit

\( S_i \) : state numbers \( i, i = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 \)

\( O_I / O_{II} / O_{III} \) : unit is operating during Stage-I/Stage-II/Stage-III

\( O_{Ir} / O_{IIr} / O_{IIIr} \) : unit is in down state and is under repair during Stage-I/Stage-II/Stage-III

\( O_{Ir} / O_{Ir} / O_{IIIr} \) : unit is in failed state and is under replacement during Stage-I/Stage-II/Stage-III

\( F_{Ir} / F_{IIr} / F_{IIIr} \) : unit is in failed state due to occurrence of a fault in the component and is under repair during Stage-I/Stage-II/
Stage-III

$F_{IIIpc}/F_{IIpc}/F_{IIIpc}$ : unit is in failed state due occurrence of a fault in the component and is under replacement during Stage-I/Stage-II/Stage-III.

$F_{IIpc}/F_{IIIpc}$ : unit is in failed state due to failure of the complete unit and is under replacement during Stage-II/Stage-III.

$F_{Ili1}/F_{IIIi2}$ : unit is in failed state and is under inspection by the service engineer/available repairman

Model-1

Transition Probabilities and Mean Sojourn Times

A state-transition diagram in fig.5.1 shows various states of transition of the system. The epochs of entry into states 0, 1, 2, 3, 4, 5, 6, 7 and 8 are regeneration points and thus these are regenerative states. The states 2, 5 and 8 are failed states. The transition probabilities are given by

\[
q_{01}(t) = a_1 \lambda_1 e^{-(\lambda_1 + \eta_1)t}; \quad q_{02}(t) = b_1 \lambda_1 e^{-(\lambda_1 + \eta_1)t};
\]

\[
q_{03}(t) = \eta_1 e^{-(\lambda_1 + \eta_1)t}; \quad q_{10}(t) = g_1(t);
\]

\[
q_{20}(t) = h_1(t); \quad q_{34}(t) = q_2 \lambda_2 e^{-(\lambda_2 + \eta_2)t};
\]

\[
q_{35}(t) = b_2 \lambda_2 e^{-(\lambda_2 + \eta_2)t}; \quad q_{3,6}(t) = \eta_2 e^{-(\lambda_2 + \eta_2)t};
\]

\[
q_{43}(t) = g_2(t); \quad q_{35}(t) = b_2 \lambda_2 e^{-(\lambda_2 + \eta_2)t};
\]

The non-zero elements $p_{ij}$ obtained as $p_{ij} = \lim_{s \to 0} q_{ij}^s(s)$ and are given by

\[
p_{01} = \frac{a_1 \lambda_1}{D}; \quad p_{02} = \frac{b_1 \lambda_1}{D}; \quad p_{03} = \frac{\eta_1}{D}; \quad p_{34} = \frac{a_2 \lambda_2}{D_0};
\]

\[
p_{35} = \frac{b_2 \lambda_2}{D_0}; \quad p_{36} = \frac{\eta_2}{D_0}; \quad p_{67} = a_3; \quad p_{68} = b_3;
\]

\[
p_{10} = p_{20} = p_{43} = p_{5,0} = p_{76} = p_{8,0} = 1;
\]

where

\[
D = \lambda_1 + \eta_1 \quad \text{and} \quad D_0 = \lambda_2 + \eta_2
\]
By these transition probabilities it can be verified that

\[ p_{01} + p_{02} + p_{03} = p_{34} + p_{35} + p_{36} = p_{67} + p_{68} = 1 \]

The mean sojourn times \((\mu_i)\) in state ‘i’ are:

\[
\begin{align*}
\mu_0 &= \frac{1}{D}; \\
\mu_1 &= \int_0^\infty G_1(t) \, dt; \\
\mu_2 &= \int_0^\infty H_1(t) \, dt; \\
\mu_3 &= \frac{1}{D_0}; \\
\mu_4 &= \int_0^\infty G_2(t) \, dt; \\
\mu_5 &= \int_0^\infty H_2(t) \, dt; \\
\mu_6 &= \frac{1}{\lambda_3}; \\
\mu_7 &= \int_0^\infty G_3(t) \, dt; \\
\mu_8 &= \int_0^\infty H_3(t) \, dt;
\end{align*}
\]

where \(D\) and \(D_0\) are already specified.
The unconditional mean time taken by the system to transit for any state ‘j’ when it is counted from epoch of entrance into state ‘i’ is mathematically stated as:

\[ m_{ij} = \int_{0}^{\infty} t q_{ij}(t) \, dt = -q_{ij}^{*}(s) \]

Thus

\[ m_{01} + m_{02} + m_{03} = \mu_0; \quad m_{1,0} = \mu_1; \quad m_{2,0} = \mu_2; \quad m_{34} + m_{35} + m_{36} = \mu_3; \]
\[ m_{43} = \mu_4; \quad m_{50} = \mu_5; \quad m_{67} + m_{68} = \mu_6; \quad m_{76} = \mu_7; \quad m_{80} = \mu_8 \]

**Mean Installation Time**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relation for \( I_i(t) \):

\[ I_0(t) = Q_{01}(t) \, I_1(t) + Q_{02}(t) \, I_2(t) + Q_{03}(t); \]
\[ I_1(t) = Q_{10}(t) \, I_0(t); \quad I(t) = Q_{20}(t) \, I_0(t); \]

Taking L.S.T. of the above relations and then solving them for \( I_0^{**}(s) \), we get

\[ I_0^{**}(s) = \frac{N_1(s)}{D_1(s)} \]

where

\[ N_1(s) = Q_{03}^{**}(s) \text{ and } D_1(s) = 1 - Q_{01}^{**}(s)Q_{10}^{**}(s) - Q_{02}^{**}(s)Q_{20}^{**}(s) \]

In the steady-state, mean installation time is given by:

\[ I_0 = \lim_{s \to 0} \left( \frac{1 - I_0^{**}(s)}{s} \right) = \frac{N_1}{D_1} \]

where

\[ N_1 = \mu_0 + \mu_1 p_{01} + \mu_2 p_{02} \text{ and } D_1 = p_{03} \]

**Expected Busy Period of the Service Engineer (Repair Time Only) during Stage-I**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( BR_i(t) \):

\[ BR_0(t) = q_{01}(t) \, BR_1(t) + q_{02}(t) \, BR_2(t) + q_{03}(t) \, BR_3(t); \]
\[ BR_1(t) = W_1(t) + q_{10}(t) \, BR_0(t); \quad BR_2(t) = q_{20}(t) \, BR_0(t); \]
\[ BR_3(t) = q_{34}(t) \, BR_4(t) + q_{35}(t) \, BR_5(t) + q_{36}(t) \, BR_6(t); \]
\( BR_4(t) = q_{43}(t) \otimes BR_3(t); \quad BR_5(t) = q_{50}(t) \otimes BR_0(t); \)

\( BR_6(t) = q_{67}(t) \otimes BR_7(t) + q_{65}(t) \otimes BR_8(t); \)

\( BR_7(t) = q_{76}(t) \otimes BR_6(t); \quad BR_8(t) = q_{8,0}(t) \otimes BR_0(t); \)

where

\( W_1(t) = G_1(t) \)

Taking L.T. of the above relations and then solving them for \( BR_0^*(s) \), we get

\[ BR_0^*(s) = \frac{N_2(s)}{D_2(s)} \]

Where

\[ N_2(s) = W_1^*(s) q_{01}^*(s)(1 - q_{34}^*(s) q_{43}^*(s))(1 - q_{67}^*(s) q_{76}^*(s)) \]

\[ D_2(s) = [1 - q_{01}^*(s) q_{10}^*(s) - q_{02}^*(s) q_{20}^*(s)] (1 - q_{34}^*(s) q_{43}^*(s)) \]

\[ [1 - q_{67}^*(s) q_{76}^*(s)] - q_{03}^*(s) q_{35}^*(s) q_{50}^*(s) [1 - q_{67}^*(s) q_{76}^*(s)] \]

\[ - q_{05}^*(s) q_{36}^*(s) q_{68}^*(s) q_{8,0}^*(s) \]

In the steady-state, the total fraction of time for which the system is under repair is given by:

\[ BR_0 = \lim_{s \to 0} s BR_0^*(s) = \frac{N_2}{D_2} \]

where

\[ N_2 = \mu_1 p_{01} (1 - p_{34}) p_{68} \]

\[ D_2 = [\mu_0 + \mu_1 p_{01} + \mu_2 p_{02}] [1 - p_{34}] p_{68} + [\mu_3 + \mu_4 p_{34} + \mu_5 p_{35}] p_{03} p_{68} + [\mu_6 + \mu_7 p_{67} + \mu_8 p_{68}] p_{03} p_{36} \]

**Expected Busy Period of the Service Engineer (Replacement Time Only) during Stage-I**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( BR_{p_i}(t) \):

\( BR_{p_0}(t) = q_{01}(t) \otimes BR_{p_1}(t) + q_{02}(t) \otimes BR_{p_2}(t) + q_{03}(t) \otimes BR_{p_3}(t); \)

\( BR_{p_1}(t) = q_{10}(t) \otimes BR_{p_0}(t); \quad BR_{p_2}(t) = W_2(t) + q_{20}(t) \otimes BR_{p_0}(t); \)
\[ \text{BRp}_3(t) = q_{34}(t) \circ \text{BRp}_4(t) + q_{35}(t) \circ \text{BRp}_5(t) + q_{36}(t) \circ \text{BRp}_6(t); \]
\[ \text{BRp}_4(t) = q_{43}(t) \circ \text{BRp}_3(t); \quad \text{BRp}_5(t) = q_{50}(t) \circ \text{BRp}_0(t); \]
\[ \text{BRp}_6(t) = q_{67}(t) \circ \text{BRp}_7(t) + q_{68}(t) \circ \text{BRp}_8(t); \]
\[ \text{BRp}_7(t) = q_{76}(t) \circ \text{BRp}_6(t); \quad \text{BRp}_8(t) = q_{8,0}(t) \circ \text{BRp}_0(t); \]

where
\[
W_2(t) = \frac{H_1}{T(t)}
\]

Taking the L.T. of the above relations and then solving them for \( \text{BRp}_0^*(s) \), we get
\[ \text{BRp}_0^*(s) = \frac{N_3(s)}{D_2(s)} \]

where
\[ N_3(s) = W_2^*(s)q_{02}^*(s)[1 - q_{34}^*(s)q_{43}^*(s)][1 - q_{67}^*(s)q_{76}^*(s)] \]
and \( D_2(s) \) is already specified.

In the steady-state, the total fraction of time for which the system is under replacement is given by:
\[ \text{BRp}_0 = \lim_{s \to 0} s \text{Rp}_0^*(s) = \frac{N_3}{D_2} \]
where
\[ N_3 = \mu_2 p_{02} (1 - p_{34}) p_{68} \text{ and } D_2 \text{ is already specified.} \]

**Mean Time to System Failure during Stage-II**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( \phi_i(t) \):
\[ \phi_3(t) = Q_{34}(t) \circ \phi_4(t) + Q_{35}(t) + Q_{36}(t); \quad \phi_0(t) = Q_{43}(t) \circ \phi_3(t); \]

Taking the L.S.T. of the above relations and then solving them for \( \phi_3^{**}(s) \), we get
\[ \phi_3^{**}(s) = \frac{N_4(s)}{D_3(s)} \]

where
\[ N_4(s) = Q_{35}^{**}(s) + Q_{36}^{**}(s) \text{ and } D_3(s) = 1 - Q_{34}^{**}(s)Q_{43}^{**}(s) \]
In the steady-state, the mean time to system failure is given by:

\[ T_3 = \lim_{s \to 0} \left[ \frac{1 - \phi_3^*(s)}{s} \right] = \frac{N_4}{D_3} \]

where

\[ N_4 = \mu_3 + \mu_4 p_{34} \quad \text{and} \quad D_3 = 1 - p_{34} \]

**Availability Analysis during Stage-II**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( A_i(t) \):

\[
\begin{align*}
A_0(t) &= q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) + q_{03}(t) \odot A_3(t); \\
A_1(t) &= q_{10}(t) \odot A_0(t); \\
A_2(t) &= q_{20}(t) \odot A_0(t); \\
A_3(t) &= M_3(t) + q_{34}(t) \odot A_4(t) + q_{35}(t) \odot A_5(t) + q_{36}(t) \odot A_6(t); \\
A_4(t) &= q_{43}(t) \odot A_3(t); \\
A_5(t) &= q_{50}(t) \odot A_0(t); \\
A_6(t) &= q_{67}(t) \odot A_7(t) + q_{68}(t) \odot A_8(t); \\
A_7(t) &= q_{76}(t) \odot A_6(t); \\
A_8(t) &= q_{80}(t) \odot A_0(t);
\end{align*}
\]

where

\[ M_3(t) = e^{-(\lambda_2 + \eta_2)t} \]

Taking the L.T. of the above relations and then solving them for \( A_3^*(s) \) get

\[ A_3^*(s) = \frac{N_5(s)}{D_2(s)} \]

where

\[ N_5(s) = M_3^*(s)[1 - q_{01}^*(s)q_{10}^*(s) - q_{02}^*(s)q_{20}^*(s)][1 - q_{67}^*(s)q_{76}^*(s)] \]

and \( D_2(s) \) is already specified.

In the steady-state the availability of the system is given by:

\[ A_3 = \lim_{s \to 0} [s \ A_3^*(s)] = \frac{N_5}{D_2} \]

where

\[ N_5 = \mu_3 p_{03} p_{68} \quad \text{and} \quad D_2 \] is already specified.
Expected Busy Period of the Service Engineer [Repair Time Only] during Stage-II

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( BR_i(t) \):

\[
\begin{align*}
BR_0(t) &= q_{01}(t)BR_1(t) + q_{02}(t)BR_2(t) + q_{03}(t)BR_3(t); \\
BR_1(t) &= q_{10}(t)BR_0(t); \\
BR_2(t) &= q_{20}(t)BR_0(t); \\
BR_3(t) &= q_{34}(t)BR_4(t) + q_{35}(t)BR_5(t) + q_{36}(t)BR_6(t); \\
BR_4(t) &= W_4(t) + q_{43}(t)BR_3(t); \\
BR_5(t) &= q_{50}(t)BR_0(t); \\
BR_6(t) &= q_{67}(t)BR_7(t) + q_{68}(t)BR_8(t); \\
BR_7(t) &= q_{76}(t)BR_6(t); \\
BR_8(t) &= q_{8,0}(t)BR_0(t);
\end{align*}
\]

where

\[
W_4(t) = G_2(t)
\]

Taking the L.T. of the above relations and then solving them for \( BR_3^*(s) \), we get

\[
BR_3^*(s) = \frac{N_6(s)}{D_2(s)}
\]

where

\[
N_6(s) = W_4^*(s)q_{34}^*(s)[1 - q_{10}^*(s)q_{10}^*(s) - q_{02}^*(s)q_{20}^*(s)][1 - q_{67}^*(s)q_{76}^*(s)]
\]

and \( D_2(s) \) is already specified.

In the steady-state, the total fraction of time for which the system is under repair is given by:

\[
BR_3 = \lim_{s \to 0} [sBR_3^*(s)] = \frac{N_6}{D_2}
\]

where

\[
N_6 = \mu_4 p_{03} p_{34} p_{68} \quad \text{and} \quad D_2 \text{ is already specified.}
\]

Expected Busy Period of the Service Engineer [Replacement Time Only] during Stage-II

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( BR_{p_i}(t) \):

\[
BR_{p_0}(t) = q_{01}(t)BR_{p_1}(t) + q_{02}(t)BR_{p_2}(t) + q_{03}(t)BR_{p_3}(t);
\]
\[ \text{BRp}_1(t) = q_{10}(t) \circ \text{BRp}_0(t); \quad \text{BRp}_2(t) = q_{20}(t) \circ \text{BRp}_0(t); \]
\[ \text{BRp}_3(t) = q_{34}(t) \circ \text{BRp}_4(t) + q_{35}(t) \circ \text{BRp}_5(t) + q_{36}(t) \circ \text{BRp}_6(t); \]
\[ \text{BRp}_4(t) = q_{43}(t) \circ \text{BRp}_3(t); \quad \text{BRp}_5(t) = W_{5}(t) + q_{50}(t) \circ \text{BRp}_0(t); \]
\[ \text{BRp}_6(t) = q_{67}(t) \circ \text{BRp}_7(t) + q_{68}(t) \circ \text{BRp}_8(t); \]
\[ \text{BRp}_7(t) = q_{76}(t) \circ \text{BRp}_6(t); \quad \text{BRp}_8(t) = q_{80}(t) \circ \text{BRp}_0(t); \]

where
\[ W_{5}(t) = H_{2}(t) \]

Taking the L.T. of the above relations and then solving them for \( \text{BRp}_3^*(s) \), we get
\[ \text{BRp}_3^*(s) = \frac{N_7(s)}{D_2(s)} \]

where
\[ N_7(s) = W_{5}(s) q_{45}(s) [1 - q_{01}(s) q_{01}(s) - q_{02}(s) q_{20}(s) [1 - q_{67}(s) q_{76}(s)]] \]
and \( D_2(s) \) is already specified.

In the steady-state, the total fraction of time for which the system is under replacement is given by:
\[ \text{BRp}_3 = \lim_{s \to 0} [s \text{BRp}_3^*(s)] = \frac{N_7}{D_2} \]

where
\[ N_7 = \mu_5 p_{03} p_{35} p_{68} \] and \( D_2 \) is already specified.

**Expected Number of Visits by the Service Engineer during Stage-II**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( V_i(t) \):
\[ V_0(t) = Q_{0}(t) \circ V_1(t) + Q_{02}(t) \circ V_2(t) + Q_{03}(t) \circ V_3(t); \]
\[ V_1(t) = Q_{10}(t) \circ V_0(t); \quad V_2(t) = Q_{20}(t) \circ V_0(t); \]
\[ V_3(t) = Q_{34}(t) \circ [1 + V_4(t)] + Q_{35}(t) \circ [1 + V_5(t)] + Q_{36}(t) \circ V_6(t); \]
\[ V_4(t) = Q_{43}(t) \circ V_3(t); \quad V_5(t) = Q_{50}(t) \circ V_0(t); \]
\[ V_6(t) = Q_{67}(t) \circ V_7(t) + Q_{68}(t) \circ V_8(t); \]
\[ V_7(t) = Q_{76}(t) \circ V_6(t); \quad V_8(t) = Q_{80}(t) \circ V_0(t) \]
Taking the L.S.T. of the above relations and solving them for $V_3^{**}(s)$, we get

$$V_3^{**}(s) = \frac{N_8(s)}{D_2(s)}$$

where

$$N_8(s) = [Q_{44}^{**}(s) + Q_{55}^{**}(s)][1 - Q_{16}^{**}(s)Q_{10}^{**}(s) - Q_{02}^{**}(s)Q_{20}^{**}(s)]\left[1 - Q_{67}^{**}(s)Q_{76}^{**}(s)\right]$$

and $D_2(s)$ is already specified.

In the steady-state, the expected number of visits per unit time by the repairman is given by

$$V_3 = \lim_{s \to 0} [s V_3^{**}(s)] = \frac{N_8}{D_2}$$

where

$$N_8 = p_{03}(p_{34} + p_{35})p_{68}$$ and $D_2$ is already specified.

**Mean Time to System Failure during Stage-III**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for $\phi_i(t)$:

$$\phi_6(t) = Q_{67}(t) \otimes \phi_7(t) + Q_{68}(t)$$

$$\phi_7(t) = Q_{67}(t) \otimes \phi_6(t)$$

Taking the L.S.T. of the above relations and solving them for $\phi_6^{**}(s)$, we get

$$\phi_6^{**}(s) = \frac{N_9(s)}{D_4(s)}$$

where

$$N_9(s) = Q_{68}^{**}(s)$$ and $D_4(s) = 1 - Q_{67}^{**}(s)Q_{76}^{**}(s)$

In the steady-state, mean time to system failure is given by:

$$T_6 = \lim_{s \to 0} \left[\frac{1 - \phi_6^{**}(s)}{s}\right] = \frac{N_9}{D_4}$$

where

$$N_9 = \mu_6 + \mu_7p_{67}$$ and $D_4 = 1 - p_{67}$
**Availability Analysis of the System during Stage-III**

Using probabilities arguments for regenerative process, we obtain the following recursive relations for $A_i(t)$:

\[
A_0(t) = q_{01}(t)A_1(t) + q_{02}(t)A_2(t) + q_{03}(t)A_3(t);
\]
\[
A_1(t) = q_{10}(t)A_0(t);
\]
\[
A_2(t) = q_{20}(t)A_0(t);
\]
\[
A_3(t) = q_{31}(t)A_4(t) + q_{32}(t)A_5(t) + q_{33}(t)A_6(t);
\]
\[
A_4(t) = q_{43}(t)A_3(t);
\]
\[
A_5(t) = q_{50}(t)A_0(t);
\]
\[
A_6(t) = M_6(t) + q_{67}(t)A_7(t) + q_{68}(t)A_8(t);
\]
\[
A_7(t) = q_{76}(t)A_6(t);
\]
\[
A_8(t) = q_{80}(t)A_0(t);
\]

where

\[
M_6(t) = e^{-\lambda_3 t}
\]

Taking the L.T. of the above relations and then solving them for $A_6^*(s)$, we get

\[
A_6^*(s) = \frac{N_{10}(s)}{D_2(s)}
\]

where

\[
N_{10}(s) = M_6^*(s)[\{1 - q_{01}^*(s)q_{10}^*(s) - q_{02}^*(s)q_{20}^*(s)\} \{1 - q_{34}^*(s)q_{43}^*(s)\} - q_{03}^*(s)q_{35}^*(s)q_{50}^*(s)]
\]

and $D_2(s)$ is already specified.

In the steady-state, the availability of the system is given by:

\[
A_6 = \lim_{s \to 0} [s A_6^*(s)] = \frac{N_{10}}{D_2}
\]

where

\[
N_{10} = \mu_6 p_{03} p_{36} \text{ and } D_2 \text{ is already specified.}
\]

**Expected Busy Period of the Repairman (Repair Time Only) during Stage-III**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for $BR_i(t)$:

\[
BR_0(t) = q_{01}(t)BR_1(t) + q_{02}(t)BR_2(t) + q_{03}(t)BR_3(t);
\]
$$BR_1(t) = q_{10}(t) \circ BR_0(t); \quad BR_2(t) = q_{20}(t) \circ BR_0(t);$$

$$BR_3(t) = q_{34}(t) \circ BR_4(t) + q_{35}(t) \circ BR_5(t) + q_{36}(t) \circ BR_6(t);$$

$$BR_4(t) = q_{43}(t) \circ BR_3(t); \quad BR_5(t) = q_{50}(t) \circ BR_0(t);$$

$$BR_6(t) = q_{67}(t) \circ BR_7(t) + q_{68}(t) \circ BR_8(t);$$

$$BR_7(t) = W_7(t) + q_{76}(t) \circ BR_6(t); \quad BR_8(t) = q_{80}(t) \circ BR_0(t);$$

where

$$W_7(t) = \overline{G_3(t)}$$

Taking the L.T. of the above relations and then solving them for $BR_6^*(s)$, we get

$$BR_6^*(s) = \frac{N_{11}(s)}{D_2(s)}$$

where

$$N_{11}(s) = W_7^*(s)q_{67}^*(s)[\{1 - q_{10}^*(s)q_{10}^*(s) - q_{02}^*(s)q_{20}^*(s)\} \{1 - q_{34}^*(s)q_{43}^*(s)\}$$

$$- q_{03}^*(s)q_{35}^*(s)q_{35}^*(s)]$$

and $D_2(s)$ is already specified.

In the steady-state, the total fraction of time for which the system is under repair is given by:

$$BR_6 = \lim_{s \to 0} [s \cdot BR_6^*(s)] = \frac{N_{11}}{D_2}$$

where

$$N_{11} = \mu_7 p_{03} p_{36} p_{67}$$

and $D_2$ is already specified.

**Expected Busy Period of the Repairman (Replacement Time Only) during Stage-III**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for $BR_{p_1}(t)$:

$$BR_{p_0}(t) = q_{01}(t) \circ BR_{p_1}(t) + q_{02}(t) \circ BR_{p_2}(t) + q_{03}(t) \circ BR_{p_3}(t);$$

$$BR_{p_1}(t) = q_{10}(t) \circ BR_{p_0}(t); \quad BR_{p_2}(t) = q_{20}(t) \circ BR_{p_0}(t);$$

$$BR_{p_3}(t) = q_{34}(t) \circ BR_{p_4}(t) + q_{35}(t) \circ BR_{p_5}(t) + q_{36}(t) \circ BR_{p_6}(t);$$
BRp_4(t) = q_{43}(t) \circ BRp_3(t); \quad BRp_5(t) = q_{50}(t) \circ BRp_0(t);

BRp_6(t) = q_{67}(t) \circ BRp_7(t) + q_{68}(t) \circ BRp_8(t);

BRp_7(t) = q_{76}(t) \circ BRp_6(t);

BRp_8(t) = W_8(t) + q_{80}(t) \circ BRp_0(t);

where

W_8(t) = \overline{H_3(t)}

Taking the L.T. of the above relations and then solving them for \(BRp_6^*(s)\), we get

\[ BRp_6^*(s) = \frac{N_{12}(s)}{D_2(s)} \]

where

\[ N_{12}(s) = W_8^*(s)q_{68}^*(s)[\{1-q_{01}^*(s)q_{10}^*(s)\} - q_{02}^*(s)q_{20}^*(s)][1-q_{34}^*(s)q_{43}^*(s)] - q_{03}^*(s)q_{35}^*(s)q_{50}^*(s)] \]

and \(D_2(s)\) is already specified.

In the steady-state, the total fraction of time for which the system is under replacement is given by:

\[ BRp_6 = \lim_{s \to 0} [s \, BRp_6^*(s)] = \frac{N_{12}}{D_2} \]

where

\[ N_{12} = \mu_8 \, p_{03} \, p_{36} \, p_{68} \] and \(D_2\) is already specified.

**Expected Number of Visits by the Repairman during Stage-III**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \(V_i(t)\):

\[ V_0(t) = Q_{01}(t) \circ V_1(t) + Q_{02}(t) \circ V_2(t) + Q_{03}(t) \circ V_3(t); \]

\[ V_1(t) = Q_{10}(t) \circ V_0(t); \quad V_2(t) = Q_{20}(t) \circ V_0(t); \]

\[ V_3(t) = Q_{34}(t) \circ V_4(t) + Q_{35}(t) \circ V_5(t) + Q_{36}(t) \circ V_6(t); \]

\[ V_4(t) = Q_{43}(t) \circ V_3(t); \quad V_5(t) = Q_{50}(t) \circ V_0(t); \]

\[ V_6(t) = Q_{67}(t) \circ [1 + V_7(t)] + Q_{68}(t) \circ [1 + V_8(t)]; \]
\[ V_7(t) = Q_{76}(t) \Rightarrow V_6(t); \quad V_8(t) = Q_{80}(t) \Rightarrow V_0(t) \]

Taking the L.S.T. of the above relations and then solving them for \( V^{**}_{6}(s) \), we get

\[ V^{**}_{6}(s) = \frac{N_{13}(s)}{D_2(s)} \]

where

\[ N_{13}(s) = [Q^{**}_{67}(s) + Q^{**}_{68}(s)](1 - Q^{**}_{01}(s)Q^{**}_{10}(s) - Q^{**}_{02}(s)Q^{**}_{20}(s)) \]

\[ 1 - Q^{**}_{34}(s)Q^{**}_{43}(s) - Q^{**}_{03}(s)Q^{**}_{35}(s)Q^{**}_{50}(s) \]

and \( D_2(s) \) is already specified.

In the steady-state, the expected number of visits per unit time by the repairman is given by:

\[ V_6 = \lim_{s \to 0} [sV^{**}_{6}(s)] = \frac{N_{13}}{D_2} \]

where

\[ N_{13} = p_{03} p_{36} [p_{67} + p_{68}] \] and \( D_2 \) is already specified.

**Profit Analysis of the System**

Expected profit of the system for the system user \((P_1)\) is given by:

\[ P_1 = C_0 [A_3 + A_6] - C_1 B R_6 - C_2 B R_6 - C_3 V_6 \]

where

- \( C_0 \) = revenue per unit up time of the system
- \( C_1 \) = cost per unit time of repair by the available repairman
- \( C_2 \) = cost per unit time of replacement by the available repairman
- \( C_3 \) = cost per visit by the available repairman

Expected profit of the system for the system provider \((P_2)\) is given by:

\[ P_2 = (S P - C P) - C_4 (B R_0 + B R_3) - C_3 (B R p_0 + B R p_3) - C_4 V_3 \]

where

- \( S P / C P \) = sale price/ cost price of the system
- \( C_4 \) = cost per unit time of repair by the service engineer
C_5 = \text{cost per unit time of replacement by the service engineer}

C_6 = \text{cost per visit by the service engineer}

**Graphical Interpretation and Conclusions**

Graphical analysis of the system has carried out at various stages of its operation for the following particular case:

\[ g_1(\cdot) = \beta_1 e^{-\beta_1 t}; \quad g_2(\cdot) = \beta_2 e^{-\beta_2 t}; \quad g_3(\cdot) = \beta_3 e^{-\beta_3 t}; \]
\[ h_1(\cdot) = \gamma_1 e^{-\gamma_1 t}; \quad h_2(\cdot) = \gamma_2 e^{-\gamma_2 t}; \quad h_3(\cdot) = \gamma_3 e^{-\gamma_3 t}. \]

Therefore we have

\[ p_{01} = \frac{a_1 \lambda_1}{\lambda_1 + \eta_1}; \quad p_{02} = \frac{b_1 \lambda_1}{\lambda_1 + \eta_1}; \quad p_{03} = \frac{\eta_1}{\lambda_1 + \eta_1}; \]
\[ p_{34} = \frac{a_2 \lambda_2}{\lambda_2 + \eta_2}; \quad p_{35} = \frac{b_2 \lambda_2}{\lambda_2 + \eta_2}; \quad p_{36} = \frac{\eta_2}{\lambda_2 + \eta_2}; \]
\[ p_{67} = a_3; \quad p_{68} = b_3; \quad p_{10} = p_{20} = p_{43} = p_{5,0} = p_{76} = p_{8,0} = 1; \]
\[ \mu_0 = \frac{1}{\lambda_1 + \eta_1}; \quad \mu_1 = \frac{1}{\beta_1}; \quad \mu_2 = \frac{1}{\gamma_1}; \]
\[ \mu_3 = \frac{1}{\lambda_2 + \eta_2}; \quad \mu_4 = \frac{1}{\beta_2}; \quad \mu_5 = \frac{1}{\gamma_2}; \]
\[ \mu_6 = \frac{1}{\lambda_3}; \quad \mu_7 = \frac{1}{\beta_3}; \quad \mu_8 = \frac{1}{\gamma_3}. \]

**Fig. 5.2** shows the behavior of MIT (I_0) with respect to failure rate (\(\lambda_1\)) during Stage-I for different values of repair rate (\(\beta_1\)) during that stage.

It can be concluded from the graph that I_0 increases with the increase in the values of \(\lambda_1\) when other parameters are fixed and has lower values for higher values of \(\beta_1\).
Fig. 5.2

Fig. 5.3 depicts the behavior of MTSF ($T_3$) of the system with respect to failure rate ($\lambda_2$) during Stage-II for three different values of deterioration rate ($\eta_2$).

It is evident from the graph that $T_3$ decreases with the increase in values of $\lambda_2$ when other parameters are fixed and decreases with the increase in the values of $\eta_2$. 

**Fig. 5.4** reveals the behavior of profit of the system user ($P_1$) with respect to failure rate ($\lambda_2$) during Stage-II of the system for different values of improvement rate ($\eta_1$).

**PROFIT OF SYSTEM USER ($P_1$) VERSUS FAILURE RATE ($\lambda_2$) FOR DIFFERENT VALUES OF IMPROVEMENT RATE ($\eta_1$)**

![Graph showing the profit of system user versus failure rate for different values of improvement rate.]

It can be concluded from the graph that $P_1$ decreases with the increase in the values of $\lambda_2$ and it increases with the increase in the values of $\eta_1$.

**Fig. 5.5**

**PROFIT OF SYSTEM USER ($P_1$) VERSUS FAILURE RATE ($\lambda_3$) FOR DIFFERENT VALUES OF REPAIR RATE ($\beta_3$)**

![Graph showing the profit of system user versus failure rate for different values of repair rate.]

**Fig. 5.5**
**Fig. 5.5** depicts the behaviour of profit of the system user ($P_1$) with respect to failure rate ($\lambda_3$) during Stage-III of the system for three different values of repair rate ($\beta_3$) at that stage.

It can be concluded from the graph that $P_1$ decreases with the increase in the values of $\lambda_3$ and has higher values for higher values of $\beta_3$.

**Fig. 5.6** reveals the behavior of profit of the system user ($P_1$) with respect to revenue per unit up time ($C_0$) of the system for different values of cost per visit ($C_3$) of the available repairman.

From the graph following conclusions can be drawn:

(i) The $P_1$ increases with the increase in the values of $C_0$ and has lower values for higher values of $C_3$.

(ii) For $C_3=100$, the $P_1$ is positive or zero or negative according as $C_0 >$ or $= 109.6693$ and hence for the system to be profitable, the revenue per unit up time ($C_0$), should be fixed greater than 109.6693 in this case.

(iii) For $C_3=200$, the $P_1$ is positive or zero or negative according as $C_0 >$ or $< 130.332$ and hence for the system to be profitable, the revenue per unit up time ($C_0$), should be fixed greater than 130.332 in this case.

(iv) For $C_3=300$, the $P_1$ is positive or zero or negative according as $C_0 >$ or $< 150.9917$ and hence for the system to be profitable, the revenue per unit up time ($C_0$), should be fixed greater than 150.9917 in this case.
unit up time \((C_0)\), should be fixed greater than 150.9917 in this case.

**Fig. 5.7** indicates the behaviour of profit of system provider \((P_2)\) with respect to failure rate \((\lambda_1)\) of the system during Stage-I for different values of improvement rate \((\eta_1)\).

![Graph of Profit of System Provider (P_2) versus Failure Rate (\lambda_1) for Different Values of Improvement Rate (\eta_1)](image)

From the graph it is clear that \(P_2\) decreases with the increase in the values of \(\lambda_1\) and it increases with the increase in the values of \(\eta_1\).

**Fig. 5.8** represents the behaviour of profit of system provider \((P_2)\) with respect to failure rate \((\lambda_2)\) during Stage-II of the system for different values of improvement rate \((\eta_1)\).
From the graph it is clear that \( P_2 \) decreases with the increase in the values of \( \lambda_2 \) and has higher values for higher values of \( \eta_1 \).

**Fig. 5.9** reveals the behaviour of profit of the system provider (\( P_2 \)) with respect to fixed profit of the system (SP-CP) for different values of cost per visit (\( C_6 \)) of service engineer.

From the graph following conclusions may be drawn:

(i) The \( P_2 \) increases with the increase in the values of (SP-CP) and \( P_2 \) has lower values for higher values of \( C_3 \).

(ii) For \( C_6 = 200 \), the \( P_2 \) is positive or zero or negative according as (SP-CP) > or = or < 45.76137 and hence, in this case, for profit to the system provider, the fixed profit (SP-CP) should be greater than 45.76137.

(iii) For \( C_6 = 400 \), the \( P_2 \) is positive or zero or negative according as (SP-CP) > or = or < 63.73309 and hence in this case, the fixed profit (SP-CP) should be fixed greater than 63.73309.

(iv) For \( C_6 = 400 \), the \( P_2 \) is positive or zero or negative according as (SP-CP) > or = or < 81.0127 and hence, in this case, the fixed profit (SP-CP) should be fixed greater than 81.0127 by the system provider.
Model-II

In this model it is assumed that on failure of the unit during warranty period service engineer will first inspect the system to know whether the fault is in a particular component or in the whole unit and also whether it is repairable/irrepairable. Then he will do the needful. The same process is followed by the available repairman after warranty period or wear out period on failure of the system.

Transition Probabilities and Mean Sojourn Times

A state transition diagram in figure 5.11 shows the various stats of transition of the system. The epochs of entering into states 0, 1, 4, 5, 10 and 11 are regenerative states. The states 2, 3, 6, 7, 8, 9, 12, 13, 14 and 15 are failed states and these are non-regenerative states. The states 1, 5 and 11 are down states. The transition probabilities are given by

\[ q_{01}(t) = p_1 \lambda_1 e^{-(\lambda_1 + \eta_1)t}; \]
\[ q_{02}(t) = a_1 q_1 \lambda_1 e^{-(\lambda_1 + \eta_1)t}; \]
\[ q_{03}(t) = b_1 q_1 \lambda_1 e^{-(\lambda_1 + \eta_1)t}; \]
\[ q_{10}(t) = q_{54}(t) = q_{11,10}(t) = g(t); \]
\[ q_{20}(t) = g_1(t); \]
\[ q_{30}(t) = h_1(t); \]
\[ q_{46}(t) = q_2 \lambda_2 e^{-(\lambda_2 + \eta_2) t}; \]
\[ q_{67}(t) = a_2 e^{-a_2 t}; \]
\[ q_{69}(t) = c_2 e^{-c_2 t}; \]
\[ q_{8,4}(t) = h_2(t); \]
\[ q_{10,11}(t) = p_3 \lambda_3 e^{-\lambda_3 t}; \]
\[ q_{12,13}(t) = a_3 e^{-a_3 t}; \]
\[ q_{12,15}(t) = c_3 e^{-c_3 t}; \]
\[ q_{14,10}(t) = h_3(t); \]
\[ q_{45}(t) = p_2 \lambda_2 e^{-(\lambda_2 + \eta_2) t}; \]
\[ q_{4,10}(t) = \eta_2 e^{-(\lambda_2 + \eta_2) t}; \]
\[ q_{68}(t) = b_2 e^{-b_2 t}; \]
\[ q_{74}(t) = g_2(t); \]
\[ q_{9,0}(t) = h(t); \]
\[ q_{10,12}(t) = q_3 \lambda_3 e^{-(\lambda_3 + \eta_3) t}; \]
\[ q_{12,14}(t) = b_3 e^{-b_3 t}; \]
\[ q_{13,10}(t) = g_3(t); \]
\[ q_{15,0}(t) = h(t); \]

The non-zero elements \( p_{ij} \) obtained as \( p_{ij} = \lim_{s \to 0} q_{ij}(s) \) and are given by

\[
\begin{align*}
p_{01} &= \frac{\lambda_1}{D_0}; & p_{02} &= \frac{a_1 \lambda_1}{D_0}; & p_{03} &= \frac{b_1 \lambda_1}{D_0}; \\
p_{04} &= \frac{\eta_1}{D_0}; & p_{45} &= \frac{\lambda_2}{D_1}; & p_{46} &= \frac{\lambda_2}{D_1}; \\
p_{67} &= a_2; & p_{68} &= b_2; & p_{69} &= c_2; \\
p_{10,11} &= p_3; & p_{10,12} &= q_3; \\
p_{12,13} &= a_3; & p_{12,14} &= b_3; & p_{12,15} &= c_3; \\
p_{10} &= p_{20} = p_{30} = p_{54} = p_{74} = p_{84} = p_{9,0} = p_{11,10} = p_{13,10} = p_{14,10} = p_{15,0} = 1 \\
\end{align*}
\]

where

\[ D_0 = \lambda_1 + \eta_1 \text{ and } D_1 = \lambda_2 + \eta_2 \]

By these transition probabilities, it can be verified that

\[
\begin{align*}
p_{01} + p_{02} + p_{03} + p_{04} &= p_{45} + p_{46} + p_{4,10} = p_{67} + p_{68} + p_{69} = p_{10,11} + p_{10,12} \\
&= p_{12,13} + p_{12,14} + p_{12,15} = q_1 = p_2 + q_2 = p_3 + q_3 = a_1 + b_1 = a_2 + b_2 + c_2 \\
&= a_3 + b_3 + c_3 = 1
\end{align*}
\]
The mean sojourn times ($\mu_i$) in state ‘i’ are:

$$\mu_0 = \frac{1}{D_0}; \quad \mu_1 = \mu_5 = \mu_{11} = \int_0^\infty G(t) \, dt; \quad \mu_2 = \int_0^\infty G_1(t) \, dt;$$

$$\mu_3 = \int_0^\infty H_1(t) \, dt; \quad \mu_4 = \frac{1}{D_1}; \quad \mu_6 = \int_0^\infty I_1(t) \, dt;$$

$$\mu_7 = \int_0^\infty G_2(t) \, dt; \quad \mu_8 = \int_0^\infty H_2(t) \, dt; \quad \mu_9 = \mu_{15} = \int_0^\infty H(t) \, dt;$$

$$\mu_{10} = \frac{1}{\lambda_3}; \quad \mu_{12} = \int_0^\infty I_2(t) \, dt; \quad \mu_{13} = \int_0^\infty G_3(t) \, dt;$$
\[ \mu_{14} = \int_0^\infty H_3(t) \, dt \]

where

\( D_0 \) and \( D_1 \) are already specified

The unconditional mean time taken by the system to transit for any state, ‘j’ when it is counted from epoch of entrance into state ‘i’ is mathematically stated as:

\[ m_{ij} = \int_0^\infty \, q_{ij}(t) \, dt = -q_{ij}^*(0) \]

Thus,

\[ m_{01} + m_{02} + m_{03} + m_{04} = \mu_0; \quad m_{10} = m_{54} = \mu_1 = \mu_5 = \mu_{11}; \]
\[ m_{20} = \mu_2; \quad m_{30} = \mu_3; \]
\[ m_{45} + m_{46} + m_{4,10} = \mu_4; \quad m_{67} + m_{68} + m_{69} = \mu_6; \]
\[ m_{74} = \mu_7; \quad m_{84} = \mu_8; \]
\[ m_{9,0} = \mu_9; \quad m_{10,11} + m_{10,12} = \mu_{10}; \]
\[ m_{12,13} + m_{12,14} + m_{12,15} = \mu_{12}; \quad m_{13,10} = \mu_{13}; \]
\[ m_{14,10} = \mu_{14}; \quad m_{15,0} = \mu_{15}; \]

**Mean Installation Time (MIT)**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( I_i(t) \):

\[ I_0(t) = Q_{01}(t) \otimes I_1(t) + Q_{02}(t) \otimes I_2(t) + Q_{03}(t) \otimes I_3(t) + Q_{04}(t); \]
\[ I_1(t) = Q_{10}(t) \otimes I_0(t); \quad I_2(t) = Q_{20}(t) \otimes I_0(t); \quad I_3(t) = Q_{30}(t) \otimes I_0(t); \]

Taking the L.S.T. of the above relations and then solving them for \( I_0**(s) \), we get

\[ I_0**(s) = \frac{N_0(s)}{D_2(s)} \]

where

\[ N_0(s) = Q_{04}**(s) \]
\[ D_2(s) = 1 - Q_{01}**(s)Q_{10}**(s) - Q_{02}**(s)Q_{20}**(s) - Q_{03}**(s)Q_{30}**(s) \]
In the steady-state, mean installation time is given by:

\[ I_0 = \lim_{s \to 0} \frac{1 - \Gamma^*_0(s)}{s} = \frac{N_0}{D_2} \]

where

\[ N_0 = \mu_0 + \mu_1 p_{01} + \mu_2 p_{02} + \mu_3 p_{03} \text{ and } D_2 = p_{04} \]

**Expected Busy Period of the Service Engineer (Components Maintenance Time Only) during Stage-I**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( E_i(t) \):

\[ E_0(t) = q_{01}(t) \circ E_1(t) + q_{02}(t) \circ E_2(t) + q_{03}(t) \circ E_3(t) + q_{04}(t) \circ E_4(t) \]

\[ E_1(t) = w_1(t) + q_{10}(t) \circ E_0(t) \]

\[ E_2(t) = q_{20}(t) \circ E_0(t) \]

\[ E_3(t) = q_{30}(t) \circ E_0(t) \]

\[ E_4(t) = q_{45}(t) \circ E_5(t) + q_{46}(t) \circ E_6(t) + q_{4,10}(t) \circ E_{10}(t) \]

\[ E_5(t) = q_{5,4}(t) \circ E_4(t) \]

\[ E_6(t) = q_{6,7}(t) \circ E_7(t) + q_{6,8}(t) \circ E_8(t) + q_{6,9}(t) \circ E_9(t) \]

\[ E_7(t) = q_{7,4}(t) \circ E_4(t) \]

\[ E_8(t) = q_{8,4}(t) \circ E_4(t) \]

\[ E_9(t) = q_{9,0}(t) \circ E_0(t) \]

\[ E_{10}(t) = q_{10,11}(t) \circ E_{11}(t) + q_{10,12}(t) \circ E_{12}(t) \]

\[ E_{11}(t) = q_{11,10}(t) \circ E_{10}(t) \]

\[ E_{12}(t) = q_{12,13}(t) \circ E_{13}(t) + q_{12,14}(t) \circ E_{14}(t) + q_{12,15}(t) \circ E_{15}(t) \]

\[ E_{13}(t) = q_{13,10}(t) \circ E_{10}(t) \]

\[ E_{14}(t) = q_{14,10}(t) \circ E_{10}(t) \]

\[ E_{15}(t) = q_{15,0}(t) \circ E_0(t) \]

where

\[ W_1(t) = \bar{G}(t) \]

Taking the L.T. of the above relations and then solving them for \( E_0^*(s) \), we get

\[ E_0^*(s) = \frac{N_1(s)}{D_3(s)} \]

where

\[ N_1(s) = W_1^*(s)q_{01}^*(s)[1 - q_{45}^*(s)q_{54}^*(s) - q_{46}^*(s)q_{67}^*(s)q_{74}^*(s) - q_{46}^*(s)] \]

\[ q_{68}^*(s)q_{84}^*(s)[1 - q_{10,11}^*(s)q_{11,10}^*(s) - q_{10,12}^*(s)q_{12,14}^*(s)q_{14,10}^*(s) \]

\[ - q_{10,12}^*(s)q_{12,14}^*(s)q_{14,10}^*(s) \]
\[
D_3(s) = [1 - q^*_{10,11}(s)q^*_{11,10}(s) - q^*_{10,12}(s)q^*_{12,13}(s)q^*_{13,10}(s) - q^*_{10,12}(s)q^*_{12,14}(s)q^*_{14,10}(s)]
\]

\[
q^*_{12,14}(s)q^*_{14,10}(s)[1 - q^*_{01}(s)q^*_{10}(s) - q^*_{02}(s)q^*_{20}(s) - q^*_{03}(s)q^*_{30}(s)]
\]

\[
q^*_{12,13}(s)q^*_{13,10}(s) - q^*_{10,12}(s)q^*_{12,14}(s)q^*_{14,10}(s)] [1 - q^*_{10}(s)q^*_{20}(s) - q^*_{03}(s)q^*_{30}(s)]
\]

\[
q^*_{10}(s) - q^*_{02}(s)q^*_{20}(s) - q^*_{03}(s)q^*_{30}(s) q^*_{67}(s)q^*_{74}(s) + q^*_{68}(s)
\]

\[
q^*_{84}(s)q^*_{46}(s) - q^*_{04}(s)q^*_{46}(s)q^*_{69}(s)q^*_{9,10}(s) [1 - q^*_{10,11}(s)]
\]

\[
q^*_{11,10}(s) - q^*_{10,12}(s)q^*_{12,13}(s)q^*_{13,10}(s) - q^*_{10,12}(s)q^*_{12,14}(s)q^*_{14,10}(s)]
\]

\[
- q^*_{04}(s)q^*_{4,10}(s)q^*_{10,12}(s)q^*_{12,15}(s)q^*_{15,0}(s)
\]

In the steady-state, the total fraction of time for which the system is under maintenance of the components only is given by:

\[
E_0 = \lim_{s \to 0} [s E^*_0(s)] = \frac{N_1}{D_3}
\]

where

\[
N_1 = \mu_1 p_{01} [p_{4,10} + p_{46} p_{49}] p_{10,12} p_{12,15}
\]

\[
D_3 = [\mu_0 + \mu_1 p_{01} + \mu_2 p_{02} + \mu_3 p_{03}] [p_{46} p_{69} + p_{4,10}] p_{10,12} p_{12,15}
\]

\[
+ \mu_4 p_{04} p_{10,12} p_{12,15} + [\mu_6 + \mu_7 p_{67} + \mu_8 p_{68} + \mu_9 p_{69}] p_{04} p_{46} p_{10,12}
\]

\[
p_{12,15} + [\mu_{10} + \mu_{11} p_{10,11} + \mu_{12} p_{10,12} + \mu_{13} p_{10,12} p_{12,13}
\]

\[
+ \mu_{14} p_{10,12} p_{12,14} + \mu_{15} p_{10,12} p_{12,15}] p_{04} p_{4,10}
\]

Expected Busy Period of the Service Engineer (Components Repair Time Only) during Stage-I

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for BR_i(t):

\[
BR_0(t) = q_{01}(t) \circ BR_1(t) + q_{02}(t) \circ BR_2(t) + q_{03}(t) \circ BR_3(t) + q_{04}(t) \circ BR_4(t);
\]

\[
BR_1(t) = q_{10}(t) \circ BR_0(t);
\]

\[
BR_2(t) = W_2(t) + q_{20}(t) \circ BR_0(t);
\]
\( \text{BR}_3(t) = q_{30}(t) \odot \text{BR}_0(t) \);

\( \text{BR}_4(t) = q_{45}(t) \odot \text{BR}_5(t) + q_{46}(t) \odot \text{BR}_6(t) + q_{4,10}(t) \odot \text{BR}_{10}(t) \);

\( \text{BR}_5(t) = q_{54}(t) \odot \text{BR}_4(t) \);

\( \text{BR}_6(t) = q_{67}(t) \odot \text{BR}_7(t) + q_{68}(t) \odot \text{BR}_8(t) + q_{69}(t) \odot \text{BR}_9(t) \);

\( \text{BR}_7(t) = q_{74}(t) \odot \text{BR}_4(t) \);

\( \text{BR}_8(t) = q_{84}(t) \odot \text{BR}_4(t) \);

\( \text{BR}_9(t) = q_{9,0}(t) \odot \text{BR}_0(t) \);

\( \text{BR}_{10}(t) = q_{10,11}(t) \odot \text{BR}_{11}(t) + q_{10,12}(t) \odot \text{BR}_{12}(t) \);

\( \text{BR}_{11}(t) = q_{11,10}(t) \odot \text{BR}_{10}(t) \);

\( \text{BR}_{12}(t) = q_{12,13}(t) \odot \text{BR}_{13}(t) + q_{12,14}(t) \odot \text{BR}_{14}(t) + q_{12,15}(t) \odot \text{BR}_{15}(t) \);

\( \text{BR}_{13}(t) = q_{13,10}(t) \odot \text{BR}_{10}(t) \);

\( \text{BR}_{14}(t) = q_{14,10}(t) \odot \text{BR}_{10}(t) \);

\( \text{BR}_{15}(t) = q_{15,0}(t) \odot \text{BR}_{0}(t) \);

where
\[
W_2(t) = G_1(t)
\]

Taking the L.T. of the above relations and then solving them for \( \text{BR}_0^*(s) \), we get
\[
\text{BR}_0^*(s) = \frac{N_2(s)}{D_3(s)}
\]

where
\[
N_2(s) = W_2^*(s)q_{02}^*(s)[1 - q_{45}^*(s)q_{54}^*(s) - q_{46}^*(s)q_{67}^*(s)q_{74}^*(s)]
\]
\[
- q_{46}^*(s)q_{68}^*(s)q_{84}^*(s)[1 - q_{10,11}^*(s)q_{11,10}^*(s)]
\]
\[
- q_{10,12}^*(s)[q_{12,13}^*(s)q_{13,10}^*(s) + q_{12,14}^*(s)q_{14,10}^*(s)]
\]

and \( D_3 \) is already specified.

In the steady-state, the total fraction of time for which the system is under repair of the components only by the service engineer is given by:
\[
\text{BR}_0 = \lim_{s \to 0} (s \text{ BR}_0^*(s)) = \frac{N_2}{D_3}
\]

where
\[
N_2 = \mu_2 p_{02} [p_{46} p_{69} + p_{4,10}] p_{10,12} p_{12,15} \text{ and } D_3 \text{ is already specified.}
**Expected Busy Period of the Service Engineer (Components Replacement Time Only) during Stage-I**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( BR_{p_i}(t) \):

\[
BR_{p_0}(t) = q_{01}(t) \odot BR_{p_1}(t) + q_{02}(t) \odot BR_{p_2}(t) + q_{03}(t) \odot BR_{p_3}(t) + q_{04}(t) \odot BR_{p_4}(t);
\]

\[
BR_{p_1}(t) = q_{10}(t) \odot BR_{p_0}(t);
\]

\[
BR_{p_2}(t) = q_{20}(t) \odot BR_{p_0}(t);
\]

\[
BR_{p_3}(t) = W_3(t) + q_{30}(t) \odot BR_{p_0}(t);
\]

\[
BR_{p_4}(t) = q_{45}(t) \odot BR_{p_5}(t) + q_{46}(t) \odot BR_{p_6}(t) + q_{410}(t) \odot BR_{p_{10}}(t);
\]

\[
BR_{p_5}(t) = q_{54}(t) \odot BR_{p_4}(t);
\]

\[
BR_{p_6}(t) = q_{67}(t) \odot BR_{p_7}(t) + q_{68}(t) \odot BR_{p_8}(t) + q_{69}(t) \odot BR_{p_9}(t);
\]

\[
BR_{p_7}(t) = q_{74}(t) \odot BR_{p_4}(t);
\]

\[
BR_{p_8}(t) = q_{84}(t) \odot BR_{p_4}(t);
\]

\[
BR_{p_9}(t) = q_{90}(t) \odot BR_{p_0}(t);
\]

\[
BR_{p_{10}}(t) = q_{10,11}(t) \odot BR_{p_{11}}(t) + q_{10,12}(t) \odot BR_{p_{12}}(t);
\]

\[
BR_{p_{11}}(t) = q_{11,10}(t) \odot BR_{p_{10}}(t);
\]

\[
BR_{p_{12}}(t) = q_{12,13}(t) \odot BR_{p_{13}}(t) + q_{12,14}(t) \odot BR_{p_{14}}(t) + q_{12,15}(t) \odot BR_{p_{15}}(t);
\]

\[
BR_{p_{13}}(t) = q_{13,10}(t) \odot BR_{p_{10}}(t);
\]

\[
BR_{p_{14}}(t) = q_{14,10}(t) \odot BR_{p_{10}}(t);
\]

\[
BR_{p_{15}}(t) = q_{15,0}(t) \odot BR_{p_0}(t);
\]

where

\[
W_3(t) = H_1(t)
\]

Taking the L.T. of the above relations and then solving them for \( BR_{p_0}^*(s) \), we get

\[
BR_{p_0}^*(s) = \frac{N_3(s)}{D_3(s)}
\]

where

\[
N_3(s) = W_3^*(s)q_{03}^*(s)[1 - q_{45}^*(s)q_{54}^*(s) - q_{46}^*(s)q_{67}^*(s)q_{74}^*(s) - q_{46}^*(s)q_{68}^*(s)\]
\]

\[
q_{84}^*(s)] [1 - q_{10,11}^*q_{11,10}^*(s) - q_{10,12}^*(s)q_{12,13}^*(s)q_{13,10}^*(s) + q_{12,14}^*(s)q_{14,10}^*(s)]
\]

and \( D_3 \) is already specified.
In the steady-state, the total fraction of time for which the system is under replacement of the component only by the service engineer is given by:

\[ \text{BRp}_0 = \lim_{s \to 0} [s \, \text{BRp}_0^*(s)] = \frac{N_3}{D_3} \]

where

\[ N_3 = \mu_3 \, p_{03} \, [p_{46} \, p_{69} + p_{4,10}] \, p_{10,12} \, p_{12,15} \]
and \( D_3 \) is already defined.

**Mean Time to System Failure of the System during Stage-II**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( \phi_i(t) \):

\[ \phi_4(t) = Q_{45}^*(t) \, \phi_5(t) + Q_{46}(t) + Q_{4,10}(t); \quad \phi_5(t) = Q_{54}^*(t) \, \phi_4(t) \]

Taking the L.S.T. of the above relations solving them for \( \phi_4^{**}(s) \), we get

\[ \phi_4^{**}(s) = \frac{N_4(s)}{D_4(s)} \]

where

\[ N_4(s) = Q_{46}^{**}(s) + Q_{4,10}^{**}(s) \]
and \( D_4(s) = 1 - Q_{45}^{**}(s)Q_{54}^{**}(s) \)

In the steady-state the mean time to system failure is given by

\[ T_4 = \lim_{s \to 0} \frac{1 - \phi_4^{**}(s)}{s} = \frac{N_4}{D_4} \]

where

\[ N_4 = \mu_4 + \mu_5 \, p_{45} \]
and \( D_4 = 1 - p_{45} \)

**Availability Analysis during Stage-II**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( A_i(t) \):

\[ A_0(t) = q_{01}(t) \, A_1(t) + q_{02}(t) \, A_2(t) + q_{03}(t) \, A_3(t) + q_{04}(t) \, A_4(t); \]
\[ A_1(t) = q_{10}(t) \, A_0(t); \]
\[ A_2(t) = q_{20}(t) \, A_0(t); \]
\[ A_3(t) = q_{30}(t) \, A_0(t); \]
\[ A_4(t) = M_4(t) + q_{45}(t) \, A_5(t) + q_{46}(t) \, A_6(t) + q_{4,10}(t) \, A_{10}(t) \]
\[ A_3(t) = q_{54}(t) \circ A_4(t); \quad A_4(t) = q_{67}(t) \circ A_7(t) + q_{68}(t) \circ A_8(t) + q_{69}(t) \circ A_9(t); \]
\[ A_7(t) = q_{74}(t) \circ A_4(t); \quad A_8(t) = q_{84}(t) \circ A_4(t); \quad A_9(t) = q_{90}(t) \circ A_0(t); \]
\[ A_{10}(t) = q_{10,11}(t) \circ A_{11}(t) + q_{10,12}(t) \circ A_{12}(t); \]
\[ A_{11}(t) = q_{11,10}(t) \circ A_{10}(t); \]
\[ A_{12}(t) = q_{12,13}(t) \circ A_{13}(t) + q_{12,14}(t) \circ A_{14}(t) + q_{12,15}(t) \circ A_{15}(t); \]
\[ A_{13}(t) = q_{13,10}(t) \circ A_{10}(t); \quad A_{14}(t) = q_{14,10}(t) \circ A_{10}(t); \quad A_{15}(t) = q_{15,0}(t) \circ A_0(t); \]

where
\[ M_4(t) = e^{-(\lambda_2 + \eta_2) t} \]

Taking the L.T. of the above relations and then solving them for \( A_4^*(s) \), we get
\[ A_4^*(s) = \frac{N_5(s)}{D_3(s)} \]

where
\[ N_5(s) = M_4^*(s)[1 - q_{101}^*(s)q_{10}^*(s) - q_{02}^*(s)q_{20}^*(s) - q_{03}^*(s)q_{30}^*(s)] \]
\[ [1 - q_{10,11}^*(s)q_{11,10}^*(s) - q_{10,12}^*(s)](q_{12,13}^*(s)q_{13,10}^*(s) + q_{12,14}^*(s)q_{14,10}^*(s)) \]

and \( D_3(s) \) is already specified.

In the steady-state, the availability of the system is given by:
\[ A_4 = \lim_{s \to 0} [sA_4^*(s)] = \frac{N_5}{D_3} \]

where
\[ N_5 = \mu_4 p_04 p_{10,12} p_{12,15} \]
and \( D_3 \) is already specified.

**Expected Busy Period of the Service Engineer (Components Maintenance Time Only) during Stage-II**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( E_i(t) \):
\[ E_0(t) = q_{01}(t) \circ E_1(t) + q_{02}(t) \circ E_2(t) + q_{03}(t) \circ E_3(t) + q_{04}(t) \circ E_4(t); \]
\[ E_1(t) = q_{10}(t) \circ E_0(t); \quad E_2(t) = q_{20}(t) \circ E_0(t); \quad E_3(t) = q_{30}(t) \circ E_0(t); \]
\[ E_4(t) = q_{45}(t) \otimes E_5(t) + q_{46}(t) \otimes E_6(t) + q_{4,10}(t) \otimes E_{10}(t); \]
\[ E_5(t) = W_5(t) + q_{54}(t) \otimes E_4(t); \]
\[ E_6(t) = q_{74}(t) \otimes E_5(t); \]
\[ E_7(t) = q_{84}(t) \otimes E_5(t); \]
\[ E_8(t) = q_{67}(t) \otimes E_7(t) + q_{68}(t) \otimes E_8(t) + q_{69}(t) \otimes E_9(t); \]
\[ E_9(t) = q_{10,11}(t) \otimes E_{10}(t); \]
\[ E_{10}(t) = q_{10,11}(t) \otimes E_{11}(t) + q_{10,12}(t) \otimes E_{12}(t); \]
\[ E_{11}(t) = q_{11,10}(t) \otimes E_{10}(t); \]
\[ E_{12}(t) = q_{12,13}(t) \otimes E_{13}(t) + q_{12,14}(t) \otimes E_{14}(t) + q_{12,15}(t) \otimes E_{15}(t); \]
\[ E_{13}(t) = q_{13,10}(t) \otimes E_{10}(t); \]
\[ E_{14}(t) = q_{14,10}(t) \otimes E_{10}(t); \]
\[ E_{15}(t) = q_{15,0}(t) \otimes E_0(t); \]

where

\[ W_5(t) = G(t) \]

Taking the L.T. of the above relations and then solving them for \( E_4^*(s) \), we get

\[ E_4^*(s) = \frac{N_6(s)}{D_3(s)} \]

where

\[ N_6(s) = W_5^*(s) q_{45}^*(s) \{ 1 - q_{01}^*(s) q_{10}^*(s) - q_{02}^*(s) q_{20}^*(s) - q_{03}^*(s) q_{30}^*(s) \} \]

\[ \times [1 - q_{10,11}^*(s) q_{11,10}^*(s) - q_{10,12}^*(s) \{ (q_{12,13}^*(s) q_{13,10}^*(s) + q_{12,14}^*(s) q_{14,10}^*(s) \} ] \]

and \( D_3(s) \) is already specified.

In the steady-state, the total fraction of time for which the system is under maintenance of components only by the service engineer is given by:

\[ E_4 = \lim_{s \to 0} [s E_4^*(s)] = \frac{N_6}{D_3} \]

where

\[ N_6 = \mu_5 p_{04} p_{45} p_{10,12} p_{12,15} \]

and \( D_3 \) is already specified.

**Expected Busy Period of the Service Engineer (Inspection Time Only) during Stage-II**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( B I_i(t) \):

\[ B I_0(t) = q_{01}(t) \otimes B I_1(t) + q_{02}(t) \otimes B I_2(t) + q_{03}(t) \otimes B I_3(t) + q_{04}(t) \otimes B I_4(t); \]
\[ B I_1(t) = q_{10}(t) \otimes B I_0(t); \]
\[ B I_2(t) = q_{20}(t) \otimes B I_0(t); \]
\[ B I_3(t) = q_{30}(t) \otimes B I_0(t); \]
\[ BI_4(t) = q_{45}(t)BI_5(t) + q_{46}(t)BI_6(t) + q_{4,10}(t)BI_{10}(t); \quad BI_5(t) = q_{54}(t)BI_4(t); \]
\[ BI_6(t) = W_6(t) + q_{67}(t)BI_7(t) + q_{68}(t)BI_8(t) + q_{69}(t)BI_9(t); \]
\[ BI_7(t) = q_{74}(t)BI_4(t); \quad BI_8(t) = q_{84}(t)BI_4(t); \quad BI_9(t) = q_{9,0}(t)BI_{10}(t); \]
\[ BI_{10}(t) = q_{10,11}(t)BI_{11}(t) + q_{10,12}(t)BI_{12}(t); \]
\[ BI_{11}(t) = q_{11,10}(t)BI_{10}(t); \quad BI_{12}(t) = q_{12,13}(t)BI_{13}(t) + q_{12,14}(t)BI_{14}(t) + q_{12,15}(t)BI_{15}(t); \]
\[ BI_{13}(t) = q_{13,10}(t)BI_{10}(t); \quad BI_{14}(t) = q_{14,10}(t)BI_{10}(t); \quad BI_{15}(t) = q_{15,0}(t)BI_{0}(t); \]

where
\[ W_6(t) = I_7(t) \]

Taking the L.T. of the above relations and then solving them for \( BI_4^*(s) \), we get
\[ BI_4^*(s) = \frac{N_7(s)}{D_3(s)} \]

where
\[ N_7(s) = W_6^*(s)q_{46}^*(s)[1 - q_{01}^*(s)q_{10}^*(s) - q_{02}^*(s)q_{20}^*(s) - q_{03}^*(s)q_{30}^*(s)] \]
\[ [1 - q_{10,11}^*(s)q_{11,10}^*(s) - q_{10,12}^*(s)[q_{12,13}^*(s)q_{13,10}^*(s) + q_{12,14}^*(s)q_{14,10}^*(s)]] \]

and \( D_3(s) \) is already specified.

In the steady-state, the total fraction of time for which the system is under inspection by the service engineer is given by:
\[ BI_4 = \lim_{s \to 0} [s \cdot BI_4^*(s)] = \frac{N_7}{D_3} \]

where
\[ N_7 = \mu_6 \ p_{04} \ p_{46} \ p_{10,12} \ p_{12,15} \] and \( D_3 \) is already specified.

**Expected Busy Period of the Service Engineer (Components Repair Time Only) during Stage-II**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( BR_i(t) \):
\[ BR_0(t) = q_{01}(t)BR_1(t) + q_{02}(t)BR_2(t) + q_{03}(t)BR_3(t) + q_{04}(t)BR_4(t); \]
$BR_1(t) = q_{10}(t) \cdot BR_0(t); \quad BR_2(t) = q_{20}(t) \cdot BR_0(t); \quad BR_3(t) = q_{30}(t) \cdot BR_0(t);$

$BR_4(t) = q_{45}(t) \cdot BR_5(t) + q_{46}(t) \cdot BR_6(t) + q_{4,10}(t) \cdot BR_{10}(t);$

$BR_5(t) = q_{54}(t) \cdot BR_4(t); \quad BR_6(t) = q_{67}(t) \cdot BR_7(t) + q_{68}(t) \cdot BR_8(t) + q_{69}(t) \cdot BR_9(t);$

$BR_7(t) = W_7(t) + q_{74}(t) \cdot BR_4(t); \quad BR_8(t) = q_{84}(t) \cdot BR_4(t);$

$BR_9(t) = q_{9,0}(t) \cdot BR_0(t); \quad BR_{10}(t) = q_{10,11}(t) \cdot BR_{11}(t) + q_{10,12}(t) \cdot BR_{12}(t);$

$BR_{11}(t) = q_{11,10}(t) \cdot BR_{10}(t); \quad BR_{12}(t) = q_{12,13}(t) \cdot BR_{13}(t) + q_{12,14}(t) \cdot BR_{14}(t) + q_{12,15}(t) \cdot BR_{15}(t);$

$BR_{13}(t) = q_{13,10}(t) \cdot BR_{10}(t); \quad BR_{14}(t) = q_{14,10}(t) \cdot BR_{10}(t); \quad BR_{15}(t) = q_{15,0}(t) \cdot BR_0(t);$

where

$W_7(t) = \overline{G_2(t)}$

Taking the L.T. of the above relations and then solving them for $BR_4^*(s)$ we get

$BR_4^*(s) = \frac{N_8(s)}{D_3(s)}$

where

$N_8(s) = W_7^*(s) \cdot q_{46}^*(s) \cdot q_{67}^*(s) \cdot [1 - q_{10}^*(s) \cdot q_{10}^*(s) - q_{20}^*(s) \cdot q_{30}^*(s) - q_{30}^*(s) \cdot q_{30}^*(s)]$

$[1 - q_{10,11}^*(s) \cdot q_{11,10}^*(s) - q_{10,12}^*(s) \cdot q_{12,13}^*(s) \cdot q_{13,10}^*(s) + q_{12,14}^*(s) \cdot q_{14,10}^*(s)]$

and $D_3$ is already specified.

In the steady-state, the total fraction of time for which the system is under repair of the components only by the service engineer is given by:

$BR_4 = \lim_{s \to 0} [s \cdot BR_4^*(s)] = \frac{N_8}{D_3}$

where

$N_8 = \mu_7 \cdot p_{04} \cdot p_{46} \cdot p_{10,12} \cdot p_{12,15}$ and $D_3$ is already specified.
Expected Busy Period of the Service Engineer (Components Replacement Time Only) during Stage-II

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for $\text{BRp}_1(t)$:

\begin{align*}
\text{BRp}_0(t) &= q_{01}(t) \odot \text{BRp}_1(t) + q_{02}(t) \odot \text{BRp}_2(t) + q_{03}(t) \odot \text{BRp}_3(t) + q_{04}(t) \odot \text{BRp}_4(t); \\
\text{BRp}_1(t) &= q_{10}(t) \odot \text{BRp}_0(t); \\
\text{BRp}_2(t) &= q_{20}(t) \odot \text{BRp}_0(t); \\
\text{BRp}_3(t) &= q_{30}(t) \odot \text{BRp}_0(t); \\
\text{BRp}_4(t) &= q_{45}(t) \odot \text{BRp}_5(t) + q_{46}(t) \odot \text{BRp}_6(t) + q_{410}(t) \odot \text{BRp}_{10}(t); \\
\text{BRp}_5(t) &= q_{54}(t) \odot \text{BRp}_4(t); \\
\text{BRp}_6(t) &= q_{67}(t) \odot \text{BRp}_7(t) + q_{68}(t) \odot \text{BRp}_8(t) + q_{69}(t) \odot \text{BRp}_{9}(t); \\
\text{BRp}_7(t) &= q_{74}(t) \odot \text{BRp}_4(t); \\
\text{BRp}_8(t) &= W_8(t) + q_{84}(t) \odot \text{BRp}_4(t); \\
\text{BRp}_9(t) &= q_{90}(t) \odot \text{BRp}_0(t); \\
\text{BRp}_{10}(t) &= q_{1011}(t) \odot \text{BRp}_{11}(t) + q_{1012}(t) \odot \text{BRp}_{12}(t); \\
\text{BRp}_{11}(t) &= q_{1110}(t) \odot \text{BRp}_{10}(t); \\
\text{BRp}_{12}(t) &= q_{1213}(t) \odot \text{BRp}_{13}(t) + q_{1214}(t) \odot \text{BRp}_{14}(t) + q_{1215}(t) \odot \text{BRp}_{15}(t); \\
\text{BRp}_{13}(t) &= q_{1310}(t) \odot \text{BRp}_{10}(t); \\
\text{BRp}_{14}(t) &= q_{1410}(t) \odot \text{BRp}_{10}(t); \\
\text{BRp}_{15}(t) &= q_{150}(t) \odot \text{BRp}_0(t); \\
\end{align*}

where

\[ W_8(t) = \overline{H_2(t)} \]

Taking the L.T. of the above relations and then solving them for $\text{BRp}_4^*(s)$ we get

\[ \text{BRp}_4^*(s) = \frac{N_9(s)}{D_3(s)} \]

where

\[ N_9(s) = W_8^*(s)q_{46}^*(s)q_{68}^*(s)[1 - q_{01}^*(s)q_{10}^*(s) - q_{02}^*(s)q_{20}^*(s) - q_{03}^*(s)q_{30}^*(s)] \]
\[ [1 - q_{1011}^*(s)q_{1110}^*(s) - q_{1012}^*(s)q_{1213}^*(s)q_{1310}^*(s) + q_{1214}^*(s)q_{1410}^*(s)] \]

and $D_3(s)$ is already specified.
In the steady-state, the total fraction of time for which the system is under replacement of the component only by the service engineer is given by:

\[ \text{BR}_{p_4} = \lim_{s \to 0} [s \text{BR}_{p_4}^*(s)] = \frac{N_9}{D_3} \]

where

\[ N_9 = \mu_8 \ p_{04} \ p_{46} \ p_{68} \ p_{10, 12} \ p_{12, 15} \text{ and } D_3 \text{ is already specified.} \]

**Expected Busy Period of the Service Engineer (Re-Installation Time Only) during Stage-II**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( \text{BR}_{f_i}(t) \):

\[
\begin{align*}
\text{BR}_{f_0}(t) &= q_{01}(t) \circ \text{BR}_{f_1}(t) + q_{02}(t) \circ \text{BR}_{f_2}(t) + q_{03}(t) \circ \text{BR}_{f_3}(t) + q_{04}(t) \circ \text{BR}_{f_4}(t); \\
\text{BR}_{f_1}(t) &= q_{10}(t) \circ \text{BR}_{f_0}(t); \quad \text{BR}_{f_2}(t) = q_{20}(t) \circ \text{BR}_{f_0}(t); \\
\text{BR}_{f_3}(t) &= q_{30}(t) \circ \text{BR}_{f_0}(t); \\
\text{BR}_{f_4}(t) &= q_{45}(t) \circ \text{BR}_{f_5}(t) + q_{46}(t) \circ \text{BR}_{f_6}(t) + q_{4,10}(t) \circ \text{BR}_{f_{10}}(t); \\
\text{BR}_{f_5}(t) &= q_{54}(t) \circ \text{BR}_{f_4}(t); \\
\text{BR}_{f_6}(t) &= q_{67}(t) \circ \text{BR}_{f_7}(t) + q_{68}(t) \circ \text{BR}_{f_8}(t) + q_{69}(t) \circ \text{BR}_{f_9}(t); \\
\text{BR}_{f_7}(t) &= q_{74}(t) \circ \text{BR}_{f_4}(t); \quad \text{BR}_{f_8}(t) = q_{84}(t) \circ \text{BR}_{f_4}(t); \\
\text{BR}_{f_9}(t) &= W_9(t) + q_{9,0}(t) \circ \text{BR}_{f_0}(t); \\
\text{BR}_{f_{10}}(t) &= q_{10,11}(t) \circ \text{BR}_{f_{11}}(t) + q_{10,12}(t) \circ \text{BR}_{f_{12}}(t); \\
\text{BR}_{f_{11}}(t) &= q_{11,10}(t) \circ \text{BR}_{f_{10}}(t); \\
\text{BR}_{f_{12}}(t) &= q_{12,13}(t) \circ \text{BR}_{f_{13}}(t) + q_{12,14}(t) \circ \text{BR}_{f_{14}}(t) + q_{12,15}(t) \circ \text{BR}_{f_{15}}(t); \\
\text{BR}_{f_{13}}(t) &= q_{13,10}(t) \circ \text{BR}_{f_{10}}(t); \quad \text{BR}_{f_{14}}(t) = q_{14,10}(t) \circ \text{BR}_{f_{10}}(t); \\
\text{BR}_{f_{15}}(t) &= q_{15,0}(t) \circ \text{BR}_{f_0}(t); \\
\end{align*}
\]

where

\[ W_9(t) = \overline{H(t)} \]
Taking the L.T. of the above relations and then solving them for $BRf_4^*(s)$, we get

$$BRf_4^*(s) = \frac{N_{10}(s)}{D_3(s)}$$

where

$$N_{10}(s) = W_0^*(s)q_{46}^*(s)q_{60}^*(s)[1 - q_{01}^*(s)q_{10}^*(s) - q_{02}^*(s)q_{20}^*(s) - q_{03}^*(s)q_{30}^*(s)]$$

$$[1 - q_{10,11}^*(s)q_{11,10}^*(s) - q_{10,12}^*(s)[q_{12,13}^*(s)q_{13,10}^*(s) + q_{12,14}^*(s)q_{14,10}^*(s)]]$$

and $D_3(s)$ is already specified.

In the steady-state, the total fraction of time for which the system is under re-installation of the unit only by the service engineer is given by:

$$BRf_4 = \lim_{s \to 0}[s BRf_4^*(s)] = \frac{N_{10}}{D_3}$$

where

$$N_{10} = \mu_0 p_{04} p_{46} p_{69} p_{10,12} p_{12,15}$$ and $D_3$ is already specified.

**Expected Numbers of Visits by the Service Engineer during Stage-II**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for $V_i(t)$:

- $V_0(t) = Q_{01}(t) \preceq V_1(t) + Q_{02}(t) \preceq V_2(t) + Q_{03}(t) \preceq V_3(t) + Q_{04}(t) \preceq V_4(t)$;
- $V_1(t) = Q_{10}(t) \preceq V_0(t)$;
- $V_2(t) = Q_{20}(t) \preceq V_1(t)$;
- $V_3(t) = Q_{30}(t) \preceq V_2(t)$;
- $V_4(t) = Q_{45}(t) \preceq V_5(t) + Q_{46}(t) \preceq [1 + V_6(t)] + Q_{4,10}(t) \preceq V_{10}(t)$;
- $V_5(t) = Q_{54}(t) \preceq V_4(t)$;
- $V_6(t) = Q_{67}(t) \preceq V_7(t)$;
- $V_7(t) = Q_{74}(t) \preceq V_4(t)$;
- $V_8(t) = Q_{84}(t) \preceq V_4(t)$;
- $V_9(t) = Q_{9,0}(t) \preceq V_0(t)$;
- $V_{10}(t) = Q_{10,11}(t) \preceq V_{11}(t) + Q_{10,12}(t) \preceq V_{12}(t)$;
- $V_{11}(t) = Q_{11,10}(t) \preceq V_{10}(t)$;
- $V_{12}(t) = Q_{12,13}(t) \preceq V_{13}(t) + Q_{12,14}(t) \preceq V_{14}(t) + Q_{12,15}(t) \preceq V_{15}(t)$;
- $V_{13}(t) = Q_{13,10}(t) \preceq V_{10}(t)$;
- $V_{14}(t) = Q_{14,10}(t) \preceq V_{10}(t)$;
- $V_{15}(t) = Q_{15,0}(t) \preceq V_0(t)$.

Taking the L.S.T. of the above relations and then solving them for $V_4^{**}(s)$, we get

$$V_4^{**}(s) = \frac{N_{11}(s)}{D_3(s)}$$
where
\[
N_{11}(s) = Q_{46}^{**}(s)[1 - Q_{10}^{**}(s)Q_{11}^{**}(s) - Q_{02}^{**}(s)Q_{20}^{**}(s) - Q_{03}^{**}(s)Q_{30}^{**}(s)]
[1 - Q_{10,11}^{**}(s)Q_{11,10}^{**}(s) - Q_{4,10}^{**}(s)Q_{4,10}^{**}(s)]
\]
and D_3 is already specified.

In the steady-state, the expected number of visits per unit time by the service engineer is given by
\[
V_4 = \lim_{s \to 0} [s V_4^{**}(s)] = \frac{N_{11}}{D_3}
\]
where
\[
N_{11} = p_{04} p_{10} p_{10,12} p_{10,15} \text{ and } D_3 \text{ is already specified.}
\]

**Mean Time to System Failure during Stage-III**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( \phi_i(t) \):
\[
\begin{align*}
\phi_0(t) &= Q_{01}(t) \Phi_1(t) + Q_{02}(t) + Q_{03}(t) + Q_{04}(t) \Phi_4(t); \\
\phi_1(t) &= Q_{10}(t) \Phi_0(t); \\
\phi_4(t) &= Q_{45}(t) \Phi_5(t) + Q_{46}(t) + Q_{4,10}(t) \Phi_{10}(t); \\
\phi_5(t) &= Q_{54}(t) \Phi_4(t); \\
\phi_{10}(t) &= Q_{10,11}(t) \Phi_{11}(t) + Q_{10,12}(t); \\
\phi_{11}(t) &= Q_{11,10}(t) \Phi_{10}(t);
\end{align*}
\]
Taking the L.S.T. of the above relations and then solving them for \( \phi_0^{**}(s) \), we get
\[
\phi_0^{**}(s) = \frac{N_{12}(s)}{D_5(s)}
\]
where
\[
N_{12}(s) = [Q_{02}^{**}(s) + Q_{03}^{**}(s)][1 - Q_{45}^{**}(s)Q_{54}^{**}(s)][1 - Q_{10,11}^{**}(s)Q_{11,10}^{**}(s)]
+ Q_{04}^{**} Q_{46}^{**}(s)[1 - Q_{10,11}^{**}(s)Q_{11,10}^{**}(s)] + Q_{04}^{**}(s)Q_{4,10}^{**}(s)Q_{10,12}^{**}(s) -
D_3(s) = [1 - Q_{01}^{**}(s)Q_{10}^{**}(s)][1 - Q_{45}^{**}(s)Q_{54}^{**}(s)][1 - Q_{10,11}^{**}(s)Q_{11,10}^{**}(s)]
\]
In the steady-state mean time to system failure is given by
\[
T_{10} = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s} = \frac{N_{12}}{D_5}
\]
where
\[
N_{12} = (\mu_0 + \mu_1) (1-p_{45}) p_{10,12} + (\mu_4 + \mu_5 p_{45}) p_{04} p_{10,12} + [\mu_{10} + \mu_{11} p_{10,11}] p_{04} p_{4,10}
\]
\[
D_5 = [1-p_{01}] [1-p_{45}] p_{10,12}
\]

Availability Analysis of the System during Stage-III

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( A_i(t) \):
\[
A_0(t) = q_{01}(t) A_1(t) + q_{02}(t) A_2(t) + q_{03}(t) A_3(t) + q_{04}(t) A_4(t);
\]
\[
A_1(t) = q_{10}(t) A_0(t);
\]
\[
A_2(t) = q_{20}(t) A_0(t);
\]
\[
A_3(t) = q_{30}(t) A_0(t);
\]
\[
A_4(t) = q_{40}(t) A_0(t);
\]
\[
A_5(t) = q_{54}(t) A_4(t);
\]
\[
A_6(t) = q_{67}(t) A_7(t) + q_{68}(t) A_8(t) + q_{69}(t) A_9(t);
\]
\[
A_7(t) = q_{74}(t) A_4(t);
\]
\[
A_8(t) = q_{84}(t) A_4(t);
\]
\[
A_9(t) = q_{9,0}(t) A_0(t);
\]
\[
A_{10}(t) = W_{10}(t) + q_{10,11}(t) A_{11}(t) + q_{10,12}(t) A_{12}(t);
\]
\[
A_{11}(t) = q_{11,10}(t) A_{10}(t);
\]
\[
A_{12}(t) = q_{12,13}(t) A_{13}(t) + q_{12,14}(t) A_{14}(t) + q_{12,15}(t) A_{15}(t);
\]
\[
A_{13}(t) = q_{13,10}(t) A_{10}(t);
\]
\[
A_{14}(t) = q_{14,10}(t) A_{10}(t);
\]
\[
A_{15}(t) = q_{15,0}(t) A_0(t);
\]

where \( W_{10}(t) = e^{-\lambda_3 t} \)

Taking the L.T. of the above relations and then solving them for \( A_{10}^*(s) \), we get
\[
A_{10}^*(s) = \frac{N_{13}(s)}{D_3(s)}
\]

where
\[
N_{13}(s) = W_{10}^*(s) [1 - q_{01}^*(s) q_{10}^*(s) - q_{02}^*(s) q_{20}^*(s) - q_{03}^*(s) q_{30}^*(s)]
\]
\[
[1 - q_{45}^*(s) q_{54}^*(s) - q_{46}^*(s) (q_{67}^*(s) q_{74}^*(s) + q_{68}^*(s) q_{84}^*(s))]
\]
\[
-W_{10}^*(s) q_{04}^*(s) q_{46}^*(s) q_{69}^*(s) q_{9,0}^*(s)
\]

and \( D_3(s) \) is already specified.
In the steady-state, the availability of the system is given by:

\[ A_{10} = \lim_{s \to 0} [s A_{10}^*(s)] = \frac{N_{13}}{D_3} \]

where

\[ N_{13} = \mu_{10} p_{04} p_{4,10} \quad \text{and} \quad D_3 \quad \text{is already specified.} \]

**Expected Busy Period of the Available Repairman (Components Maintenance Time Only) during Stage-III**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( E_i(t) \):

\[ E_0(t) = q_{01}(t) \circ E_1(t) + q_{02}(t) \circ E_2(t) + q_{03}(t) \circ E_3(t) + q_{04}(t) \circ E_4(t); \]
\[ E_1(t) = q_{10}(t) \circ E_0(t); \quad E_2(t) = q_{20}(t) \circ E_0(t); \quad E_3(t) = q_{30}(t) \circ E_0(t); \]
\[ E_4(t) = q_{45}(t) \circ E_2(t) + q_{46}(t) \circ E_6(t) + q_{4,10}(t) \circ E_{10}(t); \]
\[ E_5(t) = q_{54}(t) \circ E_4(t); \]
\[ E_6(t) = q_{67}(t) \circ E_4(t) + q_{68}(t) \circ E_8(t) + q_{69}(t) \circ E_9(t); \]
\[ E_7(t) = q_{74}(t) \circ E_4(t); \]
\[ E_8(t) = q_{84}(t) \circ E_4(t); \]
\[ E_9(t) = q_{9,0}(t) \circ E_0(t); \]
\[ E_{10}(t) = q_{10,11}(t) \circ E_{11}(t) + q_{10,12}(t) \circ E_{12}(t); \]
\[ E_{11}(t) = W_{11}(t) + q_{11,10}(t) \circ E_{10}(t); \]
\[ E_{12}(t) = q_{12,13}(t) \circ E_{13}(t) + q_{12,14}(t) \circ E_{14}(t) + q_{12,15}(t) \circ E_{15}(t); \]
\[ E_{13}(t) = q_{13,10}(t) \circ E_{10}(t); \quad E_{14}(t) = q_{14,10}(t) \circ E_{10}(t); \quad E_{15}(t) = q_{15,0}(t) \circ E_0(t); \]

where

\[ W_{11}(t) = \overline{G(t)} \]

Taking the L.T. of the above relations and then solving them for \( E_{10}^*(s) \), we get

\[ E_{10}^*(s) = \frac{N_{14}(s)}{D_3(s)} \]

where

\[ N_{14}(s) = W_{11}(s) q_{10,11}^*(s) [1 - q_{01}^*(s) q_{10}^*(s) + q_{02}^*(s) q_{20}^*(s)] \]
\[ - q_{03}^*(s) q_{30}^*(s) [1 - q_{45}^*(s) q_{54}^*(s) - q_{46}^*(s) q_{67}^*(s)] \]
\[ + q_{68}^*(s) q_{84}^*(s) \}

\[ - W_{11}(s) q_{04}^*(s) q_{46}^*(s) q_{69}^*(s) q_{9,0}^*(s) q_{10,11}^*(s) \]

and \( D_3 \) is already specified.
In the steady-state, the total fraction of time for which the system is under maintenance of components only by the available repairman is given by:

\[ E_{10} = \lim_{s \to 0} \left[ sE_{10}^*(s) \right] = \frac{N_{14}}{D_3} \]

where

\[ N_{14} = \mu_{11} p_{04} p_{10,11} \] and \( D_3 \) is already specified.

**Expected Busy Period of the Available Repairman (Inspection Time Only) during Stage-III**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( B_{I_i}(t) \):

\[
\begin{align*}
B_{I_0}(t) &= q_{01}(t)\circ B_{I_1}(t) + q_{02}(t)\circ B_{I_2}(t) + q_{03}(t)\circ B_{I_3}(t) + q_{04}(t)\circ B_{I_4}(t); \\
B_{I_1}(t) &= q_{10}(t)\circ B_{I_0}(t); \\
B_{I_2}(t) &= q_{20}(t)\circ B_{I_0}(t); \\
B_{I_3}(t) &= q_{30}(t)\circ B_{I_0}(t); \\
B_{I_4}(t) &= q_{45}(t)\circ B_{I_5}(t) + q_{46}(t)\circ B_{I_6}(t) + q_{4,10}(t)\circ B_{I_10}(t); \\
B_{I_5}(t) &= q_{54}(t)\circ B_{I_4}(t); \\
B_{I_6}(t) &= q_{54}(t)\circ B_{I_4}(t); \\
B_{I_7}(t) &= q_{74}(t)\circ B_{I_4}(t); \\
B_{I_8}(t) &= q_{84}(t)\circ B_{I_4}(t); \\
B_{I_9}(t) &= q_{9,0}(t)\circ B_{I_0}(t); \\
B_{I_{10}}(t) &= q_{10,11}(t)\circ B_{I_{11}}(t) + q_{10,12}(t)\circ B_{I_{12}}(t); \\
B_{I_{11}}(t) &= q_{11,10}(t)\circ B_{I_{10}}(t); \\
B_{I_{12}}(t) &= W_{12}(t) + q_{12,13}(t)\circ B_{I_{13}}(t) + q_{12,14}(t)\circ B_{I_{14}}(t) + q_{12,15}(t)\circ B_{I_{15}}(t); \\
B_{I_{13}}(t) &= q_{13,10}(t)\circ B_{I_{10}}(t); \\
B_{I_{14}}(t) &= q_{14,10}(t)\circ B_{I_{10}}(t); \\
B_{I_{15}}(t) &= q_{15,0}(t)\circ B_{I_{0}}(t); \\
W_{12}(t) &= \overline{I_2}(t)
\end{align*}
\]

Taking the L.T. of the above relations and then solving them for \( B_{I_{10}}^*(s) \), we get

\[ B_{I_{10}}^*(s) = \frac{N_{15}(s)}{D_3(s)} \]
where

\[ N_{15}(s) = W_{12}^*(s)q_{10,12}^*\left[1 - q_{01}^*(s)q_{10}^*(s) - q_{02}^*(s)q_{20}^*(s) - q_{03}^*(s)q_{30}^*(s)\right] \]

\[ \left[1 - q_{45}^*(s)q_{54}^*(s) - q_{46}^*(s)\{q_{67}^*(s)q_{74}^*(s) + q_{68}^*(s)q_{84}^*(s)\}\right] \]

\[- W_{12}^*(s)q_{04}^*(s)q_{46}^*(s)q_{69}^*(s)q_{9,0}^*(s)q_{10,12}^*(s) \]

and \( D_3(s) \) is already specified.

In the steady-state, the total fraction of time for which the system is under inspection by the available repairman is given by:

\[ B_{I_{10}} = \lim_{s \to 0} \left[ s \frac{N_{15}}{D_3} \right] \]

where

\[ N_{15} = \mu_{12} p_{04} p_{10,12} \] and \( D_3 \) is already specified.

**Expected Busy Period of the Available Repairman [Components Repair Time Only] during Stage-III**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( BR_i(t) \):

\[ BR_0(t) = q_{03}(t) \cap BR_1(t) + q_{02}(t) \cap BR_2(t) + q_{03}(t) \cap BR_9(t) + q_{04}(t) \cap BR_4(t); \]

\[ BR_1(t) = q_{10}(t) \cap BR_0(t); \quad BR_2(t) = q_{20}(t) \cap BR_0(t); \quad BR_3(t) = q_{30}(t) \cap BR_0(t); \]

\[ BR_4(t) = q_{45}(t) \cap BR_5(t) + q_{46}(t) \cap BR_6(t) + q_{4,10}(t) \cap BR_10(t); \]

\[ BR_5(t) = q_{54}(t) \cap BR_4(t); \]

\[ BR_6(t) = q_{67}(t) \cap BR_7(t) + q_{68}(t) \cap BR_8(t) + q_{69}(t) \cap BR_9(t); \quad BR_7(t) = q_{74}(t) \cap BR_4(t); \]

\[ BR_8(t) = q_{84}(t) \cap BR_4(t); \quad BR_9(t) = q_{9,0}(t) \cap BR_0(t); \]

\[ BR_{10}(t) = q_{10,11}(t) \cap BR_{11}(t) + q_{10,12}(t) \cap BR_{12}(t); \]

\[ BR_{11}(t) = q_{11,10}(t) \cap BR_{10}(t); \]

\[ BR_{12}(t) = q_{12,13}(t) \cap BR_{13}(t) + q_{12,14}(t) \cap BR_{14}(t) + q_{12,15}(t) \cap BR_{15}(t); \]

\[ BR_{13}(t) = W_{13}(t) + q_{13,10}(t) \cap BR_{10}(t); \]

\[ BR_{14}(t) = q_{14,10}(t) \cap BR_{10}(t); \quad BR_{15}(t) = q_{15,0}(t) \cap BR_0(t); \]
where

\[ W_{13}(t) = G_3(t) \]

Taking the L.T. of the above relations and then solving them for \( BR_{10}^*(s) \), we get

\[ BR_{10}^*(s) = \frac{N_{16}(s)}{D_3(s)} \]

where

\[ N_{16}(s) = W_{13}(s)q_{10,12}(s)q_{10,13}(s)[1 - q_{01}(s)q_{10}(s) - q_{02}(s)q_{20}(s) - q_{03}(s)q_{30}(s)] \]

\[ [1 - q_{45}(s)q_{54}(s) - q_{46}(s)[q_{67}(s)q_{74}(s) + q_{68}(s)q_{84}(s)]] \]

\[ - W_{13}(s)q_{04}(s)q_{46}(s)q_{69}(s)q_{90}(s)q_{10,12}(s)q_{10,13}(s) \]

and \( D_3(s) \) is already specified.

In the steady-state, the total fraction of time for which the system is under repair of the components only by the available repairman is given by:

\[ BR_{10} = \lim_{s \to 0} [s BR_{10}^*(s)] = \frac{N_{16}}{D_3} \]

where

\[ N_{16} = \mu_{13} p_{04} p_{10,12} p_{10,13} \] and \( D_3 \) is already specified.

**Expected Busy Period of the Available Repairman [Components Replacement Time Only] during Stage-III**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( BR_{p_1}(t) \):

\[ BR_{p_0}(t) = q_{01}(t) \odot BR_{p_1}(t) + q_{02}(t) \odot BR_{p_2}(t) + q_{03}(t) \odot BR_{p_3}(t) + q_{04}(t) \odot BR_{p_4}(t); \]

\[ BR_{p_1}(t) = q_{10}(t) \odot BR_{p_0}(t); \]

\[ BR_{p_2}(t) = q_{20}(t) \odot BR_{p_0}(t); \]

\[ BR_{p_3}(t) = q_{30}(t) \odot BR_{p_0}(t); \]

\[ BR_{p_4}(t) = q_{45}(t) \odot BR_{p_5}(t) + q_{46}(t) \odot BR_{p_6}(t) + q_{410}(t) \odot BR_{p_{10}}(t); \]

\[ BR_{p_5}(t) = q_{54}(t) \odot BR_{p_4}(t); \]

\[ BR_{p_6}(t) = q_{67}(t) \odot BR_{p_7}(t) + q_{68}(t) \odot BR_{p_8}(t) + q_{69}(t) \odot BR_{p_9}(t); \]

\[ BR_{p_7}(t) = q_{74}(t) \odot BR_{p_4}(t); \]

\[ BR_{p_8}(t) = q_{84}(t) \odot BR_{p_4}(t); \]

\[ BR_{p_9}(t) = q_{90}(t) \odot BR_{p_0}(t); \]

\[ BR_{p_{10}}(t) = q_{10,11}(t) \odot BR_{p_{11}}(t) + q_{10,12}(t) \odot BR_{p_{12}}(t); \]
BRp_{11}(t) = q_{11,10}(t) \odot BRp_{10}(t);

BRp_{12}(t) = q_{12,13}(t) \odot BRp_{13}(t) + q_{12,14}(t) \odot BRp_{14}(t) + q_{12,15}(t) \odot BRp_{15}(t);

BRp_{13}(t) = q_{13,10}(t) \odot BRp_{10}(t); BRp_{14}(t) = W_{14}(t) + q_{14,10}(t) \odot BRp_{10}(t);

BRp_{15}(t) = q_{15,0}(t) \odot BRp_{0}(t);

where

\[ W_{14}(t) = H_3(t) \]

Taking the L.T. of the above relations and then solving them for \( BRp_{10}^*(s) \), we get

\[ BRp_{10}^*(s) = \frac{N_{17}(s)}{D_3(s)} \]

where

\[ N_{17}(s) = W_{14}^*(s)q_{10,12}^*(s)q_{10,14}^*(s)[1 - q_{01}^*(s)q_{10}^*(s) - q_{02}^*(s)q_{20}^*(s)] \\
- q_{03}^*(s)q_{30}^*(s)[1 - q_{45}^*(s)q_{54}^*(s) - q_{46}^*(s)q_{67}^*(s)q_{74}^*(s)] \\
+ q_{68}^*(s)q_{84}^*(s)] - W_{14}^*(s)q_{04}^*(s)q_{46}^*(s)q_{49}^*(s)q_{9,0}^*(s)q_{10,12}^*(s)q_{10,14}^*(s) \]

and \( D_3(s) \) is already specified.

In the steady-state, the total fraction of time for which the system is under replacement of the component only by the available repairman is given by:

\[ BRp_{10} = \lim_{s \to 0} [s \ BRp_{10}^*(s)] = \frac{N_{17}}{D_3} \]

where

\[ N_{17} = \mu_{14} p_{04} p_{10,12} p_{10,14} \] and \( D_3 \) is already specified.

**Expected Busy Period of the available repairman [Re-Installation Time Only] during Stage-III**

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( BRf_i(t) \):

\[ BRf_0(t) = q_{01}(t) \odot BRf_1(t) + q_{02}(t) \odot BRf_2(t) + q_{03}(t) \odot BRf_3(t) + q_{04}(t) \odot BRf_4(t); \]

\[ BRf_1(t) = q_{10}(t) \odot BRf_0(t); \quad BRf_2(t) = q_{20}(t) \odot BRf_0(t); \]
\[ \text{BRf}_1(t) = \mu_{30}(t) \odot \text{BRf}_0(t); \]
\[ \text{BRf}_2(t) = q_{45}(t) \odot \text{BRf}_5(t) + q_{46}(t) \odot \text{BRf}_6(t) + q_{4,10}(t) \odot \text{BRf}_{10}(t); \]
\[ \text{BRf}_3(t) = q_{54}(t) \odot \text{BRf}_4(t); \]
\[ \text{BRf}_4(t) = q_{67}(t) \odot \text{BRf}_7(t) + q_{68}(t) \odot \text{BRf}_8(t) + q_{69}(t) \odot \text{BRf}_9(t); \]
\[ \text{BRf}_5(t) = q_{74}(t) \odot \text{BRf}_4(t); \]
\[ \text{BRf}_6(t) = q_{85}(t) \odot \text{BRf}_7(t) + q_{86}(t) \odot \text{BRf}_8(t) + q_{89}(t) \odot \text{BRf}_9(t); \]
\[ \text{BRf}_7(t) = q_{94}(t) \odot \text{BRf}_6(t); \]
\[ \text{BRf}_8(t) = q_{10,11}(t) \odot \text{BRf}_{11}(t) + q_{10,12}(t) \odot \text{BRf}_{12}(t); \]
\[ \text{BRf}_9(t) = q_{11,10}(t) \odot \text{BRf}_{10}(t); \]
\[ \text{BRf}_{10}(t) = q_{12,13}(t) \odot \text{BRf}_{13}(t) + q_{12,14}(t) \odot \text{BRf}_{14}(t) + q_{12,15}(t) \odot \text{BRf}_{15}(t); \]
\[ \text{BRf}_{11}(t) = q_{13,10}(t) \odot \text{BRf}_{10}(t); \]
\[ \text{BRf}_{12}(t) = q_{14,10}(t) \odot \text{BRf}_{10}(t); \]
\[ \text{BRf}_{13}(t) = q_{15,10}(t) \odot \text{BRf}_{10}(t); \]
\[ \text{BRf}_{14}(t) = q_{16,10}(t) \odot \text{BRf}_{10}(t); \]
\[ \text{BRf}_{15}(t) = \text{W}_{15}(t) + q_{15,0}(t) \odot \text{BRf}_0(t); \]

where

\[ \text{W}_{15}(t) = \overline{H(t)} \]

Taking the L.T. of the above relations and then solving them for \( \text{BRf}_{10}^*(s) \), we get

\[ \text{BRf}_{10}^*(s) = \frac{N_{18}(s)}{D_3(s)} \]

where

\[ N_{18}(s) = \text{W}_{15}^*(s) q_{10,12}^*(s) q_{10,15}^*(s) \left[ 1 - q_{01}^*(s) q_{10}^*(s) - q_{02}^*(s) q_{20}^*(s) \right] - q_{03}^*(s) q_{30}^*(s) \left[ 1 - q_{45}^*(s) q_{54}^*(s) - q_{46}^*(s) \left( q_{67}^*(s) q_{74}^*(s) + q_{68}^*(s) q_{84}^*(s) \right) \right] - \text{W}_{15}^*(s) q_{46}^*(s) q_{69}^*(s) q_{9,0}^*(s) q_{10,12}^*(s) q_{10,15}^*(s) \]

and \( D_3(s) \) is already specified.

In the steady-state, the total fraction of time for which the system is under re-installation of the unit by the available repairman is given by:

\[ \text{BRf}_{10} = \lim_{s \to 0} [s \text{BRf}_{10}^*(s)] = \frac{N_{18}}{D_3} \]

where

\[ N_{18} = \mu_{15} p_{04} p_{10,12} p_{10,15} \] and \( D_3 \) is already specified.
Expected Number of Visits by the Available Repairman during Stage-III

Using the probabilistic arguments for regenerative process, we obtain the following recursive relations for \( V_i(t) \):

\[
\begin{align*}
V_0(t) &= Q_{01}(t) \cdot V_1(t) + Q_{02}(t) \cdot V_2(t) + Q_{03}(t) \cdot V_3(t) + Q_{04}(t) \cdot V_4(t); \\
V_1(t) &= Q_{10}(t) \cdot V_0(t); \\
V_2(t) &= Q_{20}(t) \cdot V_0(t); \\
V_3(t) &= Q_{30}(t) \cdot V_0(t); \\
V_4(t) &= Q_{45}(t) \cdot V_5(t) + Q_{46}(t) \cdot V_6(t) + Q_{4,10}(t) \cdot V_{10}(t); \\
V_5(t) &= Q_{54}(t) \cdot V_4(t); \\
V_6(t) &= Q_{67}(t) \cdot V_7(t) + Q_{68}(t) \cdot V_8(t) + Q_{69}(t) \cdot V_9(t); \\
V_7(t) &= Q_{74}(t) \cdot V_4(t); \\
V_8(t) &= Q_{84}(t) \cdot V_4(t); \\
V_9(t) &= Q_{9,0}(t) \cdot V_0(t); \\
V_{10}(t) &= Q_{10,11}(t) \cdot V_{11}(t) + Q_{10,12}(t) \cdot [1 + V_{12}(t)]; \\
V_{11}(t) &= Q_{11,10}(t) \cdot V_{10}(t); \\
V_{12}(t) &= Q_{12,13}(t) \cdot V_{13}(t) + Q_{12,14}(t) \cdot V_{14}(t) + Q_{12,15}(t) \cdot V_{15}(t); \\
V_{13}(t) &= Q_{13,10}(t) \cdot V_{10}(t); \\
V_{14}(t) &= Q_{14,10}(t) \cdot V_{10}(t); \\
V_{15}(t) &= Q_{15,0}(t) \cdot V_0(t);
\end{align*}
\]

Taking the L.S.T. of the above relations and then solving them for \( V_{10}^{**}(s) \), we get

\[
V_{10}^{**}(s) = \frac{N_{19}(s)}{D_3(s)}
\]

where

\[
N_{19}(s) = Q_{10,12}^{**}(s)[1 - Q_{01}^{**}(s)Q_{10}^{**}(s) - Q_{02}^{**}(s)Q_{20}^{**}(s) - Q_{03}^{**}(s)Q_{30}^{**}(s)]
\]

\[
\left[1 - Q_{45}^{**}(s)Q_{54}^{**}(s) - Q_{46}^{**}(s)\left(Q_{57}^{**}(s)Q_{74}^{**}(s) + Q_{68}^{**}(s)Q_{84}^{**}(s)\right)\right]
\]

\[
- Q_{04}^{**}(s)Q_{46}^{**}(s)Q_{69}^{**}(s)Q_{9,0}^{**}(s)Q_{10,12}^{**}(s)
\]

and \( D_3(s) \) is already specified.

In the steady-state, the expected number of visits per unit time by the available repairman is given by:

\[
V_{10} = \lim_{s \to 0} [s \cdot V_{10}^{**}(s)] = \frac{N_{19}}{D_3}
\]

where

\[
N_{19} = p_{04} p_{4,10} p_{10,12} \quad \text{and} \quad D_3 \quad \text{is already specified.}
\]
**Profit Analysis of the System**

Expected profit of the system for the system user (P_1) is given by:

\[ P_1 = C_0 [A_4 + A_{10}] - C_1 E_{10} - C_2 B_1 E_{10} - C_3 B R_{10} - C_4 B R p_{10} - C_5 B R f_{10} - C_6 V_{10} \]

where

- \( C_0 = \) revenue per unit up time of the system
- \( C_1 = \) cost per unit time of maintenance of the system by available repairman
- \( C_2 = \) cost per unit time of inspection of the component by available repairman
- \( C_3 = \) cost per unit time of repair of the component by available repairman
- \( C_4 = \) cost per unit time of replacement of the component by available repairman
- \( C_5 = \) cost per unit time of re-installation of the unit by available repairman
- \( C_6 = \) cost per visit by the available repairman.

Expected profit of the system for the system provider (P_2) is given by:

\[ P_2 = (S P - C P) - C_7 (E_0 + E_4) - C_8 B I_4 - C_9 (B R_0 + B R_4) - C_{10} (B R p_0 + B R p_4) - C_{11} B R f_4 - C_{12} V_4 \]

where

- \( S P / C P = \) sale price/cost price of the system,
- \( C_7 = \) cost per unit time of maintenance of the system by service engineer
- \( C_8 = \) cost per unit time of inspection of the system by service engineer
- \( C_9 = \) cost per unit time of repair of the components by service engineer
- \( C_{10} = \) cost per unit time of replacement of the components by service engineer
- \( C_{11} = \) cost per unit time of re-installation of the unit by service engineer
- \( C_{12} = \) cost per visit by the service engineer

**Graphical Interpretation and Conclusions**

Graphical analysis of the system has been carried out at various stages of its operation for the following particular case:

\[ g(x) = \beta e^{-\beta x}; \quad g_1(x) = \beta_1 e^{-\beta_1 x}; \quad g_2(x) = \beta_2 e^{-\beta_2 x}; \quad g_3(x) = \beta_3 e^{-\beta_3 x}; \]
\[ h(x) = \gamma e^{-\gamma x}; \quad h_1(x) = \gamma_1 e^{-\gamma_1 x}; \quad h_2(x) = \gamma_2 e^{-\gamma_2 x}; \quad h_3(x) = \gamma_3 e^{-\gamma_3 x}; \]
\[ i_1(x) = \theta_1 e^{-\theta_1 x}; \quad i_2(x) = \theta_2 e^{-\theta_2 x} \]
Therefore we have

\[ p_{01} = \frac{p_1 \lambda_1}{\lambda_1 + \eta_1}; \quad p_{02} = \frac{a_1 q_1 \lambda_1}{\lambda_1 + \eta_1}; \quad p_{03} = \frac{b_1 q_1 \lambda_1}{\lambda_1 + \eta_1}; \quad p_{04} = \frac{\eta_1}{\lambda_1 + \eta_1}; \]

\[ p_{45} = \frac{p_2 \lambda_2}{\lambda_2 + \eta_2}; \quad p_{46} = \frac{q_2 \lambda_2}{\lambda_2 + \eta_2}; \quad p_{4,10} = \frac{\eta_2}{\lambda_2 + \eta_2}; \quad p_{67} = a_2; \]

\[ p_{68} = b_2; \quad p_{69} = c_2; \quad p_{10,11} = p_3; \quad p_{10,12} = q_3; \]

\[ p_{12,13} = a_3; \quad p_{12,14} = b_3; \quad p_{12,15} = c_3; \]

\[ p_{10} = p_{20} = p_{30} = p_{54} = p_{74} = p_{9,0} = p_{11,10} = p_{13,10} = p_{14,10} = p_{15,0} = 1 \]

\[ \mu_0 = \frac{1}{\lambda_1 + \eta_1}; \quad \mu_1 = \mu_5 = \mu_{11} = \frac{1}{\beta_1}; \quad \mu_2 = \frac{1}{\beta_1}; \quad \mu_3 = \frac{1}{\gamma_1}; \]

\[ \mu_4 = \frac{1}{\lambda_2 + \eta_2}; \quad \mu_6 = \frac{1}{\theta_1}; \quad \mu_7 = \frac{1}{\beta_2}; \quad \mu_8 = \frac{1}{\gamma_2}; \]

\[ \mu_9 = \mu_{15} = \frac{1}{\gamma}; \quad \mu_{10} = \frac{1}{\lambda_3}; \quad \mu_{11} = \frac{1}{\theta_2}; \quad \mu_{12} = \frac{1}{\beta_3}; \quad \mu_{13} = \frac{1}{\beta_3}; \]

\[ \mu_{14} = \frac{1}{\gamma_3}; \]

**Fig. 5.12** depicts the behaviour of mean installation time \((I_0)\) with respect to failure rate \((\lambda_1)\) during Stage-I of the system for different values of improvement rate \((\eta_1)\).
It can be concluded from the graph that $I_0$ increases with the increase in the value of $\lambda_1$ and has lower values for higher values of $\eta_1$.

**Fig. 5.13** shows the behaviour of MTSF ($T_4$) with respect to failure rate ($\lambda_2$) during Stage-II of the system for different values of deterioration rate ($\eta_2$).

It can be observed from the graph that $T_4$ decreases with the increase in the values of $\lambda_2$ and has lower values for higher values of $\eta_2$.

**MTSF ($T_4$) VERSUS FAILURE RATE ($\lambda_2$) FOR DIFFERENT VALUES OF DETERIORATION RATE ($\eta_2$)**

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**Fig. 5.14** represents the behaviour of MTSF ($T_{10}$) with respect to failure rate ($\lambda_3$) during Stage-III of the system for different values of repair rate ($\beta_3$).

It can be concluded from the graph that $T_{10}$ decreases with the increase in the values of $\lambda_3$ and has lower values for higher values of $\beta_3$. 
**MTSF** (*T*) **VERSUS FAILURE RATE** (**λ**) **FOR DIFFERENT VALUES OF REPAIR RATE** (**β**)

![Graph](image)

**Fig. 5.14**

**Fig. 5.15** depicts the behaviour of MTSF (*T*) with respect to failure rate (**λ**) during Stage-III of the system for different values of inspection rate (**θ**).

From the graph it can be interpreted that *T* decreases with the increase in the values of **λ** and has lower values for higher values of **θ**.

**MTSF** (*T*) **VERSUS FAILURE RATE** (**λ**) **FOR DIFFERENT VALUES OF INSPECTION RATE** (**θ**)

![Graph](image)

**Fig. 5.15**
Fig. 5.16 represents the behaviour of profit of system user \( (P_1) \) with respect to failure rate \( (\lambda_1) \) during Stage-I of the system for different values of improvement rate \( (\eta_1) \).

**PROFIT OF SYSTEM USER \( (P_1) \) VERSUS FAILURE RATE \( (\lambda_1) \)
FOR DIFFERENT VALUES OF IMPROVEMENT RATE \( (\eta_1) \)**

From the graph it can be interpreted that \( P_1 \) decreases with the increase in the values of \( \lambda_1 \) and has higher values for higher values of \( \eta_1 \).

**Fig. 5.16**

Fig. 5.17 reveals the behaviour of profit of system user \( (P_1) \) with respect to failure rate \( (\lambda_2) \) during Stage-II of the system for different value of repair rate \( (\beta_2) \).
PROFIT OF SYSTEM USER ($P_1$) VERSUS FAILURE RATE ($\lambda_2$) FOR DIFFERENT VALUES OF REPAIR RATE ($\beta_2$)

![Graph showing the relationship between $P_1$ and $\lambda_2$ for different $\beta_2$ values](image)

Fig. 5.17

It is clear from the graph that $P_1$ decreases with the increase in the values of $\lambda_2$ and has higher values for higher values of $\beta_2$.

Fig. 5.18 reveals the behaviour of profit of system user ($P_1$) with respect to failure rate ($\lambda_3$) during Stage-III of the system for different values of repair rate ($\beta_3$).

It can be concluded from the graph that $P_1$ decreases with the increase in the values of $\lambda_3$ and has higher values for higher values of $\beta_3$.

![Graph showing the relationship between $P_1$ and $\lambda_3$ for different $\beta_3$ values](image)

Fig. 5.18
Fig. 5.19 depicts the behaviour of profit of system user ($P_1$) with respect to revenue per unit up time ($C_0$) of the system for different values of cost per visit ($C_6$) of repairman.

Following can be observed from the graph:

(i) The $P_1$ increases with the increase in the values of $C_0$ and has lower values for higher values of $C_6$.

(ii) For $C_6 = 400$, the $P_1$ is positive or zero or negative according as $C_0 >$ or $= or < 81.44311$. Hence, for this case revenue per unit up time should be fixed greater than 81.44311.

(iii) For $C_6 = 600$, the $P_1$ is positive or zero or negative according as $C_0 >$ or $= or < 90.00006$. Hence, for this case the revenue per unit up time should be fixed greater than 90.00006.

(iv) For $C_6 = 800$, the $P_1$ is positive or zero or negative according as $C_0 >$ or $= or < 98.58623$. Hence, for this case the revenue per unit up time should be fixed greater than 98.58623
**Fig. 5.20** reveals the behaviour of profit of system user ($P_1$) with respect to failure rate ($\lambda_3$) during Stage-III of the system for different values of inspection rate ($\theta_2$).

**PROFIT OF SYSTEM USER ($P_1$) VERSUS FAILURE RATE ($\lambda_3$) FOR DIFFERENT VALUES OF INSPECTION RATE ($\theta_2$)**

From the graph it is concluded that $P_1$ decreases with the increase in the values of $\lambda_3$ and has higher values for higher values of $\theta_2$.

**Fig. 5.21**

**PROFIT OF SYSTEM PROVIDER ($P_2$) VERSUS FAILURE RATE ($\lambda_2$) FOR DIFFERENT VALUES OF IMPROVEMENT RATE ($\eta_1$)**

From the graph it is concluded that $P_2$ decreases with the increase in the values of $\lambda_2$ and has higher values for higher values of $\eta_1$. 
Fig. 5.21 represents the behaviour of profit of system provider ($P_2$) with respect to failure rate ($\lambda_2$) during Stage-II of the system for different values of improvement rate ($\eta_1$).

From the graph it can be interpreted that $P_2$ decreases with the increase in the values of $\lambda_2$ and has higher values for higher values of $\eta_1$.

Fig. 5.22 reveals the behaviour of profit of system provider ($P_2$) with respect to fixed profit (SP-CP) for different values failure rate ($\lambda_2$) during Stage-II of the system.

Following can be observed from the graph:

(i) The $P_2$ increases with the increase in the values of SP-CP and has lower values for higher values of $\lambda_2$.

(ii) For $\lambda_2 = 0.02$, the $P_2$ is positive or zero or negative according as SP-CP is $> \text{ or } = \text{ or } < 155.5128$. Hence for this case failure rate of the unit should be less than 155.5128.

(iii) For $\lambda_2 = 0.03$, the $P_2$ is positive or zero or negative according as SP-CP is $> \text{ or } = \text{ or } < 170.9575$. Hence for this case failure rate of the unit should be less than 170.9575.
(iv) For $\lambda_2 = 0.04$, the $P_2$ is positive or zero or negative according as $SP-CP$ is $>$ or $=$ or $<$ 183.7739. Hence for this case failure rate of the unit should be less than 183.7739.

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