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SUR LES ESPACES BITOPOLOGIQUES s-CONNEXES

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SUR LES ESPACES BITOPOLYGIQUES s-CONNEXES

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Dans un espace topologique $X$, une partie $A$ de $X$ est dite semiouvert [8] s'il existe un ensemble $O$ ouvert dans $X$ tel que $O = A \cap O$, où $O$ d'adherence de $O$ dans $X$. Un ensemble est dit semisemifermé si son complémentaire est semiouvert. L'intersection de tous les ensembles semifermés contenant $A$ est dite semiaadhérence de $A$ et est notée $\text{sel} A$; elle est toujours semifermé [1]. L'ensemble $A$ est semifermé si et seulement si $A = \text{sel} A$ [1]. Ainsi que $p \in \text{sel} A$, il faut et il suffit que tout ensemble semiouvert contenant $p$ rencontre $A$ [1]. De plus, $A \subset \text{sel} A \subset \text{cl} A$, et $A \subset B$ implique $\text{sel} A \subset \text{sel} B$. Un espace bitopologique $(X, T_1, T_2)$ est un ensemble $X$ muni de deux topologies $T_1$ et $T_2$.

1. Ensembles $<i, j>$-semiséparés. Définition 1. Deux ensembles $A$ et $B$ dans un espace bitopologique $(X, T_1, T_2)$ sont $<i, j>$-semiséparés si $A \cap T_1 = \text{cl} B = \emptyset = T_1 = \text{sel} A \cap B$, $i, j \in \{1, 2\}$.

Remarque 1. Deux ensembles $<i, j>$-semiséparés sont nécessairement disjoints. La réciproque n'est pas, en général, vraie.

Remarque 2. Soient les ensembles $A$, $B <i, j>$-semiséparés, $C \subset A$ et $D \subset B$. Alors, $C$ et $D$ sont $<i, j>$-semiséparés.

Remarque 3. Soient $A$ un ensemble $T_i$-semiouvert et $B$ un ensemble $T_j$-semiouvert. Alors, $A$ et $B$ sont $<i, j>$-semiséparés si et seulement si $A \cap B = \emptyset$.

Proposition 1. Si $A$, $B$ sont $<i, j>$-semiséparés et $A \cup B$ est biouvert, alors $A$ est $T_i$-semiouvert et $B$ est $T_j$-semiouvert.

Démonstration. On utilise les relations $(A \cup B) \cap (X - T_i - \text{cl} B) = A$ et $(A \cup B) \cap (X - T_j - \text{cl} A) = B$.

Proposition 2. Soient $(Y, T'_1, T'_2)$ un sous-espace biouvert d'un espace bitopologique $(X, T_1, T_2)$, $A \subset Y$ et $B \subset Y$. Alors, $A$ et $B$ sont $<i, j>$-semiséparés dans $X$ si et seulement si ils sont $<i, j>$-semiséparés dans $Y$.

La démonstration est immédiate.

2. Espaces bitopologiques s-connexes. Définition 2. On dit que l'espace bitopologique $(X, T_1, T_2)$ est s-connexe, s'il ne peut pas s'exprimer comme réunion de deux ensembles non vides $<i, j>$-semiséparés.

W. J. Pervin [9] a introduit la notion d'espace bitopologique connexe. On a:

Proposition 3. Un espace bitopologique s-connexe est connexe.
Remarque 4. La réciproque de la proposition 3 n’est pas, en général, vraie comme il suit de la considération suivante: X = {a, b, c, d}, T1 = {∅, {a}, {b}, {a, b}, {a, b, c}, X} et T2 = {∅, {a}, {b}, {a, b}, X}.

On démontre aisément la

Proposition 4. Pour qu’un espace bitopologique (X, T1, T2) soit s-connexe il faut et il suffit que l’il n’existe aucun sous-ensemble non vide de X qui soit à la fois T1-semiouvert et T2-semi fermé.

Proposition 5. Pour qu’un espace bitopologique (X, T1, T2) ne soit pas s-connexe il faut et il suffit que X puisse s’exprimer comme une réunion de deux ensembles non vides, disjoints A et B de sorte que A soit T1-semiouvert (resp. T2-semi fermé) et B, T2-semi ouvert (resp. T1-semi fermé).

C’est une conséquence de la proposition 4.

Définition 3. On dit qu’une partie A d’un espace bitopologique (X, T1, T2) est un ensemble s-connexe, si le sous-espace A de X est s-connexe.

Remarque 3. Tout ensemble réduit à un point est s-connexe.

Proposition 6. Soit (X, T1, T2) un espace bitopologique. Si A, B sont <i, j>-semi séparés dans X, l’ensemble E est biouvert s-connexe, E ⊂ A ∪ B, alors E ⊂ A ou E ⊂ B.


Proposition 7. Soient (X, T1, T2) un espace bitopologique, E biouvert, F bisemisouvert et E ⊂ F. Si E est s-connexe dans X alors E est s-connexe dans F.

Démonstration. Si E n’est pas s-connexe dans F, alors en vertu des théorèmes 2.4 et 2.3 dans [4], il résulte que E n’est pas s-connexe dans X.

Proposition 8. Soit (X, T1, T2) un espace bitopologique. Si E est biouvert s-connexe dans X et E ⊂ F ⊂ T1 − sel E ∩ T2 − sel E, alors F est s-connexe.

En suivant la même méthode qu’en [2], p. 201, on obtient:

Proposition 9. Dans un espace bitopologique (X, T1, T2), si deux ensembles A et B sont biouverts s-connexes et non <i, j>-semi séparés alors A ∪ B est s-connexe.

Proposition 10. Soit {Di | i ∈ Λ} une famille d’ensembles biouverts s-connexes dans un espace bitopologique (X, T1, T2). Si un membre Di de la famille n’est pas <i, j>-semi séparé d’un autre membre quelconque alors ∪Di est biouvert s-connexe.

Corollaire 1. La réunion d’une famille d’ensembles biouverts s-connexes dont l’intersection n’est pas vide, est un ensemble s-connexe.

En utilisant le Théorème 2.8 dans [4], on démontre la

Proposition 11. L'image homéomorphe d’un espace bitopologique s-connexe est s-connexe.

Définition 4. On dit qu’un espace bitopologique (X, T1, T2) est localement s-connexe si pour tout point x de X et tout ensemble O ouvert par rapport à T1 et contenant x, il existe un ensemble G biouvert s-connexe tel que x ∈ G ⊂ O.

Remarque 4. Le fait qu’un espace bitopologique (X, T1, T2) est localement s-connexe n’implique qu’il est s-connexe.
Exemple. $X = \{a, b\}$ et $T_1 = T_3 = (\emptyset, \{a\}, \{b\}, X)$.

**Remarque 5.** Soient $X = \{a, b\}$, $T_1 = (\emptyset, \{a\}, \{b\}, X)$ et $T_2 = (\emptyset, X)$. Alors $(X, T_1, T_2)$ est s-connexe mais il n’est pas localement s-connexe.

En utilisant la proposition 7 on prouve la,

**Proposition 12.** Dans un espace bitopologique localement s-connexe, tout sous-espace biouvert est localement s-connexe.

**Définition 5.** Soient $X$ un espace bitopologique localement s-connexe et $p \in X$. La s-composante de $p$ est la réunion des parties biouvertes s-connexes de $X$ contenant $p$.

**Proposition 13.** Toute s-composante d’un espace bitopologique localement s-connexe est biouverte s-connexe.

C’est une conséquence du corollaire 1.

**Proposition 14.** Soient $X$ un espace bitopologique localement s-connexe. Chaque s-composante d’une partie biouverte de $X$ est biouverte.

Cela résulte des propositions 12 et 13.

Comme dans [2], p. 203, on démontre la,

**Proposition 15.** Deux s-composantes distinctes sont $<i, j>$-semi-séparées.

**Corollaire 2.** La famille de toutes les s-composantes d’un espace bitopologique localement s-connexe $X$ est une partition de $X$.

**Proposition 16.** Toute s-composante localement s-connexe d’un espace bitopologique est bifermé.

Cela résulte du corollaire 2 et la proposition 13.

Comme dans [2], p. 216, on obtient :

**Proposition 17.** Soit $(X, T_1, T_2)$ un espace bitopologique localement s-connexe. Si $G$ est biouvert dans $X$ et si $A$ est une s-composante dans $G$ alors $T_1 - \text{Fr}(A) \cap G = \emptyset$.

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CONNECTEDNESS OF A STRONGER TYPE IN TOPOLOGICAL SPACES

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Abstract: Two subsets A and B are separated in a topological space X iff the closure of A is disjoint from B and the closure of B is disjoint from A. Any two separated sets are disjoint but not conversely. In the present paper the concept of semi-separated sets is defined. It is seen that any two separated sets are semiseparated and any two semiseparated sets are disjoint. However, the converses need not be true. This concept is utilized and a new type of connectedness (which comes out to be stronger than connectedness) is introduced and its basic properties are obtained.

1. INTRODUCTION.

In a topological space X, a set A is termed semiopen [5] if for some open set O, 0 ≤ A ≤ cl O, where cl O denotes the closure of O in X. Each open set is semiopen but the converse may not be true and any union of semiopen sets is semiopen [5]. A set is semiclosed if its complement is semiopen. The intersection of all the semiclosed sets containing A is called the semiclosure of A. Denote it by sc A. It is observed [2] that, A ≤ sc A ≤ cl A; A ≤ B implies sc A ≤ sc B; p ≤ sc A iff each semiopen set containing p meets A; sc A is always semiclosed and A is semiclosed iff A = sc A. A function f : X → Y is semicontinuous [5] if the inverse image of every open set is semiopen. Every continuous function is semicontinuous but not conversely [5].

A function f : X → Y is irresolute [1] if the inverse image of every semiopen set is semiopen. The concepts of continuous and irresolute are independent and that every irresolute function is semicontinuous but not conversely [1]. Two subsets A and B in a topological space X are said to be separated [4] iff A ∩ cl B = ∅ = cl A ∩ B. Any two separated sets are disjoint but in general the converse may not be true.

2. SEMI-SEPARATED SETS.

DEFINITION 1. Two sets A and B of a topological space X are termed semiseparated iff A ∩ sc B = ∅ = sc A ∩ B.

Remark 1. Since A ≤ sc A, A ≤ cl A, it follows that any two separated sets are semiseparated and any two semiseparated sets are disjoint. However, the converses may be false. For,

Example 1. Let X = {a, b, c, d}, and T = (Ø, {a}, {b}, {a, b}, X) be the topology on X. The sets {a, d}, {b, c} are semiseparated but not separated whereas {a, b}, {c, d} are disjoint but not semiseparated.

Theorem 1. If A, B are semiseparated and if C ⊆ A, D ⊆ B then C, D are semiseparated.
Proof. Since \( C \subseteq A \), therefore \( \text{scl } C \subseteq \text{scl } A \), similarly \( \text{scl } D \subseteq \text{scl } B \).

Theorem 2. Any two semiopen sets are semiseparated iff they are disjoint.

Proof: Necessity: For any two semiseparated sets are disjoint (Remark 1).

Sufficiency: For, \( A \cap \text{scl } B \neq \emptyset \Rightarrow p \in A \cap \text{scl } B \Rightarrow A \cap B \neq \emptyset \), if \( A \) is semiopen.

Theorem 3. If \( A, B \) are semiseparated and \( A \cup B \) open, then \( A \) and \( B \) are semiopen.

We shall require the following lemma:

Lemma 1. \([6]\). If \( U \) be open and \( A \) semiopen in \( X \) then \( U \cap A \) is semiopen in \( X \).

Proof of the theorem: Since \( A, B \) are semiseparated and \( B \subseteq \text{scl } B \), it follows that \( (A \cup B) \cap (X \cap \text{scl } B) = A \). Now, \( A \cup B \) being open and \( \text{scl } B \) semiclosed it follows by lemma 1 that \( A \) is semiopen. Similarly, \( B \) is semiopen.

Theorem 4. Let \( Y \) be an open subspace of a topological space \( X \), and \( A, B \) are subsets of \( Y \). Then \( A, B \) are semiseparated in \( X \) iff they are semiseparated in \( Y \).

The following lemma will be required:

Lemma 2. \([7]\). If \( A \subseteq Y \subseteq X \) and if \( Y \) is semiopen in \( X \) then \( \text{scl } A \cap Y \subseteq \text{scl } A \cap Y \). The equality holds if \( Y \) is open.

Proof of the theorem: By lemma 2 it follows that, \( \text{(semiclosure of } A \text{ in } Y \cap B) \cup (A \cap \text{semiclosure of } B \text{ in } Y) = (\text{scl } A \cap Y \cap B \cup (A \cap Y \cap \text{scl } B) = (\text{scl } A \cap B) \cup (A \cap \text{scl } B) \). The theorem is now evident.

Remark 2. In the above theorem, \( Y \) to be open is essential. For, consider, \( Y = \{b, c, d\} \) in the space of example 1. Then \( Y \) is semiopen but not open in \( X \). The sets \( \{b, c\} \) and \( \{d\} \) are semiseparated in \( X \) but not semiseparated in \( Y \).

Theorem 5. If the sets \( A \) and \( B \) are both semiopen or both semiclosed then the sets \( A - B \) and \( B - A \) are semiseparated.

Proof. If \( A \) is semiclosed, then we have, \( \text{scl } (A-B) \cap (B-A) = \text{scl } A \cap (X-B) \cap B \cap (X-A) \cap A \cap (X-A) = \emptyset \). And, if \( B \) is semiopen then we have \( \text{scl } (A-B) \cap (B-A) \subseteq (X-B) \cap B = \emptyset \). Consequently, the theorem follows.

3. CONNECTEDNESS OF A STRONGER TYPE:

A space \( X \) is connected iff it is not the union of two nonempty
separated sets \[3\]. In this section we define a new type of connectedness and study some of its basic properties.

Definition 2. A topological space \(X\) is \(s\)-connected iff it is not the union of two nonempty semiseparated sets.

Theorem 6. Every \(s\)-connected space \(X\) is connected.

Proof. For, if \(X\) is not connected it is a union of two nonempty separated sets. Since any two separated sets are semiseparated (remark 1), it follows that \(X\) is not \(s\)-connected.

Remark 3. The converse of theorem 6 need not be true. For, the space of example 1 is connected but it is not \(s\)-connected.

Theorem 7. A space \(X\) is \(s\)-connected iff no nonempty proper subset of \(X\) is both semippen and semiclosed.

Proof. Necessity: Let \(A\) be a nonempty proper subset of \(X\) both semippen and semiclosed. Then \(X-A\) is nonempty and semippen. By theorem 2, \(A\) and \(X-A\) are semiseparated. Consequently, \(X\) is not \(s\)-connected.

Sufficiency: Suppose that \(X\) is not \(s\)-connected. Let \(G, H\) be two nonempty semiseparated sets such that \(X = G \cup H\). By theorem 3, \(G\) and \(H\) are semippen and they are disjoint being semiseparated (Remark 1). Thus, \(G = X-H\). Hence, \(G\) is a nonempty proper subset of \(X\) which is both semippen and semiclosed.

Theorem 8. A space \(X\) is not \(s\)-connected iff it is the union of two nonempty disjoint semippen (resp. semiclosed) sets.

This is a consequence of theorem 7.

Definition 3. In a topological space \(X\), a set is \(s\)-connected iff it is a \(s\)-connected subspace of \(X\).

Remark 4. By theorem 7 it follows that each singleton set and any indiscrete space are \(s\)-connected. Any discrete space, which contains more than one point is not \(s\)-connected.

Remark 5. A non \(s\)-connected space may have an open \(s\)-connected set. For, \((a)\) is an open \(s\)-connected set in the space of example 1.

Theorem 9. Let the sets \(A, B\) be nonempty and semiseparated. Then \(A \cup B\) is not \(s\)-connected iff it is open.

This follows from Theorem 4.

Theorem 10. If \(E\) be an open \(s\)-connected set and \(A, B\) are semiseparated in \(X\) such that \(E \subseteq A \cup B\), then \(E \subseteq A\) or \(E \subseteq B\).

Proof. \(E = (E \cap A) \cup (E \cap B)\). Assume \(E \cap A \neq \emptyset \neq E \cap B\). By theorem 1,
E ∩ A, and E ∪ B are semiseparated in X and since E is open, they are semiseparated in E by theorem 4. Therefore E is not s-connected. This is a contradiction. And so, E ∩ B = ∅ or E ∩ A = ∅. Consequently, E ⊆ A or E ⊆ B.

Theorem 11. Let E be open, F semiopen in X and E ⊆ F. If E is s-connected in X then so it is in F.

The following lemmas will be needed:

Lemma 3.[6] If U be open and A semiopen in X then U ∩ A is semiopen in U.

Lemma 4.[6] If Y ⊆ X and A is semiopen in Y then A is semiopen in X iff Y is semiopen in X.

Proof. Suppose that E is not s-connected in F. Let A, B be nonempty semiseparated sets in a subspace E of F such that A ∪ B = E. Since E is open in F, A and B are semiseparated in F by theorem 4. And so, by theorem 3, A and B are semiopen in F. Since F is semiopen it results by lemma 4 that A, B are semiopen in X. Now by lemma 3, E ∩ A = A and E ∩ B = B are semiopen in the subspace E of X. Evidently, A, B are disjoint. Hence by theorem 8, E is not s-connected in X. This is absurd. Hence, E is s-connected in F.

Theorem 12. Let E be an open s-connected set and E ⊆ F ⊆ scl E. Then F is s-connected.

Proof. Suppose that F is not s-connected. Let A, B be nonempty semiseparated sets in F such that A ∪ B = F. Since E ⊆ F ⊆ scl E, it follows that F is semiopen. Therefore, by theorem 11, E is s-connected in F. Evidently, E is open in F. Hence by theorem 10, E ⊆ A or E ⊆ B. Let E ⊆ A. Now, since B ⊆ scl E, scl E ⊆ scl A and B ⊆ F, it follows on utilising lemma 2 that B = scl E ∩ B ⊆ scl A ∩ F ∩ B ⊆ (semiclass of A in F) ∩ B = ∅, since A and B are semiseparated in F. Consequently B = ∅, a contradiction. Hence F is s-connected.

Corollary 1. If E is an open s-connected set then scl E is s-connected and the interior of scl E is open s-connected.

Theorem 13. If A, B are open, s-connected and non semiseparated sets in X then A ∪ B is s-connected.

Proof. Assume that A ∪ B is not s-connected. Let C, D be nonempty and semiseparated in A ∪ B such that A ∪ B = C ∪ D. Since A ∪ B is open, by theorem 4, C and D are semiseparated in X. Now A ∩ C, A ∩ D are semiseparated in A by theorems 1 and 4. Since, A = (A ∩ C) ∪ (A ∩ D), we have, A ∩ D = ∅ or A ∩ C = ∅. Therefore, ACC or ACD. In the same way, BCC or BCD. If ACC and BCC, then A ∪ B ⊆ C, so that C ∩ A ∩ D which implies that D = ∅, a contradiction. Thus, ACC ∋ BCD. Similarly, ACD ∋ BCD. Consequently, by theorem 1, A and B are semiseparated in X. This is contrary to the hypothesis. Hence, A ∪ B is s-connected.
Theorem 14. If \( \{D_\lambda \mid \lambda \in \Lambda\} \) is a family of open s-connected sets such that one of them, \( D_\lambda \), is not semiseparated from every other member, then \( \bigcup D_\lambda \) is s-connected.

**Proof.** Let \( E = \bigcup D_\lambda \). Assume that \( E \) is not s-connected. Then there exist nonempty sets \( A \) and \( B \), which are semiseparated in \( E \) such that \( E = A \cup B \). Since \( E \) is open, \( A, B \) are semiseparated in \( X \) by theorem 4. Now by theorem 10, \( D_\lambda \subset A \) or \( D_\lambda \subset B \). Let \( D_\lambda \subset A \). Since \( D_\lambda \) and \( D_\lambda \) are not semiseparated for any \( \lambda \), \( D_\lambda \cup D_\lambda \) are open s-connected by

Theorem 13. We assert that for each \( \lambda \), \( D_\lambda \cup D_\lambda \subset A \). (For, if for some \( \lambda = \lambda_B \), \( D_\lambda \cup D_\lambda \subset B \) then \( D_\lambda \subset A \) and, \( D_\lambda \subset B \) imply that \( D_\lambda \) and \( D_\lambda \) are semiseparated which is absurd. Hence, \( B \subset A \). Therefore \( B = \emptyset \), a contradiction. Hence, \( E \subset A \). s-connected.

Corollary 2. If \( \{D_\lambda \mid \lambda \in \Lambda\} \) is a nonempty family of open s-connected sets such that \( \bigcap D_\lambda \neq \emptyset \), then \( \bigcup D_\lambda \) is s-connected.

It follows from remark 1 that any two non-disjoint sets are non semiseparated. Theorem 14 is now applicable.

Corollary 3. If \( \{A_n\}_{n \geq 0} \) be an infinite sequence of open s-connected sets such that \( A_{n+1} \cap A_n \neq \emptyset \) for each \( n \geq 0 \), then \( \bigcup_{n=0}^{\infty} A_n \) is s-connected.

**Proof.** By induction on \( n \), the set \( B_n = \bigcup_{i=0}^{n} A_i \) is open s-connected for all \( n \) by corollary 2. The sets \( B_n \) have a nonempty intersection. Hence, \( \bigcup_{n=0}^{\infty} A_n \) is s-connected by corollary 2.

It is known [4; theorem 3, p. 200] that if \( C \) is connected and \( C \cap A \neq \emptyset \neq C \setminus A \) then \( C \cap \text{Fr}(A) \neq \emptyset \). We have.

Theorem 15. If \( C \) is open s-connected in \( X \) and \( C \cap A \neq \emptyset \neq C \setminus A \) then \( C \cap \text{scl} (X \setminus A) \neq \emptyset \).

**Proof.** By virtue of the s-connectedness of the set \( C \) and \( C = (C \cap A) \cup (C \setminus A) \), the sets \( C \cap A \) and \( C \setminus A \) are not semiseparated in \( C \). C being open, these sets are not semiseparated in \( X \) by theorem 4. That is, \( [\text{scl} (C \cap A) \cap (C \setminus A)] \cup [(C \cap A) \cap \text{scl}(C \setminus A)] \neq \emptyset \). Since \( \text{scl} (C \cap A) \subset \text{scl} A \), \( (C \setminus A) \subset \text{scl} (X \setminus A) \), \( \text{scl}(C \setminus A) \subset \text{scl} (X \setminus A) \) and \( A \subset \text{scl} A \), it follows that
\( \text{scl } (X-A) \neq \emptyset. \)

**Theorem 16.** Let \( f : X \rightarrow Y \) be onto and semicontinuous. If \( X \) is s-connected then \( Y \) is connected.

**Proof.** If \( Y \) is not connected then it is the union of two nonempty disjoint open sets \( A \) and \( B \). Since \( f \) is semicontinuous, \( f^{-1}(A), f^{-1}(B) \) are semiopen, nonempty, disjoint and their union is \( X \). Hence by theorem 8, \( X \) is not s-connected.

**Theorem 17.** Let \( f : X \rightarrow Y \) be onto and irresolute. If \( X \) is s-connected then \( Y \) is s-connected.

The proof is analogous to that of Theorem 16 and utilises theorem 8.

**Theorem 18.** s-connectedness is a topological property.

The proof is similar to that of theorem 16, and utilises theorem 8 and Lemma 5, given below:

**Lemma 5.[6].** If \( f : X \rightarrow Y \) be 1-1, open and continuous and if \( B \) is semiopen in \( Y \) then \( f^{-1}(B) \) is semiopen in \( X \).

4. S-COMPONENTS.

A topological space \( X \) is said to be locally connected [8] if for every point \( x \in X \) and every open set \( G \) containing \( x \) there exists an open connected set \( G \) such that \( x \in G \cap O \).

**Definition 4.** A space \( X \) is locally s-connected iff for every point \( x \in X \) and every open set \( G \) containing \( x \), there exists an open s-connected set \( G \) such that \( x \in G \cap O \).

**Theorem 19.** Every locally s-connected space is locally connected.

This follows from Theorem 6.

**Remark 5.** The converse of Theorem 19 may be false. For, the space of example 1 is locally connected but not locally s-connected.

**Remark 6.** Locally s-connected space need not be s-connected. For, any finite discrete space which contains more than one point is locally s-connected but not s-connected.

**Theorem 20.** Every open subspace \( Y \) of a locally s-connected space \( X \) is locally s-connected.

**Proof.** Let \( p \in Y \) and \( O \) be open in \( Y \) such that \( p \in O \). Since \( X \) is locally s-connected there is an open s-connected set \( G \) in \( X \) such that \( p \in G \cap O \). Now \( G \subset O \cap Y \) and so by Theorem 11, \( G \) is s-connected in \( Y \).
Evidently, $G$ is open in $Y$. Consequently, $Y$ is locally $s$-connected.

Definition 5. Let $X$ be a locally $s$-connected space and $p \in X$. The $s$-component of $p$ is the union of all open $s$-connected sets which contain the point $p$.

Theorem 21. Each $s$-component is open and $s$-connected.

This follows from Corollary 2.

Theorem 22. Each $s$-component of an open set of a locally $s$-connected space is open.

This is a consequence of Theorems 20 and 21.

Theorem 23. Two distinct $s$-components are semiseparated.

Proof. If the $s$-components $S_1$ and $S_2$ are not semiseparated then by Theorems 21 and 13, $S_1 \cup S_2$ is open $s$-connected. And so, $S_1 \cup S_2 \subset S_1$ and $S_1 \cup S_2 \subset S_2$. That is, $S_1 = S_2$.

Corollary 4. The family of all the $s$-components, in a locally $s$-connected space $X$, is a partition of $X$.

Proof. This follows from Theorem 23 and the fact that each point of $X$ is contained in some $s$-component.

Theorem 24. Each $s$-component is closed.

Proof. Each $s$-component is open by Theorem 21. The theorem now follows in view of Corollary 4.

Theorem 25. If $G$ is open in a locally $s$-connected space $X$ and if $A$ be a $s$-component of $G$ then $F_r(A) \cap G = \emptyset$.

Proof. $A$ is open in $X$ by Theorem 22. Since $A$ is closed in $G$ by Theorem 24, and $G$ is open in $X$, it follows that $G-A$ is open in $X$. Now, $F_r(A) \cap G = \text{cl} A \cap \text{cl}(X-A) \cap G = \text{cl} A \cap (X-A) \cap G = (\text{cl} (A-A)) \cap G = \emptyset$.

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