CHAPTER 2

MARKOV CHAIN MODEL FOR TRAFFIC SHARING IN COMPUTER NETWORKS

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2.1.1 INTRODUCTION

In present days of high profile electronic media and Internet there are several Internet service providers growing drastically day-by-day and, as a consequence, the traffic load is getting higher and higher. The quality of service (QoS) is worst affected due to flow of heavy traffic and service providers to think of maintaining a quality level under traffic constraints. Two computers are said to be interconnected if they are able to exchange information. Networks of computers are being widely used throughout the world by scientific, commercial and business organizations in the form of local area networks (LAN), wide area networks (WAN) and Internet. The transmission of data, through a network, bears a kind of uncertainty due to several network problems like congestion, delay, non-connectivity, hardware failure etc. This leads to study of performance of a network using a probability model. Lee and Li (1991) used a Markov chain model for the study of performance of a nonblocking space-division packet switch, given that the traffic intensities at the
switch are nonuniform. Authors used finite-state Markov chain as an underlying process to govern the time variation of traffic for the entire switch. Kim and Lee (1995) picked up the Markov chain model to study the performance of knockout switch based multistage computer networks in the context to high-speed packet switching. Yuan and Lygeros (2005) used stochastic differential equations using Markov chain for the study of the problem of mean square exponential stabilization, in a computer network.

The Internet service distribution is based on the service providers either be the government or non-government organizations. In market, a large number of private companies are involved offering Internet services to the users. This generates a competition in the market about the blocking chances and quality of services (QoS). A user prefers an Internet operator providing better QoS throughout over a definite span of time.

Naldi (1998) discussed a general Internet traffic flow problem in terms of mathematical aspect. In one more contribution, Naldi (1999) developed a probabilistic model for the Internet users, depending upon the measurement of traffic flow, under the varying model parameters. While the environment is based on several operators and the network is often over loaded with users, Naldi (2002) derived a traffic sharing model among operators.

An operator means either Internet service providers (ISP) or network owners. The blocking in a network may be due to congestion (traffic overflow), insufficient number of modems, inefficient hardware for transmissions, or due to inadequate care and services. Operators used to have investments for the betterment and to have marketing strategies for the popularity in terms of attracting customers. On the other hand, preferences opted by customers depend upon the quality of services in terms
of cost, reliability and faster connectivity. While the Internet is free of cost, it increments the chance of congestion (blocking probability) in the network and when service cost is high, the customer refuses to prefer an operator. Therefore, a trade-off is used to establish among operators in terms of cost, congestion, profit and services. This chapter takes into account a Markov chain model for describing the users behavior when only two operators are in competition to provide the better quality of Internet services and a user is suppose to attempt twice only to an operator at a time.

2.2.1 MARKOV CHAIN MODEL

Let $O_1$ and $O_2$ be two operators, in a competitive market. The network diagram of operators with users is described in fig. 1.1.

![Diagram](image-url)
2.2.2 ASSUMPTIONS IN MODEL FOR SHARING INTERNET

1. Only two operators are in competition for the traffic share.

2. A user can begin with either of operators with the probability $p$ (other with $1 - p$) depending upon the better quality of services. This, we say, is the initial preference to an operator.

3. When first attempt of connectivity is failed, the user attempts one more to the same operator, and thereafter, switch over to the next one where two more consecutive attempts are likely. This we say “two-call” basis attempts for the effort of connectivity.

4. While the second attempt is over on an operator, the user has two choices either (a) to shift to the next operator or (b) to abandon the process and leave the computer with probability $P_a$.

5. The blocking probabilities are $L_i$ for operator $O_i$ and $L_2$ for $O_2$.

2.2.3 TRANSITION MECHANISM IN MODEL AND PROBABILITIES

Rule 1: User attempts to $O_i$ with initial probability $p$ (based on QoS the $O_i$ provides).

Rule 2: If fails, then reattempts to $O_i$. 
**Rule 3:** User may succeed to \( O_1 \) in one attempt or in the next. Since the blocking probability for \( O_1 \) in one attempt is \( L_1 \), therefore, blocking probability for \( O_1 \) in the next attempt is:

\[
P[O_1 \text{ blocked in an attempt}] \cdot P[O_1 \text{ blocked in next attempt / previous attempt to } O_1 \text{ was blocked}] = (L_1, L_1) = L_1^2
\]

The total blocking probability is \( (L_1 + L_1^2) \) inclusive of both attempts. Hence, success probability for \( O_1 \) is \( [1 - (L_1 + L_1^2)] \). Similar could be derived for operator \( O_2 \) in the form \( [1 - (L_2 + L_2^2)] \).

**Rule 4:** User shifts to \( O_2 \) if call blocks in both attempts to \( O_1 \) and does not abandon the attempting process. The transition probability is:
\[ P [ O_i \text{ blocked in an attempt}] \cdot P [ O_i \text{ blocked in next attempt } / \text{ previous attempt to } O_i \text{ was blocked}] \cdot P [\text{does not abandon attempting process}] \\
= L_i^2 (1 - P_A) \]

**Rule 5:** User earliest abandons the system only after two attempts to an operator which is a compulsive assumption with this model. This leads to probability that user abandons process after two attempts over \( O_i \) is:

\[ P [ O_i \text{ blocked in attempt}] \cdot P [ O_i \text{ blocked in next attempt } / \text{ previous attempt to } O_i \text{ was blocked}] \cdot P [\text{abandon the attempting process}] \\
= L_i^2 P_A. \]

Similar could be derived for \( O_2 \) interchanging \( L_1 \) by \( L_2 \).

### 2.2.4 MARKOV CHAIN TRANSITION PROBABILITY MATRIX

Define a markov chain \( \{X^{(n)} , n = 0,1,2,........\infty\} \) where \( X^{(n)} \) describes the \( n^{th} \) attempt by a user to connect (or succeed) a call while transitioning over four states \( O_1, O_2, A \) and \( Z \) described below:

**State 1:** Operator \( O_1 \),  
**State 2:** Operator \( O_2 \)

**State 3:** Success (call it \( A \)),  
**State 4:** Abandon (call it \( Z \))

The \( A \) and \( Z \) are absorbing states and unit-step transition probability matrix for the markov chain \( X^{(n)} \) is:
<table>
<thead>
<tr>
<th>States</th>
<th>$O_1$</th>
<th>$O_2$</th>
<th>$Z$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_1$</td>
<td>$L_1$</td>
<td>$L_1^2(1-P_A)$</td>
<td>${1-(L_1+L_1^2)}$</td>
<td>$L_1^2P_A$</td>
</tr>
<tr>
<td>$O_2$</td>
<td>$L_2^2(1-P_A)$</td>
<td>$L_2$</td>
<td>${1-(L_2+L_2^2)}$</td>
<td>$L_2^2P_A$</td>
</tr>
<tr>
<td>$Z$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$A$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The initial probabilities, based on QoS of operators (at $n = 0$), are

\[
P[X^{(0)} = O_1] = p; \quad P[X^{(0)} = O_2] = 1-p
\]

\[
P[X^{(0)} = Z] = 0; \quad P[X^{(0)} = A] = 0
\]  \hspace{1cm} \text{\ldots\ldots\ldots (2.2.4.1)}

### 2.2.5 PROBABILITIES OF ATTEMPTS

At $n=1$,

\[
\begin{align*}
P[X^{(1)} = O_1] &= P[X^{(0)} = O_1] \cdot P[X^{(0)} = O_1; X^{(0)} = O_1] = pL_1, \\
P[X^{(1)} = O_2] &= P[X^{(0)} = O_2] \cdot P[X^{(0)} = O_2; X^{(0)} = O_2] = (1-p)L_2
\end{align*}
\]

\hspace{1cm} \text{\ldots\ldots (2.2.5.1)}

Define some symbols

\[
C = L_1^3(1-P_A); \quad D = L_2^3(1-P_A); \quad M = (L_1 L_2 CD) \quad \text{\ldots\ldots (2.2.5.2)}
\]

The probabilities for the attempts $n > 1$ are

\[
n = 2, \begin{align*}
P[X^{(2)} = O_1] &= P[X^{(1)} = O_1] \cdot P[X^{(1)} = O_1; X^{(1)} = O_1] = (1-p)L_1D, \\
P[X^{(2)} = O_2] &= P[X^{(1)} = O_2] \cdot P[X^{(1)} = O_2; X^{(1)} = O_2] = pL_1C
\end{align*}
\]

\hspace{1cm} \text{\ldots\ldots (2.2.5.3)}

\[
n = 3, \begin{align*}
P[X^{(3)} = O_1] &= P[X^{(2)} = O_1] \cdot P[X^{(2)} = O_1; X^{(2)} = O_1] = (1-p)L_1L_2D, \\
P[X^{(3)} = O_2] &= P[X^{(2)} = O_2] \cdot P[X^{(2)} = O_2; X^{(2)} = O_2] = pL_1L_2C
\end{align*}
\]

\hspace{1cm} \text{\ldots\ldots (2.2.5.4)}
\[
\begin{align*}
\text{if } n &= \text{ Type } A : \quad P[X^{(i)} = O_1] = P[X^{(4n-i)} = O_1] = pM^nL_1; \\
\text{if } n &= \text{ Type } B : \quad P[X^{(i)} = O_1] = P[X^{(4n-i)} = O_1] = (1-p)M^n/C; \\
\text{if } n &= \text{ Type } C : \quad P[X^{(i)} = O_1] = P[X^{(4n)} = O_1] = pM^n; \\
\text{if } n &= \text{ Type } D : \quad P[X^{(i)} = O_1] = P[X^{(4n-2)} = O_1] = (1-p)M^n/L_1C; \quad (n > 0)
\end{align*}
\]

Similarly, the general expressions of \(n\)\textsuperscript{th} attempt for operator \(O_2\) are:

\[
\begin{align*}
\text{if } n &= \text{ Type } A : \quad P[X^{(i)} = O_2] = P[X^{(4n-i)} = O_2] = (1-p)M^nL_2 \\
\text{if } n &= \text{ Type } B : \quad P[X^{(i)} = O_2] = P[X^{(4n-i)} = O_2] = pM^n/D \\
\text{if } n &= \text{ Type } C : \quad P[X^{(i)} = O_2] = P[X^{(4n)} = O_2] = (1-p)M^n \\
\text{if } n &= \text{ Type } D : \quad P[X^{(i)} = O_2] = P[X^{(4n-2)} = O_2] = pM^n/L_2D \\
\end{align*}
\]

**Note 2.2.1:** If \(n\)\textsuperscript{th} attempt is type A then \((n-1)\)\textsuperscript{th} attempt will be type C because

\[
\text{Type } A \Rightarrow (4n+1) \text{ and } [(4n+1) - 1] = 4n = \text{ Type } C
\]
The following table has a display of relationship among \(n^\text{th}\), \((n - 1)^\text{th}\) and \((n - 2)^\text{th}\) attempts:

<table>
<thead>
<tr>
<th>(n^\text{th}) call attempt</th>
<th>((n - 1)^\text{th}) call attempt</th>
<th>((n - 2)^\text{th}) call attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td>if (n = ) Type A</td>
<td>(n - 1) = Type C</td>
<td>(n - 2) = Type B</td>
</tr>
<tr>
<td>if (n = ) Type B</td>
<td>(n - 1) = Type D</td>
<td>(n - 2) = Type A</td>
</tr>
<tr>
<td>if (n = ) Type C</td>
<td>(n - 1) = Type B</td>
<td>(n - 2) = Type D</td>
</tr>
<tr>
<td>if (n = ) Type D</td>
<td>(n - 1) = Type A</td>
<td>(n - 2) = Type C</td>
</tr>
</tbody>
</table>

This is to note that \((4n - 3)\) generates similar series of attempts as \((4n + 1)\) for \(n > 0\). The further detail of this is given in appendix B.

### 2.3.0 QUALITY OF SERVICE (QoS)

User is satisfied when call is complete in minimum number of attempts either by the first or by the second operator. In model, it is assumed that user can stick to an operator maximum upto two attempts and, thereafter, attempts for the next one. User be of two types (See Naldi(2002)):

1. **Faithful user**: who has faith to a particular operator and attempts to that until succeeds, otherwise abandons.

2. **Impatient user**: who is impatient about the success of the call, irrespective of operators \(O_1\) or \(O_2\), and continues efforts for both as per conditions described in the model.

The quality of service is a function of blocking probability faced by the user.

The faithful user faces the blocking probability \(L_1^1\) in one attempt, \(L_1^2\) in next
attempt for \( O_1 \). Similarly, \( L_2 \) and \( L_2^2 \) happens respectively for \( O_2 \). Therefore, the average blocking probability \( B_f \) for faithful user is
\[
B_f = pL_1^2 + (1-p)L_2^2
\]  \hspace{1cm} (2.3.0.1)

The impatient user suffers from varying average blocking probability \( B_i^{(n)} \) at the \( n^{th} \) attempt by
\[
B_i^{(n)} = w_1 L_1^2 + w_2 L_2^2
\]
where
\[
w_1 = P[X^{(n-1)} = O_1] / \{P[X^{(n-1)} = O_1] + P[X^{(n-1)} = O_2]\}
\]
\[
w_2 = P[X^{(n-1)} = O_2] / \{P[X^{(n-1)} = O_1] + P[X^{(n-1)} = O_2]\};
\]
and \( w_1, w_2 \) are varying weights over \( n, (w_1 + w_2 = 1) \).

Then,
\[
B_i^{(n)} = \frac{P[X^{(n-1)} = O_1] L_1^2 + P[X^{(n-1)} = O_2] L_2^2}{P[X^{(n-1)} = O_1] + P[X^{(n-1)} = O_2]} \quad \text{(See Naldi (2002))}
\]

This could be expressed into four cases based on \( n \) and table 2.2.0:
\[
B_i^{(n=Type A)} = pL_1^2 + (1-p)L_2^2 \quad \hspace{1cm} (2.3.0.2)
\]
\[
B_i^{(n=Type B)} = \frac{L_1 L_2 \left\{(1-p)L_1D + pL_2C\right\}}{(1-p)L_2D + pL_1C} \quad \hspace{1cm} (2.3.0.3)
\]
\[
B_i^{(n=Type C)} = \frac{(1-p)L_2^2D + pL_2^2C}{(1-p)D + pC} \quad \hspace{1cm} (2.3.0.4)
\]
\[
B_i^{(n=Type D)} = \frac{pL_1^3 + (1-p)L_2^3}{pL_1 + (1-p)L_2} \quad \hspace{1cm} (2.3.0.5)
\]

2.3.1 **Comparison of Faithful and Impatient Users**

(1) \( B_i^{(n=Type A)} < B_f \) always \hspace{1cm} (2.3.1.1)
The operator $O_1$ is benefited much when its faithful users are in higher probability than impatient users. For type C attempts, probability of faithful users is high. But for type A, it happens if only one operator is under cent percent preference which is rare in a competitive market.

For operator $O_1$ (Type C)
For operator $O_1$ (Type B)

Fig. 1.6

Fig. 1.7

Fig. 1.8

Fig. 1.3 to fig.1.8 reveals the reducing probability of initial preference over blocking probability for $O_1$ when attempts are type B and D. Faithful users looses faith when blocking probability is high. While $L_2$ increases (i.e. blocking probability of competitor $O_2$), the faithful user bears high chance (more faith) with $O_1$, even when increasing $L_1$.

2.3.2 TRAFFIC SHARING

A call may complete (successful) in any of $n$ attempts through operator $O_1$ when it is not blocked. Suppose it completes through $O_1$ at the $n^{th}$ attempt, then probability is

$$P_{1}^{(n)} = P[X^{(n-1)} = O_1] \cdot P[X^{(n)} = Z / X^{(n-1)} = O_1]$$
\[ P[X^{(n-1)} = O_1] \{ 1 - (L_1 + L_2^2) \} \]  \hspace{1cm} \text{(2.3.2.1)}

This could be derived in four different ways using table 2.2.0 :

if \( n = \text{Type A} : P_1^{(n+1)} = P[X^{(4n+1)-1} = O_1] \{ 1 - (L_1 + L_2^2) \} = \left[ (1 - p) M^{n+1} \right] \{ 1 - (L_1 + L_2^2) \} \]

if \( n = \text{Type B} : P_1^{(n)} = P[X^{(4n)-1} = O_1] \{ 1 - (L_1 + L_2^2) \} = \left[ (1 - p) M^{n-1} / (L_1 C) \right] \{ 1 - (L_1 + L_2^2) \} \]

if \( n = \text{Type C} : P_1^{(n)} = P[X^{(4n-1)} = O_1] \{ 1 - (L_1 + L_2^2) \} = \left[ (1 - p) M^{n-1} / C \right] \{ 1 - (L_1 + L_2^2) \} \]

if \( n = \text{Type D} : P_1^{(n-2)} = P[X^{(4n-2)-1} = O_1] \{ 1 - (L_1 + L_2^2) \} = \left[ p M^{n-1} L_1 \right] \{ 1 - (L_1 + L_2^2) \} \]

\hspace{1cm} \text{(2.3.2.2)}

The appendix B describes the attempt number and its type.

Similarly, when the call is complete through operator \( O_2 \) at the \( n^{th} \) attempt

\[ P_2^{(n)} = P[X^{(n-1)} = O_2] \cdot P[X^{(n)} = Z / X^{(n-1)} = O_2] \]

\[ = P[X^{(n-1)} = O_2] \{ 1 - (L_2 + L_2^2) \} \]  \hspace{1cm} \text{(2.3.2.3)}

if \( n = \text{Type A} : P_2^{(n+1)} = \left[ (1 - p) M^{n+1} \right] \{ 1 - (L_2 + L_2^2) \} \]

if \( n = \text{Type B} : P_2^{(n)} = \left[ p M^{n-1} / (L_2 D) \right] \{ 1 - (L_2 + L_2^2) \} \]

if \( n = \text{Type C} : P_2^{(n)} = p M^{n-1} / D \{ 1 - (L_2 + L_2^2) \} \]

if \( n = \text{Type D} : P_2^{(n-2)} = \left[ (1 - p) M^{n-1} L_2 \right] \{ 1 - (L_2 + L_2^2) \} \]  \hspace{1cm} \text{(2.3.2.4)}

The cumulative probability that a call completes through \( O_1 \) within the first \( n \) attempts is

\[ \bar{P}_1^{(n)} = P \text{ [Call completes in } n \text{ attempts through } O_1 \text{]} \]

\[ = \sum_{i=1}^{n} P_1^{(i)} \]

\[ = \sum_{i=1}^{n} P[X^{(i-1)} = O_1] \cdot P[X^{(i)} = Z / X^{(i-1)} = O_1] \]
\[
= \sum_{i=1}^{n} P[X^{(i)} = O_1, \{1 - (L_1 + L_2^2)\}] \\
= \{1 - (L_1 + L_2^2)\} \sum_{i=0}^{n-1} P[X^{(i)} = O_1] \\
= \{1 - (L_1 + L_2^2)\} \left[ \sum_{i=\text{Type } A}^{n-1} P[X^{(i)} = O_1] + \sum_{i=\text{Type } B}^{n-1} P[X^{(i)} = O_1] \right] \\
+ \sum_{i=\text{Type } C}^{n-1} P[X^{(i)} = O_1] + \sum_{i=\text{Type } D}^{n-1} P[X^{(i)} = O_1] \\
= \{1 - (L_1 + L_2^2)\} \left[ \sum_{j=0}^{n-1} pM^j L_1 + \sum_{j=1}^{n-1} (1 - p)M^j / C + \sum_{j=0}^{n-1} pM^j + \sum_{j=1}^{n-1} (1 - p)M^j / \{L_1 C\} \right] \\
\]

\[\text{......... (2.3.2.5)}\]

Summing over \(n\), we get the following four expressions of cumulative probability for \(O_1\).

(a) when \(n = \text{Type } A\),

\[
\overline{P}_{1A}^{(n)} = \{1 - (L_1 + L_2^2)\} \left[ pL_1C(1 - M^{n+1}) + (1 - M^*p)\{L_1pC + (1 - p)M(1 + L_1)\} \right] \\
\]

(b) when \(n = \text{Type } B\),

\[
\overline{P}_{1B}^{(n)} = \{1 - (L_1 + L_2^2)\} \left[ (1 - M^*)\{pL_1C(1 + L_1) + (1 - p)M\} + (1 - p)L_1M \{1 - M^{n+1} \} \right] \\
\]

(c) when \(n = \text{Type } C\),

\[
\overline{P}_{1C}^{(n)} = \{1 - (L_1 + L_2^2)\} \left[ (1 + L_1) \left[ pL_1C + (1 - p)M \{1 - M^*\} \right] \right] \\
\]

(d) when \(n = \text{Type } D\),

\[
\overline{P}_{1D}^{(n)} = \{1 - (L_1 + L_2^2)\} \left[ \frac{1}{M^*} \left[ pL_1C(1 - M^{n+1}) + (1 - p)M \{1 - M^{n+1} \} \right] \right] \\
\]
Similarly, the general expression for cumulative probability over $O_2$ is

$$
\overline{P}_2^{(\alpha)} = \{1 - (L_2 + L_2^2)\} \left[ \sum_{t=Type A}^{n-1} P[X^{(t)} = O_2] + \sum_{t=Type B}^{n-1} P[X^{(t)} = O_2] \right] \\
+ \sum_{t=Type C}^{n-1} P[X^{(t)} = O_2] + \sum_{t=Type D}^{n-1} P[X^{(t)} = O_2] \right]

= \{1 - (L_2 + L_2^2)\} \left[ \sum_{j=0}^{n-1} (1 - p)M^j L_2 + \sum_{j=0}^{n-1} pM^j / D + \sum_{j=0}^{n-1} (1 - p)M^j + \sum_{j=0}^{n-1} pM^j / \{L_2 D\} \right]

\ldots \ldots \text{(2.3.2.6)}

(e) when $n = \text{Type A},$

$$
\overline{P}_{2A}^{(\alpha)} = \left\{1 - (L_2 + L_2^2)\right\}(1 - p)\frac{L_2 D(1 - M^{n+1})}{(1 - M)\frac{L_2 D}{L_2 D}} + (1 - M^*)\left\{L_2^2 D(1 - p) + pM(1 + L_2)\right\}

(f) when $n = \text{Type B},$

$$
\overline{P}_{2B}^{(\alpha)} = \left\{1 - (L_2 + L_2^2)\right\}(1 - M^*)\left\{(1 - p)L_2 D(1 + L_2) + pM\right\} + pL_2 M(1 - M^{n-1})

(g) when $n = \text{Type C},$

$$
\overline{P}_{2C}^{(\alpha)} = \left\{1 - (L_2 + L_2^2)\right\}(1 + L_2)\left\{(1 - p)L_2 D + pM\right\}(1 - M^*)

(h) when $n = \text{Type D},$

$$
\overline{P}_{2D}^{(\alpha)} = \left\{1 - (L_2 + L_2^2)\right\}(1 + L_2)\left\{(1 - p)L_2 D(1 - M^*) + pM\right\}(1 - M^{n-1})

The probabilities $\overline{P}_1^{(\alpha)}$ and $\overline{P}_2^{(\alpha)}$ give the idea of actual traffic share between the two operators, as a function of $p$, $L_1$ and $L_2$. The change in initial preferences $p$ affects the traffic share when blocking probabilities $L_1$ and $L_2$ are same.
When \( L_1 > L_2 \), the stability of expressions \( P_1^{(n)} \) and \( P_2^{(n)} \) occur after \( n > 1 \).

The \( P_2^{(n)} \) is not much affected over the increasing attempts rather it has a little increment. Only \( P_1^{(n)} \) has a sharp down fall at the first attempt and stability thereafter. With the increase of blocking probability, the operator \( O_1 \) with high
reputation (or QoS) in the market bears a sudden loss in traffic share. When $L_1 < L_2$, both operators bear a loss but the reputed one has more. This seems the call is likely to complete in a few attempts and one can replace $\overline{P}_1^{(s)}$ and $\overline{P}_2^{(s)}$ to their limiting values.

### 2.3.3 LIMITING BEHAVIOUR

If attempts $n > \infty$, cumulative probability that a call completes through an operator in large number of attempts is:

$$\overline{P}_1 = \left[ \lim_{n \to \infty} \overline{P}_1^{(s)} \right] = \frac{(1 - (L_1 + L_2))(1 + L_1)}{(1 - M)L_1 C} \left[ pL_1 C + (1 - p)M \right] \quad \text{......... (2.3.3.1)}$$

$$\overline{P}_2 = \left[ \lim_{n \to \infty} \overline{P}_2^{(s)} \right] = \frac{(1 - (L_2 + L_1))(1 + L_2)}{(1 - M)L_2 D} \left[ (1 - p)L_2 D + pM \right] \quad \text{......... (2.3.3.2)}$$

### 2.3.4 ERROR AT n$^{th}$ ATTEMPT

Define term $E_n$ as error at the $n^{th}$ attempt by $O_i$ like:

$$E_n = \left| \frac{\overline{P}_1}{\overline{P}_1^{(s)}} - 1 \right| \quad \text{T = 18686} \quad \text{......... (2.3.4.1)}$$

if $n = \text{Type A}$: $(E_{n,A})_{O_i} = \frac{(1 + L_1)(pL_1 C + (1 - p)M)}{\left[ pL_1 C(1 - M^{s-1}) + (1 - p)M \right] \left[ (1 - M^{s-1}) + pL_1 M(1 - M^{s-1}) \right]} - 1$

if $n = \text{Type B}$: $(E_{n,B})_{O_i} = \frac{pL_1 C(1 + L_1) + (1 - p)M(1 - M^{s-1})}{\left[ pL_1 C(1 - M^{s-1}) + (1 - p)M \right] \left[ (1 - M^{s-1}) + pL_1 M(1 - M^{s-1}) \right]} - 1$

if $n = \text{Type C}$: $(E_{n,C})_{O_i} = \frac{1}{(1 - M^{s-1})} - 1$

if $n = \text{Type D}$: $(E_{n,D})_{O_i} = \left| \frac{pL_1 C + (1 - p)M}{\left[ pL_1 C(1 - M^{s-1}) + (1 - p)M \right] (1 - M^{s-1})} - 1 \right| \quad \text{......... (2.3.4.2)}$

55
Similarly for $O_1$, we have

if $n = \text{Type A}$

$$A_1 = \frac{\left(1 + L_2\right)\left[\left(1 - p\right)L_2 D + pM\right]}{\left(1 - p\right)L_2 D\left(1 - M^{n+1}\right) + \left(1 - M^n\right)L_2^2 \left(1 - p\right)D + pM \left(1 + L_2\right)} - 1$$

if $n = \text{Type B}$: $B_1 = \frac{\left(1 + L_2\right)\left[\left(1 - p\right)L_2 D + pM\right]}{\left(1 - p\right)L_2 D\left(1 + L_2\right) + pM \left(1 - M^n\right) + pL_2 M \left(1 - M^{n+1}\right)} - 1$

if $n = \text{Type C}$: $C_1 = \frac{1}{\left(1 - M^n\right)} - 1$

if $n = \text{Type D}$: $D_1 = \frac{\left[\left(1 - p\right)L_2 D + pM\right]}{\left(1 - p\right)L_2 D\left(1 - M^n\right) + pM \left(1 - M^{n+1}\right)} - 1$

\[\ldots (2.3.4.3)\]

### 2.3.5 MINIMUM NUMBER OF ATTEMPTS

Using (2.3.4.2) and (2.3.4.3), for the prefix level of error, the minimum number of attempts for $O_1$ (for Type A only) could be obtained like

$$n_{\text{min},A} = \frac{\log\left[\left(\alpha_2 + \alpha_3\right) - \alpha_1 / \left[1 + \left(E_{n,A}\right)_1\right]\right] - \log\left(\alpha_2 M + \alpha_3\right)}{-\log M} \quad \ldots (2.3.5.1)$$

where

$$\alpha_1 = \left(1 + L_1\right)\left[pL_4 C + \left(1 - p\right)M\right]$$

$$\alpha_2 = pL_4 C$$

$$\alpha_3 = pL_4^2 C + \left(1 - p\right)M \left(1 + L_1\right)$$

Similarly, for operator $O_2$ (Type A only), we have

$$n_{\text{min},A} = \frac{\log\left[\left(B_2 + B_3\right) - B_1 / \left[1 + \left(E_{n,A}\right)_2\right]\right] - \log\left(B_2 M + B_3\right)}{-\log M} \quad \ldots (2.3.5.2)$$
where \[ B_1 = (1 + L_2)[(1 - p)L_2D + pM] \]

\[ B_2 = (1 - p)L_2D \]

\[ B_3 = (1 - p)L_2^2D + pM(1 + L_2) \]

### 2.3.6 $p$ AS FUNCTION OF $L_1, L_2$ AND $\bar{P}_1$ (or $\bar{P}_2$)

We compute the initial share $p$ in terms of $L_1, L_2$ and limiting probability for both operators

\[
(p)_{O_1} = \frac{ML_1^3 + 2ML_1^2 + \{\bar{P}_1C(1-M)\}L_1 - M}{\{M - 2C\}L_1^3 + \{2M\}L_1^2 + L_1C - M - CL_1^2} \tag{2.3.6.1}
\]

\[
(p)_{O_2} = \frac{DL_2^4 + 2DL_2^3 + \{\bar{P}_2D(1-M)\}L_2 - D}{DL_2^4 + \{2D - M\}L_2^3 + \{2M\}L_2^2 - L_2D + M} \tag{2.3.6.2}
\]

The fig. 1.15 to fig. 1.18 presents the variation of expressions (2.3.6.1) and (2.3.6.2) over $L_1$, keeping $L_2, P_A$ and $\bar{P}_1$ fixed. These graphs are iso-share curves indicating the amount of initial share desired to maintain the same level of final share $\bar{P}_1$ for operator $O_1$.

![Fig. 1.15](image1)

![Fig. 1.16](image2)
The maximum 35% blocking probability \( L_1 = 0.35 \) for the operator \( O_1 \) is allowed if extraordinarily he bears the cent percent initial share. In particular, if final share is 80\% \( (\bar{P}_1 = 0.8) \), then under 15\% blocking probability \( L_1 = 0.15 \) the \( O_1 \) must have to maintain initial share not less than 83\%. If call above same with 30\% blocking \( L_1 = 0.30 \), \( O_1 \) can only maintain the same when exceptionally \( p = 1.0 \) which is a rare situation in competitive market. This seems with a small increase in blocking probability the operator has to perform hard to maintain the same level of initial share(or QoS) under fixed \( L_1, L_2 \).

**Table 2.3.1 (based on fig. 1.15 and 1.18)**

<table>
<thead>
<tr>
<th></th>
<th>( \bar{P}_1 )</th>
<th>( p )</th>
<th>( p - \bar{P}_1 )</th>
<th>( R.I. )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_1 = 0.15 )</td>
<td>0.70</td>
<td>0.80</td>
<td>0.85</td>
<td>0.90</td>
</tr>
<tr>
<td>( p )</td>
<td>0.74</td>
<td>0.84</td>
<td>0.89</td>
<td>0.94</td>
</tr>
<tr>
<td>( p - \bar{P}_1 )</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>( R.I. )</td>
<td>5%</td>
<td>5%</td>
<td>4.7%</td>
<td>4.4%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( \bar{P}_1 )</th>
<th>( p )</th>
<th>( p - \bar{P}_1 )</th>
<th>( R.I. )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_1 = 0.15 )</td>
<td>0.70</td>
<td>0.80</td>
<td>0.85</td>
<td>0.90</td>
</tr>
<tr>
<td>( p )</td>
<td>0.71</td>
<td>0.81</td>
<td>0.86</td>
<td>0.91</td>
</tr>
<tr>
<td>( p - \bar{P}_1 )</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>( R.I. )</td>
<td>1.4%</td>
<td>1.2%</td>
<td>1.1%</td>
<td>1.1%</td>
</tr>
</tbody>
</table>
Where relative increase $= R.I. = \left( \frac{P_i - \bar{P}_i}{\bar{P}_i} \right) \times 100$

From table 2.3.1, it is apparent that to maintain the predecided level of final traffic share, the operator $O_1$ has to make 4%-5% extra increase in its initial share when $L_1 = 0.15$ and $L_2 = 0.01$. Similarly, operator has to put 1%-2% extra increment in initial share to level the preset final share when $L_1 = 0.15$ and $L_2 = 0.40$.

2.3.7 SHARE LOSS

The traffic share is directly a function of Quality of Service therefore better service of competitor leads to a loss of traffic by the earlier. Define the traffic share loss of $O_1$ by

$$ (\Delta p)_{O_1} = p - \bar{P}_1 = p - \left[ \frac{1 - L_1 - 2L_2}{(1 - M)L_1C} \left[ pL_1C + (1 - p)M \right] \right] \quad \ldots \ldots \text{(2.3.7.1)} $$

and

$$ (\Delta p)_{O_2} = p - \bar{P}_2 = p - \left[ \frac{1 - L_2 - 2L_2}{(1 - M)L_2D} \left[ (1 - p)L_2D + pM \right] \right] \quad \ldots \ldots \text{(2.3.7.2)} $$

The $O_1$ has an advantage over the initial share if $(\Delta p)_{O_1} < 0$ which gives $p < (p_{\text{lim}})_{O_1}$ where

$$ p = (p_{\text{lim}})_{O_1} = \left[ \frac{1}{1 + r} \right]; \quad r = \frac{L_1C(2 + L_1) - ML_1C}{1 - L_1 - 2L_2} \frac{M}{M} \quad \ldots \ldots \text{(2.3.7.3)} $$

Similarly, the operator $O_2$ has advantage over the initial share if $(\Delta p)_{O_2} < 0$ which provides $p < (p_{\text{lim}})_{O_2}$ where

$$ p = (p_{\text{lim}})_{O_2} = \left[ \frac{1}{1 + s} \right]; \quad s = \left[ \frac{(1 - M)}{\left[ 1 - L_2 - 2L_2 \right]} - \frac{M}{L_2D} \right] \quad \ldots \ldots \text{(2.3.7.4)} $$
The fig. 1.19 to 1.22 present the variation of \((p_{\text{lim}})_{O_i}\) over \(L_1\) of expression (2.3.7.3) keeping \(L_2\) fixed. If the limiting share \((p_{\text{lim}})_{O_i}\) is prefixed, the operator \(O_i\) has to be aware for an upper limit of blocking probability \(L_1\) in order to maintain the no loss level of traffic.

For example, using above figure, if \((p_{\text{lim}})_{O_i} = 0.4\) upper limits of blocking for \(O_i\) are \(L_1 = 0.09\) (when \(L_2 = 0.20\)), \(L_1 = 0.13\) (when \(L_2 = 0.30\)), \(L_1 = 0.17\) (when \(L_2 = 0.40\)), \(L_1 = 0.25\) (when \(L_2 = 0.50\)). The operator \(O_i\) cannot bear the blocking level more than these if wants to have a gain in traffic share. This seems that \(O_i\) is benefited in share when competitor’s blocking probability is high.
2.4.0 CASE OF MORE THAN TWO OPERATORS

The following shall be assumptions in this case:

(i) Let there are \( K \) operators \( O_1, O_2, O_3, \ldots, O_K \) in a competitive market with the initial selection probabilities
\[ P[X^{(0)} = O_i] = p_i \ (i = 1, 2, 3, \ldots, K) . \]

(ii) The user attempts to the \( i^{th} \) operator twice (two-call-basis) and, thereafter, either shifts to the next one or abandons the process with probability \( P_A \).

(iii) During the process of attempt, blocking probabilities are \( L_1, L_2, L_3, \ldots, L_K \).

(iv) The transition probabilities are
\[ P[X^{(n)} = O_i \mid X^{(n-1)} = O_j] = p_{ij} \ , \ \ i = j = 1, 2, \ldots, K \]

Such that \( \sum_{j=1}^{k} p_{ij} = [(1 - L_i) + L_i^2 (1 - P_A)] \)

(v) The \( A \) and \( Z \) are two absorbing states for abandon and success of process respectively.

The transition probability matrix is given below

\[
\begin{array}{cccccccc}
\text{States} & & & & & & & & \\
O_1 & O_2 & \ldots & O_k & Z & A \\
O_1 & p_{11} & p_{12} & \ldots & p_{1k} & 1 - (L_1 + L_2^2) & L_1^2 P_A \\
O_2 & p_{21} & p_{22} & \ldots & p_{2k} & 1 - (L_2 + L_3^2) & L_2^2 P_A \\
O_3 & p_{31} & p_{32} & \ldots & p_{3k} & 1 - (L_3 + L_4^2) & L_3^2 P_A \\
& \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
O_K & p_{K1} & p_{K2} & \ldots & p_{KK} & 1 - (L_K + L_K^2) & L_K^2 P_A \\
Z & 0 & 0 & 0 & 1 & 0 \\
A & 0 & 0 & 0 & 0 & 1 \\
\end{array}
\]
One can define the markov chain, for this case, as \( \{X^{(n)}, n = 0,1,2,3, \ldots, \infty\} \) where \( X^{(n)} \) describes \( n^{th} \) attempt by a user to connect while transitioning over \((K + 2)\) states \( O_1, O_2, O_3, \ldots, O_K, Z \) and \( A \).

### 2.5.0 CONCLUSIONS

The proposed markov chain model analyzes the scenario of Internet traffic while to share between two operators. The faithful user to an operator gains significantly when blocking probability to competitor gets high. The probability of success in \( n^{th} \) attempt could be easily obtained mathematically using this model and the saturation level occurs immediately after the first attempt \((n > 1)\). The limiting probabilities are enough to explain the behavior pattern of connectivity and the final share. In order to maintain a preset level of final traffic share (when blocking probability is fixed) the operator has to keep a higher expectation level (1%-5% extra). Under assumption of two-call attempts, the increase in blocking probability of an operator saturates his expectation little early as \( L_1 = 0.35(L_2 = 0.01) \) expects cent percent initial share of customers in market. Therefore, an operator can not bear more than 35% blocking chances in order to maintain the 70% initial share of customers. The increase in blocking probability of competitor creates a relax scenario to the operator. If competitor is at 20% blocking level, the operator can not go beyond 10% blocking if wants to maintain 40% initial traffic share. In this way, the markov chain model explains the operator-consumer behavior pattern in sharing the Internet traffic.