CHAPTER (I)

θ-SEMI CONTINUOUS MAPPINGS

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CHAPTER I

$\theta$-SEMI CONTINUOUS MAPPINGS

1.0 INTRODUCTION:

In 1963, Levine introduced the concept of semi-open sets and used this concept to define the concept of semi-continuity which is weaker than the continuity [6]. In 1943 Fomin introduced the concept of $\theta$-continuity which is also weaker than the continuity [4]. Further, Crossley and Hildebrand introduced the concept of irresolute mappings [1]. It was shown to be stronger than semi-continuity but independent of the continuity. Recently, in 1986 F. H. Khedr and T. Noiri introduced a class of $\theta$-irresolute mappings [5]. A continuous mapping and similarly an irresolute mapping may fail to be the $\theta$-irresolute mapping, in general. The concept of closure semi-continuity due to P. Das [3] is weaker than the semi-continuity. This concept is also studied by Noiri [9], therein it is called $\theta$-semi continuity which is weaker than the present $\theta$-semi continuity.
The present chapter is devoted to a study of a new class of mappings termed $\theta$-semi continuous. The concepts of $\theta$-semi continuity and semi-continuity are found to be independent of each other. The $\theta$-semi continuity is weaker than the both $\theta$-continuity and $\theta$-irresoluteness while stronger than the closure semi-continuity. Some of the equivalent conditions for a mapping to be $\theta$-semi continuous are given and some of its basic properties are also studied.

1.1 TERMINOLOGY :

In a topological space $X$, the closure (interior) of a set $E$ is denoted by $\overline{E}$ ($\text{Int } E$). A subset $E$ is defined to be semi-open if there exists some open set $U$ such that $U \subseteq E \subseteq \overline{U}$ [6]. Complements of semi-open sets are semi-closed sets. The semi-closure and semi-interior of a set $E$ are denoted by $\text{scI } E$ and $\text{sint } E$, respectively. Any point $x$ is said to be in the $\theta$-closure of a set $E$, denoted by $x \in \text{clg } E$, if $\overline{U} \cap E \neq \emptyset$, for every open set $U$ containing $x$ [10]. Further a point $x$ is said to be a $\theta$-interior point of $E$, denoted by $x \in \text{intg } E$, if there exists an open set $U$ such that $x \in U$, $\overline{U} \subseteq E$. Analogously, $x \in \text{sclg } E$ if $\overline{U} \cap E \neq \emptyset$.
for every semi-open set \( U \) containing \( x \), and \( x \notin \operatorname{int}_E \) if there exists a semi-open set \( U \) such that \( x \notin U \). \( \operatorname{cl}_U \subseteq E \) [5].

1.2 \( \theta \)-SEMI CONTINUITY:

**DEFINITION 1.2.1**: A mapping \( f : X \rightarrow Y \) is said to be \( \theta \)-semi continuous at \( x \in X \), if for each open set \( V \) with \( f(x) \in V \), there exists a semi-open set \( U \) with \( x \in U \), such that \( f(\operatorname{cl}_U) \subseteq \operatorname{cl}_V \). If \( f \) is \( \theta \)-semi continuous at every \( x \in X \), then \( f \) is said to be \( \theta \)-semi continuous on \( X \).

**DEFINITION 1.2.2** [4]: A mapping \( f : X \rightarrow Y \) is said to be \( \theta \)-continuous at \( x \in X \), if for each neighbourhood \( M \) of \( f(x) \) there exists a neighbourhood \( N \) of \( x \) such that \( f(\operatorname{cl}_N) \subseteq \operatorname{cl}_M \).

**THEOREM 1.2.1**: \( \theta \)-continuity implies \( \theta \)-semi continuity.

**PROOF**: Since each open is semi open, the proof is obvious. For the converse, an example is given below.

**EXAMPLE 1.2.1**: Let \( X = (a, b, c) \) with topology \( T = (X, \emptyset, (b), (c), (b, c)) \).
and a mapping \( f : X \rightarrow X \) be defined as

\[
f(a) = b, \quad f(b) = b, \quad f(c) = c.
\]

Here, at the point \( a \), the mapping \( f \) is \( \Theta \)-semi continuity but not \( \Theta \)-continuous.

**Definition 1.2.3 [3]**: A mapping \( f : X \rightarrow Y \) is said to be closure semi-continuous on \( X \) if for every \( x \in X \), open set \( V \) containing \( f(x) \), there exists some semi-open set \( U \) such that

\[
f(sclU) \subseteq clV.
\]

Recall that the closure semi-continuity is equivalent to the \( \Theta \)-semi continuity in the sense of Noiri [9]. Henceforth \( \Theta \)-semi continuity in the sense of Noiri [9] will be called closure semi-continuity.

**Theorem 1.2.2**: Every \( \Theta \)-semi continuous mapping is closure semi-continuous mapping.

**Proof**: Let a mapping \( f : X \rightarrow Y \) be \( \Theta \)-semi continuous. Then for each open set \( V \) with \( f(x) \in V \), there exists a semi-open set \( U \) with \( x \in U \), such that \( f(clU) \subseteq clV \). Now, \( sclU \subseteq clU \). Therefore, \( f(sclU) \subseteq clV \), and hence, \( f \) is closure semi-continuous.
The converse of the above theorem 1.2.2 is not true in general, as may be seen by the following example.

**Example 1.2.2**: Let $X = \{a, b, c, d\}$ with topology $T = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$

and a mapping $f : X \rightarrow X$ be defined as

$f(a) = b, f(b) = a, f(c) = d, f(d) = c$.

Then the mappings $f$ is closure semi-continuous but not $\emptyset$-semi continuous.

**Theorem 1.2.3**: If $f : X \rightarrow Y$ is a continuous mapping, then $f$ is $\emptyset$-semi continuous.

**Proof**: Let a mapping $f : X \rightarrow Y$ be continuous. Then for each $x \in X$ and open set $V$ with $f(x) \in V$, there exists an open set $U$ with $x \in U$ such that $f(U) \subseteq V$. Again, by continuity $f(\text{cl}U) \subseteq \text{cl} f(U)$. Therefore, $f(\text{cl}U) \subseteq \text{cl}V$.

Since each open set is semi open the result follows.

The converse of the above theorem 1.2.3 is not true in general as is shown by the example given below.
EXAMPLE 1.2.3: Let $X = \{a, b, c, d\}$ with topology $T = \{X, \emptyset, \{a, d\}, \{b, c\}\}$ and $Y = \{p, q, r\}$ with topology $T' = \{Y, \emptyset, \{q\}, \{r\}, \{q, r\}\}$.

Let a mapping $f$ be defined as:

\[ f(a) = p = f(b), \quad f(c) = q, \quad f(d) = r. \]

Here, $f$ is $\emptyset$-semi continuous but not continuous.

DEFINITION 1.2.4 [6]: $f : X \rightarrow Y$ is semi-continuous if the inverse image of every open subset of $Y$ is semi-open in $X$.

DEFINITION 1.2.5 [1]: $f : X \rightarrow Y$ is said to be irresolute if the inverse image of every semi-open subset of $Y$ is semi-open in $X$.

In the above example 1.2.3 the mapping $f$ is $\emptyset$-semi continuous. But it is not semi-continuous, and hence, not a irresolute mapping.

EXAMPLE 1.2.4: Let $X = \{a, b, c, d\}$ with topology $T = \{X, \emptyset, \{a, b\}, \{c\}, \{a, b, c\}\}$ and $Y = \{p, q, r\}$ with topology $T' = \{Y, \emptyset, \{p\}, \{q\}, \{p, q\}\}$.
Let a mapping $f : X \rightarrow Y$ be defined by

$$f(a) = q = f(b), \quad f(c) = p = f(d)$$

In this example the mapping $f$ is irresolute and hence is semi-continuous, but not $\Theta$-semi continuous.

In the view of the examples 1.2.3 and 1.2.4, it follows that the concept of semi-continuity is independent of the $\Theta$-semi continuity. However, the following results are concerned with the conditions under which the $\Theta$-semi continuity becomes the semi-continuity and vice versa.

**THEOREM 1.2.4** : Let $(Y, T)$ be a regular space and a mapping $f : X \rightarrow Y$ be $\Theta$-semi continuous. Then $f$ is semi-continuous.

**PROOF** : Let $x \in X$ and $V$ be an open set containing $f(x)$. Since the space $Y$ is regular there exists some open set $W$ such that $f(x) \in W \subseteq \text{cl}W \subseteq V$. Again, $f$ is $\Theta$-semi continuous. Therefore, for $f(x) \in W$, there exists some semi-open set $U$ containing $x$ such that $f(\text{cl}U) \subseteq \text{cl}W \subseteq V$. Hence, the mapping $f$ is semi-continuous.
**DEFINITION 1.2.6** (11): A space $X$ is said to be $K$-regular if for any $x \in X$ and semi-open set $V$ with $x \in V$ there exists an open set $U$ such that $x \in U \subseteq \text{cl}U \subseteq V$.

**THEOREM 1.2.5**: If $f : X \to Y$ is a semi-continuous mapping and space $X$ is $K$-regular, then $f$ is $\Theta$-semi continuous.

**PROOF**: Let $x \in X$ and $W$ be any open set containing $f(x)$. Then by semi-continuity there exists a semi-open set $V$ such that $x \in V$, $f(V) \subseteq W$. In the $K$-regular space $X$ we have $x \in U \subseteq \text{cl}U \subseteq V$, for some open set $U$. The open set $U$ is semi-open and such that $f(\text{cl}U) \subseteq \text{cl}W$. Therefore, the mapping $f$ is $\Theta$-semi continuous.

**THEOREM 1.2.6**: If a mapping $f : X \to Y$ is $\Theta$-semi continuous and $Y$ is a $K$-regular space, then $f$ is irresolute.

**PROOF**: Let any $x \in X$ and $V$ be any semi-open set in $Y$ containing $f(x)$. Since the space $Y$ is $K$-regular, there exists an open set $U$ such that $f(x) \in U \subseteq \text{cl}U \subseteq V$. Further, the mapping $f$ is $\Theta$-semi continuous. Therefore, for the open set $U$ there exists some semi-open set $W$ in $X$ such that
\( x \in W \) and \( f(\text{cl}W) \subseteq \text{cl}U \), and hence, \( f(W) \subseteq V \). Therefore, the mapping \( f \) is irresolute.

**Definition 1.2.7 [5]**: A mapping \( f : X \rightarrow Y \) is said to be \( \theta \)-irresolute if for each point \( x \in X \), and for each semi-open set \( V \) containing \( f(x) \), there exists a semi-open set \( U \) with \( x \in U \), such that \( f(\text{cl}U) \subseteq \text{cl}V \).

**Theorem 1.2.7**: Every \( \theta \)-irresolute mapping is \( \theta \)-semi continuous.

By definitions themselves, the proof is straightforward. However, the converse is not true, in general, for we have the example 1.2.3.

**Lemma 1.2.1 [8]**: In extremally disconnected space \( X \), for any semi-open subset \( V \), \( \text{cl}V = \text{sc}V \).

**Theorem 1.2.6**: If a mapping \( f : X \rightarrow Y \) is semi-continuous and \( X \) is extremally disconnected, then \( f \) is \( \theta \)-semi continuous.

**Proof**: Let \( x \in X \) and \( V \) be any open set containing \( f(x) \). Since \( f \) is semi-continuous, \( f^{-1}(V) \) is semi-open in \( X \). Again, since \( X \) is extremally disconnected, \( \text{cl}(f^{-1}(V)) = \text{sc}l(f^{-1}(V)) \). Moreover since \( f \) is semi-continuous, \( \text{sc}l(f^{-1}(V)) \)
\( \subseteq f^{-1}(clV) \). Put \( f^{-1}(V) = U \). Then \( sc \cup U = clU \), and \( f(clU) \subseteq clV \). Hence, \( f \) is a \( \theta \)-semi continuous mapping.

However, we have the following implications diagram:

\[
\begin{array}{ccccc}
\text{lrr} & \longrightarrow & \text{S.C.} & \longleftrightarrow & \text{C} \\
\text{\theta-S.C.} & \downarrow & & \downarrow & \\
\text{\theta-lrr} & \longrightarrow & \text{C.S.C.} & \longleftrightarrow & \text{\theta-C.}
\end{array}
\]

In diagram, we have abbreviated as follows:

\( \text{lrr} = \) irresolute, \( \text{S.C.} = \) semi-continuous, \( \text{C} = \) continuous, \( \theta-\text{S.C.} = \theta\)-semi continuous, \( \theta-\text{lrr} = \theta\)-irresolute, \( \text{C.S.C.} = \) closure semi-continuous, \( \theta-\text{C.} = \theta\)-continuous.

It may be mentioned here, that the concepts of irresolute, \( \theta\)-irresolute and continuous mappings are independent of each other. Also, the concept of \( \theta\)-semi continuity is independent of each of the concepts-irresoluteness and semi-continuity.
THEOREM 1.2.9. : For a mapping \( f : X \rightarrow Y \), the following conditions are equivalent.
(a) \( f \) is \( \theta \)-semi continuous.
(b) \( f^{-1}(V) \subseteq \text{sint}_g(f^{-1}(\text{cl}_V)) \), for every open \( V \) in \( Y \).
(c) \( f(\text{sclg}A) \subseteq \text{clg}(f(A)) \), for every \( A \subseteq X \).
(d) \( \text{sclg}(f^{-1}(B)) \subseteq f^{-1}(\text{clg}(B)) \), for every \( B \subseteq Y \).
(e) \( f^{-1}(\text{int}_gB) \subseteq \text{sint}_g(f^{-1}(B)) \), for every \( B \subseteq Y \).

PROOF : (a) \( \Rightarrow \) (b). Let \( V \) be any open set in \( Y \). For any \( x \in X \) if \( f(x) \in V \), then \( x \in f^{-1}(V) \subseteq f^{-1}(\text{cl}_V) \). Since \( f \) is \( \theta \)-semi continuous, there exists a semi-open set \( U \) such that \( x \in U \) and \( \text{cl}_U \subseteq f^{-1}(\text{cl}_V) \). This means \( x \in \text{sint}_g(f^{-1}(\text{cl}_V)) \). Therefore, \( f^{-1}(V) \subseteq \text{sint}_g f^{-1}(\text{cl}_V) \).

(b) \( \Rightarrow \) (c). Let \( A \) be any subset of \( X \) and \( x \in \text{sclg}A \). Then \( \text{cl}_U \cap A \neq \emptyset \) for every semi-open \( U \) containing \( x \). Take any open set \( V \) with \( f(x) \in V \). Then by (b), \( f^{-1}(V) \subseteq \text{sint}_g(f^{-1}(\text{cl}_V)) \). Hence, \( x \in \text{sint}_g(f^{-1}(\text{cl}_V)) \). Therefore, there exists some semi-open \( W \) such that \( x \in W \subseteq \text{cl}_W \subseteq f^{-1}(\text{cl}_V) \).

Since \( x \in \text{sclg}A \), \( \text{cl}_W \cap A \neq \emptyset \), and hence, \( \text{cl}_V \cap f(A) \neq \emptyset \). Therefore, \( f(x) \in \text{clg}(f(A)) \). Thus \( f(x) \in \text{clg}(f(A)) \), if \( x \in \text{sclg}A \) and hence \( f(\text{sclg}A) \subseteq \text{clg}A \).
(c) \implies (d). Let \( B \) be any subset of \( Y \) and \( x \in X \) be such that \( x \in \text{sclgf}^{-1}(B) \). Then \( f(x) \in f(\text{sclg} f^{-1}(B)) \) and by (c), \( f(x) \in f(\text{sclgf}^{-1}(B)) \subseteq \text{clg} (ff^{-1}(B)) \subseteq \text{clg} B \) and so \( x \in f^{-1}(\text{clg} B) \). Therefore, \( \text{sclgf}^{-1}(B) \subseteq f^{-1}(\text{clg} B) \).

(d) \implies (e). Let \( B \) be any subset of \( Y \) and \( x \in X \) be such that \( x \in f^{-1}(\text{intg} B) \). Then \( f(x) \in \text{intg} B \) and so \( x \in \text{sclgf}^{-1}(Y - B) \). \( \text{sclgf}^{-1}(Y - B) = \text{sclg} (X - f^{-1}(B)) \). Therefore, there exists some semi-open set \( U \) such that \( x \in U \) and \( \text{cl} U \cap (X - f^{-1}(B)) = \emptyset \). This means \( x \in f^{-1}(\text{clg} B) \subseteq f^{-1}(B) \). Therefore, \( x \in \text{sintgf}^{-1}(B) \). It follows that \( f^{-1}(\text{intg} B) \subseteq \text{sintgf}^{-1}(B) \).

(e) \implies (a). Let \( x \in X \) and \( V \) be any open set such that \( f(x) \in V \), i.e., \( x \in f^{-1}(V) \). Recall that for open set \( V \), \( V \subseteq \text{intg} \text{cl} V \). Now, by (e), \( x \in f^{-1} (\text{intg} (\text{cl} V)) \subseteq \text{sintg} f^{-1} (\text{cl} V) \). Therefore, \( f^{-1} (V) \subseteq f^{-1} (\text{intg} (\text{cl} V)) \subseteq \text{sintg} (f^{-1} (\text{cl} V)) \). Therefore, there exists some semi-open \( U \) such that \( x \in U \subseteq \text{cl} U \subseteq f^{-1} (\text{cl} V) \). Therefore, \( f (\text{cl} U) \subseteq f (\text{cl} V) \).

1.3 Some Other Results:

Definition 1.3.1 [7]: A space \( X \) is said to be semi-\( T_2 \) if for any two distinct points \( x \) and \( y \) of \( X \), there exists semi-open sets \( U \) and \( V \) such that \( x \in U \) and \( y \in V \) and \( \text{cl} U \cap \text{cl} V = \emptyset \).
THEOREM 1.3.1: If a mapping $f: X \rightarrow Y$ is $	heta$-semi continuous and injective and if space $Y$ is Urysohn then the space $X$ is semi-$T'_{2}$.

PROOF: Let $x_{1}$ and $x_{2}$ be any two distinct points of $X$. Since the mapping $f$ is injective and $Y$ is Urysohn, there exist open set $V_{i}$ such that $f(x_{i}) \in V_{i}$, $i = 1, 2$ and $\text{cl}V_{1} \cap \text{cl}V_{2} = \emptyset$. Since mapping $f$ is $\theta$-semi continuous, there exist semi-open sets $U_{1}$ and $U_{2}$ containing $x_{1}$ and $x_{2}$, respectively, such that $f(\text{cl}U_{i}) \subseteq \text{cl}V_{i}$. Therefore, $\text{cl}U_{1} \cap \text{cl}U_{2} = \emptyset$, and hence $X$ is semi $T'_{2}$.

The composition of $\theta$-semi continuous mappings is not necessarily $\theta$-semi continuous, as may be seen by the example given below.

EXAMPLE 1.3.1: Let $X = \{a, b, c, d\}$, with topology $T_{X} = \{X, \emptyset, \{a, d\}, \{b, c\}\}$

$Y = \{a, b, c\}$, with topology

$T_{Y} = \{Y, \emptyset, \{b\}, \{c\}, \{b, c\}\}$

$Z = \{a, b, c\}$ with topology

$T_{Z} = \{Z, \emptyset, \{a\}, \{b, c\}\}$

Mapping $f: X \rightarrow Y$ be defined as $f(a) = f(b) = a$, $f(c) = b$, $f(d) = c$. 
and mapping \( g : Y \rightarrow Z \) be defined as: \( g(a) = c, \)
\( g(b) = b, \) \( g(c) = a. \)

Here, the mapping \( f \) is \( \theta \)-semi continuous
at \( a \in X \) and the mapping \( g \) is \( \theta \)-semi continuous
at \( f(a) = a \in Y. \) But the mapping \( g \circ f : X \rightarrow Z \)
is not \( \theta \)-semi continuous at \( a \in X. \)

**THEOREM 1.3.2**: If mapping \( f : X \rightarrow Y \) and
\( g : Y \rightarrow Z \) are \( \theta \)-semi continuous and if the
space \( Y \) is \( K \)-regular then the mapping \( g \circ f \) is
\( \theta \)-semi continuous.

**PROOF**: Let \( x \in X \) and \( W \) be any open set in \( Z \) such
that \( (g \circ f)(x) \in W. \) Since the mapping \( g \) is
\( \theta \)-semi continuous, there exists a semi-open set
\( V_1 \) such that \( \text{cl}V_1 \subseteq g^{-1}(\text{cl}W). \) Now, \( Y \) being \( K \)-
regular there exists an open set \( V \) containing
\( f(x) \) such that \( V \subseteq \text{cl}V \subseteq V_1. \) Therefore, \( \text{cl}V \subseteq \)
\( g^{-1}(\text{cl}W). \) Again, since \( f \) is \( \theta \)-semi continuous
there exists a semi-open subset \( U \) in \( X \) such that
\( x \in U \subseteq \text{cl}U \subseteq f^{-1}(\text{cl}V). \) Therefore, \( (g \circ f) \)
\( (\text{cl}U) \subseteq \text{cl}W. \) Consequently, \( g \circ f \) is a \( \theta \)-semi
continuous mapping.

**THEOREM 1.3.3**: If a mapping \( f : X \rightarrow Y \) is \( \theta \)-
semi continuous and a mapping \( g : Y \rightarrow Z \) is
semi-continuous and if space \( Y \) is \( K \)-regular, then
the mapping \( g \circ f \) is \( \theta \)-semi continuous.
**PROOF**: In view of the theorem 1.2.5 the mapping \( g \) becomes \( \Theta \)-semi continuous and hence, by the above theorem 1.3.2 the mapping \( g \circ f \) is \( \Theta \)-semi continuous.

**THEOREM 1.3.4**: If a mapping \( f : X \rightarrow Y \) is \( \Theta \)-irresolute and mapping \( g : Y \rightarrow Z \) is \( \Theta \)-semi continuous, then \( g \circ f \) is a \( \Theta \)-semi continuous mapping.

**PROOF**: Let for any \( x \in X \), \( W \) be any open subset of \( Z \) containing \((g \circ f)(x)\). Since \( g \) is \( \Theta \)-semi continuous, there exists a semi-open subset \( V \) of \( Y \) such that \( f(x) \in \text{cl}V \subseteq g^{-1}(\text{cl}W) \). Again, since \( f \) is \( \Theta \)-irresolute, there exists a semi-open subset \( U \) of \( X \) such that \( x \in \text{cl}U \subseteq f^{-1}(\text{cl}V) \). Thus \( x \in \text{cl}U \subseteq f^{-1}(g^{-1}(\text{cl}W)) = (g \circ f)^{-1}\text{cl}W \). Therefore, it follows that the mapping \( g \circ f \) is \( \Theta \)-semi continuous.

**LEMMA 1.3.1 (1)**: In a space for any open set \( U \) and semi-open set \( V \), the set \( U \bigcap A \) is semi-open.

**THEOREM 1.3.5**: Let a mapping \( f : X \rightarrow Y \) be \( \Theta \)-semi continuous and \( A \) be any open subset of \( X \). Then the restriction \( f/A \) is also \( \Theta \)-semi continuous.
Proof: Let for any $x \in A$. $V$ be an open subset containing $f(x)$. Since $f$ is $\Theta$-semi continuous, there exists a semi-open set $U$ in $X$ such that $x \in U \subseteq \text{cl}U \subseteq f^{-1}(\text{cl}V)$. Obviously, $x \in U \cap A \subseteq \text{cl}_A(U \cap A) \subseteq (\text{cl}U) \cap A \subseteq f^{-1}(\text{cl}V) \cap A$. Let $f/A = g$. Then $x \in U \cap A \subseteq \text{cl}_A(U \cap A) \subseteq g^{-1}(\text{cl}V)$. Since $U \cap A$ is semi-open, it follows that $g$ is $\Theta$-semi continuous at every point of $A$.

Theorem 1.3.6: Let $f : X \to Y$ be a mapping and $g : X \to X \times Y$ be its graph mapping given by $g(x) = (x, f(x))$ for every $x \in X$. Then $f$ is $\Theta$-semi continuous iff $g$ is $\Theta$-semi continuous.

Proof: First suppose that the graph mapping $g$ is $\Theta$-semi continuous. Let $x \in X$ and $V$ be any open set in $Y$ containing $f(x)$. Then $X \times V$ is an open subset of $X \times Y$ containing $g(x)$. Since $g$ is $\Theta$-semi continuous there exists a semi-open set $U$ of $X$ containing $x$ such that $g(\text{cl}U) \subseteq \text{cl}(X \times V) = X \times \text{cl}V$. Since $g$ is the graph mapping of $f$, we have $f(\text{cl}U) \subseteq \text{cl}V$. Therefore $f$ is $\Theta$-semi continuous.

For the converse part, suppose mapping $f$ is $\Theta$-semi continuous. Let $x \in X$ and $V$ be any open set in $X \times Y$ containing $g(x)$. We may assume that $V = V_1 \times V_2$ where $V_1$ and $V_2$ are open subsets. Then $g(x) = (x, f(x)) \in V_1 \times V_2$. Since
mapping $f$ is $\theta$-semi continuous there exists a semi-open set containing $x$ such that $f(\text{cl}U) \subseteq \text{cl}V_2$. Now, $g$ being the graph mapping of $f$ we have $g(\text{cl}U) = (\text{cl}U \times f(\text{cl}U)) \subseteq (\text{cl}U \times \text{cl}V_2)$ i.e. $g(\text{cl}U) \subseteq \text{cl}(U \times V_2)$. Further, for open set $V_1$ and semi-open set $U$, the set $W = U \bigcap V_1$ is semi-open containing $x$. Hence $g(\text{cl}W) \subseteq g(\text{cl}U) \subseteq \text{cl}(V_1 \times V_2) = \text{cl}V$. Consequently $g$ is a $\theta$-semi continuous mapping.

**Theorem 1.3.7:** Let mappings $f_1 : X_1 \rightarrow Y_1$ and $f_2 : X_2 \rightarrow Y_2$ be $\theta$-semi continuous and let $f : X \rightarrow Y$, where $X = X_1 \times X_2$ and $Y = Y_1 \times Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then $f$ is $\theta$-semi continuous.

**Proof:** Let $x \in X$, and $x = (x_1, x_2)$ where $x_1 \in X_1$, $x_2 \in X_2$. Let $W$ be any open set containing $f(x)$. Then there exist open sets $U_1$ and $U_2$ containing $f(x_1)$ and $f(x_2)$, respectively such that $U_1 \times U_2 \subseteq W$. Since mappings $f_1$ and $f_2$ are $\theta$-semi continuous, there exist semi-open sets $V_1$ and $V_2$ containing $x_1$ and $x_2$ respectively such that $f_1(\text{cl}V_1) \subseteq \text{cl}U_1$ ($i = 1, 2$). Then $V = V_1 \times V_2$ is semi-open set containing $x$ such that $f(\text{cl}V) = f(\text{cl}(V_1 \times V_2)) = f_1\text{cl}(V_1) \times f_2(\text{cl}V_2) \subseteq \text{cl}U_1 \times \text{cl}U_2 \subseteq \text{cl}W$. Therefore, the mapping $f$ is $\theta$-semi continuous.
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