PREFACE

In 1943, Fomin initiated a study of $\theta$-continuous mappings. These are found to be useful in the study of Hausdorff non-regular spaces and turn out to be a natural tool for studying almost compact spaces of Alexandroff and Urysohn, since $\theta$-continuous image of an almost compact space is almost compact. Initially $\theta$-continuous mappings were introduced to obtain some results about $H$-closed extensions of topological spaces. Moreover, in the recent past, an interest in formulating a series of concepts in terms of semi-open sets has appeared considerably not only to attain some new concepts but also to make the semi-open sets convenient for having wide applications therein to stand firmly along side the open sets in the context of general topology.

These considerations enforced us to carry out a study of new mappings which are derived by taking semi-open sets into consideration with $\theta$-continuous mappings.

A chapterwise summary of the present thesis entitled 'A study of some new mappings associated with $\theta$-continuity' is as follows:
In chapter 0, firstly, our attempt is to provide a few lines on the development of Topology laying emphasis on its historical roots, its standing today as one of the most significant modern branches of mathematics, and a way there of characterizing a topological property as a class of topological spaces. Secondly, recalling a significance of "Continuous mapping" in developing the study of topological structures, our aim is to discuss some of the 'nice' mappings - revealed in the literature centered around the continuity and to classify them on the basis of the way with which they may be seen arrived.

In chapter 1, we introduce a concept of 8-semi continuous mappings. It is known that the concepts of irresolute mappings and continuous mappings are independent of each other and further, each is strictly stronger than that of semi-continuous mappings. Also, every semi-continuous mapping is cloure semi-continuous. It may be mentioned, here, that the 8-semi continuity in the sense of Noiri (Indian J. Pure. Appl. Math 21(5) (1990), 410 - 415) coincides with the closure semi-continuity. The 8-semi
continuity is introduced in such a way that it is found to be stronger than the closure semi-continuity. Further, it is found to be weaker than each of the concepts $\Theta$-irresoluteness and $\Theta$-continuity. Also, the $\Theta$-semi continuity is shown to be independent of each of the concepts of irresoluteness and semi-continuity. A number of simple but interesting characterizations of $\Theta$-semi continuity have been noticed. It is also characterized by its graph. In fact, a mapping \( f : X \rightarrow Y \) is $\Theta$-semi continuous iff its graph mapping \( g \) is $\Theta$-semi continuous. Some of its basic algebraic properties Viz., composition, restriction etc. have been also examined.

In chapter II, we are concerned with a new class of mappings called weakly $\Theta$-irresolute mappings. It has been shown that the concept of $\Theta$-irresoluteness and similarly that of irresoluteness is stronger than the concept of weak $\Theta$-irresoluteness which is further stronger than the closure semi-continuity. The semi-continuity, the weak $\Theta$-irresoluteness and $\Theta$-semi continuity studied earlier in chapter I, are independent of each other. A continuous mapping and hence,
$\theta$-continuous mappings may fail to be a weakly $\theta$-irresolute mapping. The concept of weak-$\theta$-irresoluteness is found to be independent of the continuity and $\theta$-continuity. Various characterizations of a mapping to be weakly-$\theta$-irresolute have been obtained. Some basic properties of weakly $\theta$-irresolute mappings viz., composition, restriction, retraction, graph etc. have been also discussed.

In chapter III, a new class of mappings termed strongly closure semi-continuous has been introduced. This concept is found to be weaker than the irresoluteness but stronger than the both closure semi-continuity and semi weak continuity. Further, the semi continuity, weak $\theta$-irresoluteness studied in chapter II, $\theta$-semi continuity studied in chapter I, and strong closure semi-continuity are found to be independent of each other. Further, the strong closure semi-continuity is also found to be independent of the continuity. Various conditions equivalent to the condition for a mapping to be strongly closure semi-continuous have been observed. The graph $G(f)$ of a strongly closure
semi continuous mapping $f$ of a space into a Hausdorff space is shown to be strongly semi-closed.

In chapter IV, a new class of mappings $\Theta_k$-continuous mappings is presented. The idea with which the concept of $\Theta$-continuity of Fomin may be seen derived from the continuity is utilized, here, to derive the $\Theta_k$-continuity from the $K$-continuity. A mapping $f : X \rightarrow Y$ is called $\Theta_k$-continuous if for each $x \in X$, and semi-open set $V$ containing $f(x)$ there exists an open set $U$ with $x \in U$ such that $f(\text{cl}U) \subseteq \text{cl}V$. We investigate the interrelation among the $\Theta_k$-continuity and other known concepts. The $\Theta_k$-continuity is shown to be weaker than the $K$-continuity but stronger than both the $\Theta$-irresolubleness and $\Theta$-continuity. However, the $\Theta_k$-continuity is found to be independent of each of the concepts irresolubleness and continuity. Some of the conditions with which the $\Theta_k$-continuity is implied by some weaker or independent concepts, have been observed. Several characterizations of $\Theta_k$-continuity are established. Some of the basic properties of $\Theta_k$-continuous mappings are also examined.
In chapter V, our interest is to introduce a new class of mappings what are called weakly K-continuous mappings. This concept may be seen arrived at in two ways. In one way, one may have an idea of generalizing the concept of K-continuity just as weak continuity generalizes the continuity, and in other way, in the requirement of a weak continuous mapping one may replace open set at range place by semi-open set. A mapping \( f : X \rightarrow Y \) is said to be weakly K-continuous if for each \( x \in X \) and semi-open set \( V \), containing \( f(x) \) there exists an open set \( U \) containing \( x \), such that \( f(U) \subseteq \text{cl}V \). The concept of weak K-continuity is found to be stronger than each of the concepts-weak semi-continuity, weak-continuity and weak \( \theta \)-irresoluteness but is weaker than the K-continuity. However, each of the concepts-irresoluteness, continuity, \( \theta \)-continuity, semi-continuity and \( \theta \)-semi continuity is independent of the concept of weak K-continuity. The \( \theta \)-semi continuity is studied earlier in the chapter I, while weak \( \theta \)-irresoluteness in the chapter II. Various characterizations of weak K-continuity have been obtained and some other basic properties viz: composition, restriction,
graph etc. have been discussed. A result which is analogous to a well known result concerning two continuous mappings of a space to a $T_2$-space, is also obtained. If $f, g : X \rightarrow Y$ are weakly $K$-continuous on $X$ and $Y$ is semi $T_2'$, then the set $A = \{x \in X : f(x) = g(x)\}$ is closed in $X$: and in particular, if $A$ is dense in $X$ then $f = g$ on $X$. The image of a compact space under a weakly $K$-continuous surjective mapping is found to be $S$-closed.

In chapter VI, our purpose is to bring forth some more generalized results concerning with $\Theta_k$-continuous mappings while it is extended to a bitopological situation. A mapping $f : (X, P_1, P_2) \rightarrow (Y, Q_1, Q_2)$ is said to be pairwise $\Theta_k$-continuous if for each $x \in X$ and for each $Q_i$-semi-open set containing $f(x)$ there exists a $P_i$-open set $U$ containing $x$ such that $f(P_j-\text{cl}U) \subseteq Q_j-\text{cl}V$, where $i, j = 1, 2, i \neq j$. The concept of pairwise $\Theta_k$-continuity is formulated in such a way that it is found to be weaker than the bi $K$-continuity but stronger than the $\Theta$-pairwise continuity. It is also found to be independent of each of the concepts of bi-semi continuity and
bi-irresoluteness. Various characterizations of the pairwise $\Theta_k$-continuity have been established. Some of its basic properties like composition, retraction, graph etc. have been examined.

It may be mentioned, here, that we have given the concerned bibliography at the end of each chapter rather than at the end of the thesis, so as to make the task of reference to it more convenient. Also, the relevant terminology and notations are being given in each chapter to make them convenient for ready reference with an excuse for their duplication naturally occurred in the present thesis. Further, every statement (definition, theorem, example, etc.) has been numbered as an ordered triple $(p,q,r)$, where $r$ denotes the number of the associated statement of the section $q$ of the chapter $p$.

Finally, I may add that the following papers based on the work presented in the thesis have been submitted for publication and some of them have already been accepted for publication.

1. $g$-semi continuous mappings,

2. Weakly $\theta$-irresolute,
   [Under communication].

3. Strongly closure semi-continuous mappings,
   [Accepted for publication in the Journal of the Indian Academy of Math (1992)].

4. $\theta_k$-continuous mappings,
   [Under communication].

5. Weakly-$K$-continuous mappings,
   [Under communication].

6. Pairwise $\theta_k$-continuous mappings,
   [Under communication].