The co-existence of extensive tax evasion and widespread corruption within the tax collecting body is not a recent phenomenon. But in recent times, it has scaled such an alarming height that few governments can risk to ignore it. It is endangering the very functioning of the government on both economic and social fronts. The government is unable to garner sufficient resources for its welfare activities. Another fall-out of this aspect, which may be considered to be more serious and harmful in some sense, is that the social fabric is being weakened because of its ill-effect on social values and norms. This has been especially true in developing countries. The existing tax system in these countries have become so discriminatory (even if this may be so unintentionally) to those few people who are still honest, it will not be farfetched to say that the temptation to join the band-wagon is hard to resist. Very soon, the specie of honest taxpayers might become extinct.

Many of the governments seem to have waken up to the problem. In recent times, they have been trying to tackle
the problem in right earnest. The need for effectively arresting its growth and keeping it within a reasonable limit has been accorded very high priority in any tax policy in these countries. After appreciating the gravity of the situation, the governments have tried to tackle this problem in various ways. Amongst the ways and/or options used by various governments, we may mention the following: lowering the tax rate, increasing the penalty and/or the superpenalty rate\(^2\), bringing in incentive-generating policy in the tax system, increasing vigilance over the affairs of the taxpayers (TPs) as well as tax auditors (TAs).

Before any judgment is passed on the merits and/or demerits of the policies, it will be wise to first undertake a detailed study of the anatomy of such a corrupt society to understand the interplay of forces within it. Once we get a clear insight of the mechanism of the ways in which various forces play against/along with each other in such a society, we will be in a better position to devise ways to deal with this problem. It may then become possible for one to prescribe a correct therapy first to prevent this dreadful disease (which has already got into and done enough damage to the body of the society) from growing in size and secondly to reduce its size to some tolerable limit (if it is not possible to be completely wiped out).

In this chapter, our attempt will be to get a clearer insight of the mechanisms of the interplay of various forces
in such a society. We shall also be studying questions of
the following kind: why will certain type of policies be
more successful in combating this problem whereas others may
fail. In fact, we try to devise ways for the government to
attain its optimizing position (i.e. of maximal expected net
revenue) within the existing set-up.

Using the same model of the previous chapter, we shall
try to study the anatomy of such a society. The only change
being that in the previous chapter, we had worked with a
conjectural or anticipated q (q being the probability of
super audit) but here we shall try to determine the actual q
which will maximize the government's revenue collection.
The problem of devising the optimum audit policy (inclusive
of super audit) will be studied in two possible alternative
situations. In both the situations, the government is
assumed to be fully aware of the prevalence of corruption in
the tax collecting machinery.

In the first situation, it is assumed that the
government assigns the job of studying the seriousness of
the problem vis-a-vis the degree of corruption prevailing,
the pattern of income declaration by the TPs, of the
exposure level by the TAs and other related aspects of the
problem to some research organizations/institutes and/or
some eminent scholars. In particular, it is assumed that
besides other relevant information, the government obtains
estimates of \( \alpha \) (the proportion of income declared by a TP),
e (the exposure level by a TA) and \( \beta \) (the proportion of corrupt officials present). Using these estimates and other available information, the government tries to determine an optimum audit policy\(^3\) (superaudit included). For our exercise, an optimum audit policy is the one which produces the maximum expected net revenue for the government. Thus, in our first situation, the problem has been reduced to that of determining an optimum audit policy taking the other parameters involved to be exogenously given.

In fact, the first situation may be seen in the following light as well. The government enters the stage after the other two agents in the game namely the TP and the TA have already played their parts. The two agents, after taking into account information regarding the tax policy of the government for the current year as well as that of past few years, are assumed to have struck a deal and agreed the bribe sum, \( B^* \) to be transacted between them. This implies that the level of declaration, \('a'\) and of exposure \('e'\) have already been fixed. It is at this juncture that the government enters and tries to determine the optimum audit policy so that in the given circumstances, the expected net revenue is maximized.

In the second situation, the government commits itself to a prior announced audit policy. The government also realizes that the TP and the TA will adjust their behaviors to its a prior announced policy. In fact, here we shall
endogenise the behavioral pattern of the TP as well as the TA in accordance to the declared audit (inclusive of superaudit) policy of the government. Thus in this case, the job of the government is to devise an optimum audit policy, after endogenising the likely behavioral pattern of the TP and the TA to its audit policy.

In the next section, the model has been spelt out. The optimum audit and superaudit probabilities have been determined. Some comparative statics results have been done, thereafter. Finally in the concluding section, we summarize the results of the chapter.

The Model
We shall be working on the same model of the previous chapter with the two following points being kept in mind:

1. Instead of the expected or conjectural value of the probability of superaudit, here we attempt to work out the actual optimum q-value.

2. The government incurs cost in both the activities of auditing and superauditing. The average auditing cost denoted by $C_1$ is taken to be an increasing convex function of the probability of audit i.e. $C_1 = C_1(p)$ with $C_1', C_1'' > 0$ and $C_1(0) = 0, C_1'(0) = 0, C_1'(1) = \infty$.

The average superauditing cost denoted by $C_2$ is taken to be an increasing convex function of both $p$ and $q$ i.e. $C_2 = C_2(p,q)$ with $C_2^j, C_2^jj > 0$ where $j = p, q$ and
Here we are assuming that with greater \( p \) and/or \( q \), much larger infrastructural cost is required due to the enhancement in size of operation. Also, along with the increase in number of cases being covered, one is likely to encounter new cases with greater complexities, the solving of which would require additional expenditure. Taking account of these considerations, the marginal cost involved in each additional audit and/or superaudit is being taken to be increasing.

We will write down the expected net tax revenue, \( ER \) of the government from a representative TP who declares only a fraction, \( \alpha \) of his true income \( Y \) to the Government. The magnitude of \( ER \) will vary with situations in which the concerned TP is likely to land up. The possible situations may be listed out as follows [Refer to appendix I]:

1. He gets audited by a corrupt auditor who himself gets superaudited.
2. gets audited by a corrupt auditor who is not superaudited.
3. gets audited by an honest auditor who, in spite of being honest, gets superaudited.
4. gets audited by an honest auditor who is not superaudited, and finally
5. he is not audited.
Let $ER_i$ $(i = 1, 2, 3, 4, 5)$ be the expected net tax revenue to the government from the TP at $i^{th}$ situation with actual income $Y$. Now, we can write down the following expressions:

$$ER_1 = \theta aY + \pi \theta (1-\alpha)Y + A_0 \theta (1-e)Y - C_1(p) - C_2(q)$$

$$ER_2 = \theta aY + (1-r)\pi \theta (e-\alpha)Y - C_1(p)$$

$$ER_3 = \theta aY + (1-r)\pi \theta (1-\alpha)Y - C_1(p) - C_2(q)$$

$$ER_4 = \theta aY + (1-r)\pi \theta (1-\alpha)Y - C_1(p)$$

$$ER_5 = \theta aY$$

So, the expected net revenue, $ER$ (or in short $R$) to the government from a representative TP will be given by

$$R = p(qR_1 + (1-q)R_2) + (1-p)\{qR_3 + (1-q)R_4\} + (1-p)R_5$$

$$= p[\theta \beta \{qR_1 + (1-q)R_2\} + (1-\beta)\{qR_3 + (1-q)R_4\}] + (1-p)R_5$$

$$= p[\theta \beta Y\{\alpha + (1-r)\pi (e-\alpha) + q\pi (1-\alpha - (1-r)(e-\alpha)) + qA_0(1-e)\} - \beta (C_1 + qC_2)$$

$$+ (1-\beta)\{qR_3 + (1-q)R_4\}] + (1-p)\theta aY$$

$$= p[\theta aY + (1-r)\pi \theta (1-\alpha)Y - C_1 - qC_2 + \beta (1-r)\pi \theta Y(1-e)$$

$$+ \beta \pi \theta Y(1-e + r(e-\alpha)\pi + A_0(1-e))] + (1-p)\theta aY$$

$$= \theta aY + p\{(1-r)\pi \theta (1-\alpha)Y - C_1 - qC_2\}$$

$$+ p\beta \pi \theta Y(1-e + q(A_0 + \pi) - \pi (1+r)) + \pi \theta r(e-\alpha)]$$ (1)

We may also represent equation (1) as $R = f(p, q)$

The optimal audit policy

in the first game situation:

Now, if we go back to our first construct, the problem at hand is to determine $p$ and $q$ which will maximize $ER$ [the details have been stated earlier].

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The first order condition for maximization will require that each of the partial derivatives of ER with respect to \( p \) and \( q \) respectively should be equated to zero, i.e.

\[
\frac{\partial R}{\partial p} = f_1(p, q) = 0 \quad ; \quad \frac{\partial R}{\partial q} = f_2(p, q) = 0.
\]

The second order conditions for maximization requires that

\[
f_1p < 0 \quad ; \quad f_2q < 0
\]

and

\[
f_{1p}f_{2q} > f_{pq}^2
\]

where \( f_{1p} = \frac{\partial^2 R}{\partial p^2} \); \( f_{2q} = \frac{\partial^2 R}{\partial q^2} \)

\[
f_{pq} = \frac{\partial^2 R}{\partial q \partial p} = \frac{\partial^2 R}{\partial p \partial q} = f_{qp}
\]

Thus, the first order condition for maximization provides

\[
\frac{\partial R}{\partial p} = (1-r)\pi \theta (1-\alpha)Y - C_1 - qC_2 - \beta \pi \theta \{ \gamma (1-r)(1-e) - q(1-e+r(e-\alpha)) \}
\]

\[
+ \beta qA_0 \theta (1-e)Y - pqC_1/\partial p - pqC_2/\partial q = 0
\]

or

\[
f_1(p, q) = 0 \quad (2)
\]

\[
\frac{\partial R}{\partial q} = p[-C_2 + \beta \pi \theta \{ (1-e) + r(e-\alpha) \} Y + \beta A_0 \theta (1-e)Y] - pqC_2/\partial q = 0
\]

\[
\Rightarrow (1-e + r(e-\alpha)) \pi + A_0(1-e) \beta \theta Y - C_2(p, q) - q(\delta C_2/\delta q) = 0
\]

\[
\Rightarrow (1-e)(\pi + A_0) + \pi r(e-\alpha) \beta \theta Y - C_2 - q(\delta C_2/\delta q) = 0
\]

or

\[
f_2(p, q) = 0 \quad (3)
\]

As we argue in the appendix, for any \( p \in [0,1] \), there exists a unique \( q(p) \in (0,1) \) such that \( f_2(p, q(p)) = 0 \). Also, we have that for any \( q(p) \in [0,1] \) there exist a
unique $p' \in (0,1)$ such that $f_1(p',q(p)) = 0$ where $p' = \tau \circ q(p)$.

Now, consider the composite function $\tau \circ q : [0,1] \rightarrow [0,1]$

Clearly this function is continuous. Hence by Brouwer's

fixed point theorem, there exists a fixed point $p^* \in [0,1]$

i.e. $\tau \circ q(p^*) = p^*$

Define $q^* = q(p^*)$.

Then, $(p^*, q^*)$ is a unique solution to (2) and (3). We
shall assume that $(p^*, q^*)$ is a point of stability.

Comparative Statics

In our context, we have two possible situations from the

stability condition of $(p^*, q^*)$ [Refer to the Appendix II].

Case (a) : $f_1q < 0$, $f_2p < 0$ and $f_1f_2q - f_1qf_2p > 0$

Case (b) : $f_1q > 0$, $f_2p < 0$ and $f_1f_2q + f_1qf_2p < 0$

In both the cases we find that

$$f_1f_2q - f_1qf_2p > 0$$

For notational convenience, let $K = f_1f_2q - f_1qf_2p > 0$.

From equations (2) and (3), we may put down $p^*$ and $q^*$
as

$$p^* = p(q^*; \alpha, \epsilon, r, \beta, \theta, \pi, A_0)$$

$$q^* = q(p^*; \alpha, \epsilon, r, \beta, \theta, \pi, A_0)$$

Now, we proceed to obtain the following comparative statics results.

The effect of a change in any parameter, $\lambda$ on $p^*$ and $q^*$
may be found from the following expressions:

$$\frac{\partial p^*}{\partial \lambda} = -f_1f_2q + f_1qf_2p \cdot K$$

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and \( \frac{\partial q^*}{\partial \lambda} = -f_{1p}f_{2\lambda} + f_{1\lambda}f_{2p} / K \)

i) Change in \( n \):

We have \( f_{1n} = \Theta Y[(1-r)(1-\alpha-\beta(1-e)) + \beta q(1-e+r(e-\alpha))] > 0 \)

\( f_{2n} = \beta \Theta Y(1 - e + r(e-\alpha)) > 0 \)

Case (a) : \( \partial p^* / \partial n > 0 \) according as \( f_{1n}f_{2q} < f_{1q}f_{2n} \)

\( \partial q^* / \partial n > 0 \) according as \( f_{1p}f_{2n} < f_{1n}f_{2p} \)

Case (b) : \( \partial p^* / \partial n > 0 \)

\( \partial q^* / \partial n > 0 \) according as \( f_{1p}f_{2n} < f_{1n}f_{2p} \)

Thus, we see that the available information at this stage enable us to attribute a clear-cut sign to only a single expression that of \( \partial p^* / \partial n \) of case (b) which is found to be positive. Thus, we have found a situation where an increase in penalty rate must be accompanied by a greater probability of audit to retain optimality which could never have been the case in a corrupt-free system. In the usual literature we find the existence of some trade off between \( p \) and \( n \) as a deterring measure.

ii) Change in \( \theta \):

Here, \( f_{1\theta} = Y[\pi(1-r)(1-\alpha-\beta(1-e)) + \beta q((1-e)(\pi + A_0) + r(e-\alpha))] > 0 \)

\( f_{2\theta} = \beta \pi Yr(e-\alpha) > 0 \)

Here also, at the present stage, a definite sign is determinable for the expression of \( \partial p^* / \partial \theta \) of case (b) only.
which is found to be positive. From the point of view of policy, this result is quite significant, especially in present juncture when the trend in many countries is towards lower taxes for greater tax collection. The governments or the policy makers in these countries have recently become very critical about high taxes. But for a system which is in case (b) situation, our result says that ceteris paribus a change in tax rate (whether it is a case of being raised or being lowered) disturbs the optimality of the system. It further shows that if a hike in tax rate is accompanied by an increase in probability of audit, the optimality of the system can be retained.

iii) Change in $A_0$

Here

$$f_{1A_0} = \beta q(1-e)\theta Y > 0$$

$$f_{2A_0} = \beta (1-e)\theta Y > 0$$

Here also the sign can be determined only for a single case that of case (b) which is found to be positive. Thus, in a situation of case (b), an increase in $A_0$ must be accompanied by a corresponding increase in $p^*$ for optimality of the system.

iv) Change in $\alpha$

Here,

$$f_{1\alpha} = \theta Y\{-\pi(1-r) - \beta qr\} < 0$$

$$f_{2\alpha} = -\pi r \beta \theta Y < 0$$

Here also, for the situation depicted by only case (b), we could say that $p^*$ needs to be decreased along with increase in $\alpha$ for optimality.
v) Change in \( e \)

Here, \( f_{le} = \beta \theta Y\{\pi(1-q) - r(2\pi-q)\} \).

Clearly, \( f_{le} > 0 \) according as \( q(\pi, r) < \pi(1-2r) \).

\[ f_{2e} = -\beta \theta Y\{\pi(1-r) + Ao\} < 0. \]

It can be shown that if \( f_{le} > 0 \), then \( \partial q^*/\partial e < 0 \) in both cases (a) and (b) whereas \( \partial q^*/\partial e > 0 \) in case (a). The sign of \( \partial p^*/\partial e \) is not obtainable for case (b) at this stage. Thus in the case of positive \( f_{le} \), we have results, quite close to one's intuition. But in case of negative \( f_{le} \), we are able to attribute sign to a single expression that of \( \partial p^*/\partial e \) of case (b), which is found to be negative. This latter result which is quite contrary to intuition, may have arisen from the prevailing specific relationship between \( \pi, q \) and \( r \) in the system.

In the comparative statics exercise, we have not discussed the cases of these parameters, \( \lambda \) for which the sign of \( f_{1\lambda} \), \( i=1,2 \) is ambiguous. The effect of change in these parameters on \( p^* \) or \( q^* \) could not be determined because of the ambiguity in sign of \( f_{1\lambda} \).

The optimal audit policy in the second game situation:

Now, let us take up the problem of determining the optimum audit policy in the other set-up. Here, it is being assumed that the government while devising its audit policy has already incorporated the behavioral pattern of the other
two players namely the TP and TA. The government believes that the compliance level, \( \alpha \) of the TP is a positive linear function in \( p \) and \( q \) i.e. 
\[
\alpha = \alpha(p,q) \text{ with } \alpha_p, \alpha_q > 0 \text{ and } \alpha_{ij} = 0 \text{ for } i,j = p,q \n\]
with \( \alpha(0,q) = 0 \); \( \alpha(p,1) = 1 \).

In case of the exposure level of the TA, the government is assumed to believe that it is an increasing positive function of \( q \) of the following nature :
\[
e = e(q) ; \quad e_q, e_{qq} > 0 \text{ with the following corner values } \quad e(0) = \alpha ; \quad e(1) = 1 ; \quad e_q(0) = 0 ; \quad e_q(1) = \alpha.
\]

In fact the following \( e \)-function satisfies all the desired properties
\[
e = 1 - (1-\alpha)((1-q^2))^{1/2} \quad [\text{Here, } \alpha \text{ is a parameter}].
\]

From our earlier study, we know that \( \alpha \) and/or \( e \) is influenced by various factors amongst which the prevailing tax rate, penalty, incentive rate, super-penalty may be mentioned besides the above two factors namely probability of audit and superaudit. Here, we have just picked up \( p \) and \( q \) factors to keep our exercise within manageable limit.

Besides, our objective is to determine an optimum audit policy. The role of the other parameters in the system has been studied in the comparative statics.

Incorporating these behavioral functions \( \alpha(p,q) \) and \( e(q) \) in place of the predetermined pair \( (\alpha,e) \) in (1), we have
\[
ER = \theta \alpha(p,q)Y + p[(1-r)(1-\alpha(p,q))]\pi\theta Y - c_1(p) - qC_2(q)
\]
Again, first order condition for maximization provides

$$\frac{\partial \mathcal{E}_R}{\partial p} = 0$$

$$
(1-r)\pi \theta (1-\alpha) Y - C_1 - qC_2 - \beta \theta p Y \{(1-r)(1-e) - q\} \left(1-e+r(e-\alpha)\right) \\
+ \beta qA_0 \theta (1-e) Y + (1-p\pi(1-r+\beta qr)) \theta Y p - pC_1' = 0
$$

$$
\Rightarrow \theta Y[p(1-r)(1-\alpha) - \beta \{(1-e)(\pi(1-r) - q(\pi A_0)) - qr\pi(e-\alpha)\}] \\
+ (1-p\pi(1-r+\beta qr)) \theta Y p - pC_1' - C_1 - qC_2 - qC_2p = 0
$$

or

$$\phi_1(p, q) = 0$$

$$\frac{\partial \mathcal{E}_R}{\partial q} = 0$$

$$
\Rightarrow p[-C_2 + \beta \theta Y\{(1-e+r(e-\alpha)) + A_0(1-e)\} - qC_2q] \\
+ \theta Y[p(1-r)\pi + \beta pqr]\alpha_q + \beta\{p\pi((1-r) - q(1-r))e_q - qA_0e_q\} = 0
$$

$$
\Rightarrow p[-C_2 + \beta \theta Y\{(1-e)(\pi + A_0) + \pi r(e-\alpha)\}] + \theta Y[p\pi(1-r + \beta qr)]\alpha_q \\
+ p\theta Y\{(1-r)(1-q) - qA_0\}e_q - pqC_2q = 0
$$

or

$$\phi_2(p, q) = 0$$

As we argue in the appendix for any \( q \in (0, 1) \), there exists a unique \( p(q) \in (0, 1) \) such that \( \phi_1(p(q), q) = 0 \). Also, we have that for any \( p(q) \in [0, 1] \) there exist a unique \( q' \in (0, 1) \) such that \( \phi_2(p(q), q') = 0 \) where \( q' = \tau\cdot p(q) \).

Now, consider the composite function \( \tau\cdot p : [0, 1] \rightarrow [0, 1] \). Clearly this function is continuous. Hence by Brouwer's fixed point theorem, there exists a fixed point \( q^* \in [0, 1] \) i.e. \( \tau\cdot p(q^*) = q^* \)

Define \( p^* = p(q^*) \)

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Then, \((p^*, q^*)\) is a unique solution to
\[
\phi_1(p, q) = 0
\]
\[
\phi_2(p, q) = 0.
\]
Here also, we assume that \((p^*, q^*)\) is a point of stability. Again, from the slope condition of stability at \((p^*, q^*)\), the following two possible cases arise [Refer to the Appendix II].

Case (a) : \(\phi_{1q} < 0, \phi_{2p} < 0\) and \(\phi_{1p}\phi_{2q} - \phi_{1q}\phi_{2p} > 0\)

Case (b) : \(\phi_{1q} > 0, \phi_{2p} < 0\) and \(\phi_{1p}\phi_{2q} + \phi_{1q}\phi_{2p} < 0\)

In both the cases we find that
\[
\phi_{1p}\phi_{2q} - \phi_{1q}\phi_{2p} > 0
\]
For notational convenience, let \(H = \phi_{1p}\phi_{2q} - \phi_{1q}\phi_{2p} > 0\)

**Comparative Statics:**

Similar to the analysis in the game situation 1, here also, the effect of a change in any parameter \(\lambda\) on \(p^*\) and \(q^*\), may be obtained from the following expressions:

\[
\frac{\partial p^*}{\partial \lambda} = -\phi_{1p}\phi_{2q} + \phi_{1q}\phi_{2\lambda} / H
\]
\[
\frac{\partial q^*}{\partial \lambda} = -\phi_{1p}\phi_{2\lambda} + \phi_{1q}\phi_{2p} / H
\]

**i) Change in \(\pi\):**

Here, we have
\[
\phi_{1\pi} = \theta Y\{(1-\alpha)(1-\alpha) - \beta(1-e)(1-r-q) + \beta qr(e-\alpha)\} - p(1-r+\beta qr)\theta Y_{\alpha}/\partial p
\]
\[
= \theta Y\{(1-\alpha) - \beta(1-e) - p\delta \alpha / \partial p\} + \beta q(1-e + r(e-\alpha + p\beta \alpha / \partial p))\}
\]
\[
= \{1 - \tilde{a}(p) - \beta(1-e)(1-q) + \beta qr(e-\alpha \tilde{p}(p))\} \theta Y; \quad [\tilde{a}(p) = \alpha(1+c \alpha : p)]
\]
Let \(\tilde{a}(p) \leq e\), then \(1-\tilde{a}(p) \geq (1-e)\),
and hence \( 1-\tilde{\alpha}(p) \geq \beta(1-q)(1-e) \)

In this case, we have \( \partial \psi_1/\partial \pi > 0 \)
Further, \( \phi_{2\pi} \)
\[
= p\theta Y[\beta\{1-e+r(e-\alpha)\} -(1-r+r \epsilon)(\partial \epsilon/\partial q) + \beta(1-r)(1-q)\epsilon e/\partial q]
\]
\[
= p\theta Y[1-e -q\epsilon e/\partial q + rq\epsilon(e-\alpha)/\partial q
\]
\[
+ (1-r)\{\beta(\epsilon e/\partial q) -\partial \epsilon/\partial q\} + \beta r(e-\alpha)\}
\]

It is easily seen that if \( \beta(\epsilon e/\partial q) \geq \partial \epsilon/\partial q \)

\[\text{i.e. } \beta \geq (\partial \epsilon/\partial q) / (\epsilon e/\partial q), \text{ then } \phi_{2\pi} \text{ is positive.}
\]

If both \( \phi_{1\pi} \) and \( \phi_{2\pi} \) are positive, then we have exactly the similar situation as in earlier set up. For such a situation, we may write down the following:

Case (a) : \( \partial p^*/\partial \pi > 0 \) according as \( \phi_{1\pi} \phi_{2q} < \phi_{1q} \phi_{2\pi} \)

and \( \partial q^*/\partial \pi > 0 \) according as \( \phi_{1p} \phi_{2\pi} < \phi_{1\pi} \phi_{2p} \)

Case (b) : \( \partial p^*/\partial \pi > 0 \)

and \( \partial q^*/\partial \pi > 0 \) according as \( \phi_{1p} \phi_{2\pi} < \phi_{1\pi} \phi_{2p} \)

Thus, in this set up also, we are able to attribute a sign to only \( \partial p^*/\partial \pi \) of case (b). We have, thus, shown that even in the second set up, some situations exist where increase in severity of penalty requires simultaneous increase in probability of audit to retain the optimality of the system.
ii) Change in $\theta$:

Here, $\phi_{1\theta}$

$$= Y[(1-r)(1-\alpha-\beta(1-e)) + \beta(q(1-e)(\pi+\lambda_0) + \pi\theta) + \{(1-\pi(\beta q r)\alpha_p)$$

$$= \{pC_1 + qC_2 + qC_2p\}/\theta > 0 \quad \text{[using equation (2a)].}$$

and $\phi_{2\theta} = p\beta Y((1-e)(\pi+\lambda_0) + \pi\theta(e-\alpha)) + Y(1-p\pi(1-r+b rq))\frac{\partial \alpha}{\partial q}$

$$- p\beta Y((q\lambda_0 - \pi(1-r)(1-q))(\partial e/\partial q)$$

$$= p[C_2 + qC_2q]/\theta > 0 \quad \text{[using equation (3a)].}$$

Here also, the only expression for which sign is attributable at this stage that of $\partial p^*/\partial \theta$ is found to be positive. Thus, even in this second set up, we find that a hike in $\theta$ if accompanied by corresponding increase in $p$ will not disturb the optimality of the system.

iii) Change in $\lambda_0$

Here $\phi_{1\lambda_0} = \beta q \theta Y > 0$

$$\phi_{2\lambda_0} = p\beta Y(1 - e - q(\partial e/\partial q)) = p\beta Y(1 - \tilde{e}(q)) > 0$$

Again at this stage, we can attribute the sign to only a single expression that of $\partial p^*/\partial \lambda$ of case (b) which is found to be positive.

CONCLUSION AND SUMMARY

We started out with the objective of trying to become more knowledgeable about the anatomy of a corrupt tax system and its workings. Two possible game situations (as specified in the text) were considered for this purpose. The task of the government was to play the game to maximize its returns. For this we set about to determine the optimum audit policy.
(inclusive of the superaudit policy) of the government which maximizes the expected net revenue in the given tax game situations.

In the first situation, it is assumed that the other two players (i.e. the TP and the TA) of the tax game have already positioned themselves in best places in accordance to the prevailing game conditions before the government enters into the playing arena. That is to say that the TP and TA, respectively have already fixed the level of $\alpha$ and $\epsilon$, after taking account of the available information regarding the tax situations. At this stage, the government enters the arena and seeks for that strategic move in the form of an audit policy which will take him to his goal of maximizing the expected net revenue. A pair $(p^*, q^*)$ which constitutes the optimum audit policy of the government for the given tax system is determined.

In the second situation of the tax game, the task of the government is to frame a rule of the game (in fact, it is to devise an audit policy) which will serve him the best. While framing the rule, he is supposedly fully aware and takes account of the fact that the rule (or the audit policy) will set a certain game pattern from the other players of the game. Specifically the level of $\alpha$ and $\epsilon$ are assume to be functions of audit and superaudit frequencies of the nature as described in the text. The inter dependency of the actions of players in the game have been
recognized while determining the optimum audit policy. Here again we could determine an optimum pair \( (p^*, q^*) \) which will take the government to the position of maximum expected not revenue.

It is quite natural for players to seek new moves if there is any change in the playing conditions of the game under consideration. Similarly, in our tax game too, we find that the optimal values of \( p \) and \( q \) or in short the optimal audit policy needs to be changed along with change in values of tax parameters of the system. The adjustments, required in the values of \( p \) and/or \( q \) with respect to changes in individual tax parameters are worked out in our comparative statics exercise.

In case (b) of first game situation, optimality requires a higher \( p^* \) for greater \( \theta, \pi \) or \( A_0 \). Again in case (b), the government should reduce audit if he gets to know that the compliance level of the TP has improved. We could also specify the conditions under which an improvement in exposure level requires higher \( p^* \) and lower \( q^* \) for optimality [case a(i)]. In case b(i), we have \( \delta q^*/\delta e < 0 \) whereas in case b(ii), we have \( \delta p^*/\delta e < 0 \). In other cases we could not determine the direction of adjustment required in \( p \) and/or \( q \). In short we may note that the adjustment required in \( p \) or \( q \) in response to change in any parameter change is situation specific.

Even in the second game situation, for certain specific
cases, we could specify the directions of the required adjustment in \( p^* \) and/or \( q^* \) in response to change in some of the parameters. But one point which may be noted is that as the model got more complex, the working within the system becomes more and more blurred and fewer and fewer clear statements can be made about the anatomy of the system.

As a matter of fact, we could only specify the required adjustments in \( p \) and/or \( q \) for few cases. Nonetheless our study has thrown up quite significant points regarding the anatomy of a corrupt system and also how difficult it becomes for the government to devise a correct and precise audit policy and modify its policy in case of changed situations.

The fact that there is need for a change in \((p^*, q^*)\) with the parameters does not convey any significant message on its own as far as policy formulation is concerned for the government. From the above, the only information that may be inferred is that if the change is for higher (lower) \( p^* \) and/or \( q^* \), then to reach the new optimum position, the government will need to incur a greater (less) expenditure in its auditing and/or superauditing activities. The more relevant question is that whether it is worthwhile to go in for the policy changes (the variation in tax parameters along with the required accompanying changes in \( p \) and/or \( q \) to reach the new optimum point). In fact, what is needed is to examine whether this new optimum point represents a
position of larger expected net revenue than the earlier one. If the answer is positive, then the policy changes are clearly justified in spite of the additional work-load and/or unpleasantness involved in implementing it.

We have tried to answer the above query in the appendix attached at the end of this chapter. The movement of $ER^*$ (i.e. the maximum expected net revenue) with respect to various parameters and existing corruption level have been studied. In majority of the cases, ER was found to be increasing with the parameters. It should be noted that the increase in ER, obtained in the above exercise is only when the changes in the parameters are accompanied by the appropriate changes in audit policy so that the optimality conditions remain satisfied. It should also be seen that with some parameters, the above result holds good only when the existing conditions are favorable for the result.

In the end, it can be said that a revenue maximizing government has quite a lot of options to choose from, to ensure a higher tax revenue collection. All the theoretically possible choices of policy may not be politically feasible nor socially acceptable. Also, some theoretically very sound policy which are very effective at certain time and place might just not work at another place and time. The system might simply refuse to move towards the desired direction even after the tax reform. So, the choice of policy formulations should be region specific with
due consideration given to the prevailing political and social atmosphere. In fact, a right mix of policy reforms might prove complementary to each other and in the process enforcing and enhancing each other's effect in pushing the system towards the maximum attainable revenue position and also lifting it to a much higher efficiency level.
Notes

1. The small number of honest tax payers reflects the low tax ethics prevailing amongst the citizens in that system. It may not be wrong to say that even out of the few honest TPs, majority of them may be so, not by choice but due to their positions and major source of their income being of the withholding kind. As discussed in the introductory chapter, each individual might come out with his own justifications regarding his act. Besides the ready availability of the corrupt auditors tempts and makes it easier for the TP to enter the corrupt world. The auditors may either argue that they are not getting their due returns for their work or that they are not able to sustain a reasonably good living hence, the system has forced them to indulge in bribe taking etc. The existence of the large number of corrupt auditors may also be attributed to the low tax ethics prevailing in today’s society.

2. Penalty is the fine imposed on tax evader and the superpenalty is the fine imposed on corrupt auditor for taking bribe.

3. Whenever we talk of optimum audit policy, it is inclusive of the superaudit policy as well. Also, the term optimum is used in the sense of maximum expected net tax revenue.

4. The effect of q on e is direct whereas q has at most an indirect effect on α. For an increase in q will imply that the probability of a TA, getting superaudited has increased and the superaudited TA might turn out to be either an honest one or one who is not associated with our TP and hence he, in either case remains unaffected.

5. \[ f_1(p, q) = \pi \theta Y \{1 - \alpha - \beta (1 - e)(1 - r) + \beta q (1 - e + r(e - \alpha))\} - \{c_1(p) + \partial c_1 / \partial p + q(c_2(q))\} \]

\[ f_{11} = \partial f_1 / \partial p = -2(\partial c_1 / \partial p) - p(\partial^2 c_1 / \partial p^2) < 0 \text{ for all } p. \]

Hence \( f_1 \) is monotonic in \( p \).
6. Since \((p^*, q^*)\) solves \(f_1(p, q) = 0, f_2(q) = 0\) and also that \(f_{11} < 0, f_{22} < 0, f_{11}f_{22} - f_{12}f_{21} > 0\) as \(f_{12} = f_{21} = 0\).

7. \[
\frac{dq^*}{\partial \theta} = \{(1-e)(\pi + A_o) + \pi r(e-\alpha)\} \beta Y / -f_{22} > 0 \text{ and }
\frac{dq^*}{\partial A_o} = (1-e) \beta \theta Y / -f_{22} > 0
\]

8. This is quite a weak assumption. This is so because \(1-r < 1\), hence \((1-r)^{-1} > 1\). Hence \(p\pi = (1-r)^{-1}\) is quite plausible.

9. \((p^*, q^*)\) solves the equations \(\phi_1(p, q) = 0\) and \(\phi_2(p, q) = 0\) Also the pair satisfies the second order conditions for maximization i.e.
\[
\phi_{11} < 0, \phi_{22} < 0 \text{ and } \phi_{11} \phi_{22} - \phi_{12}^2 > 0
\]
Here,
\[
\phi_{11} = -2\pi \theta Y (1-r+\beta rq)(\partial \alpha / \partial p) - 2(\partial c_1 / \partial p) - p(\partial^2 c_1 / \partial p^2) < 0
\]
\[
\phi_{22} = -p[2(\partial c_2 / \partial q) + q(\partial^2 c_2 / \partial q^2) + 2\beta Y\{(\pi (1-r)+A_o)(\partial e / \partial q)
+ \pi r(\partial \alpha / \partial q)\}] < 0
\]
and \(\phi_{12} = \phi_{21} = -(1-r+\beta rq)(\partial \alpha / \partial q) - c_2 - q \partial c_2 / \partial q
- \beta \pi \theta Y\{pr(\partial \alpha / \partial p) - (1-r)(1-q) \partial e / \partial q}\}

10. The government may consider the act of under exposure by its own officials to be far more serious offense than that of under-reporting by the TPs. Also, it may view these corrupt officials as the root cause of all the malpractice, prevailing in the tax system. In such circumstances, it is quite likely that \(A_o \geq \pi\).
Tree diagram showing the possible situations arising out of the interaction between the government and the TP.
APPENDIX II

AI The existence of a unique \( p(q) \) such that \( f_1(p(q),q) = 0 \)

From equation (2), we have

i) \( f_1(0,q) = [(1-r)(1-\alpha-\beta(1-e)) + \beta q(1-e+r(\alpha-e))]qY \)
   \[ + \beta q A_0 (1-e)Y > 0 \]

ii) \( f_1(1,q) = [(1-r)(1-\alpha-\beta(1-e)) + \beta q((1-e)(\pi+A_0) + r(\alpha-e))]qY \)
   \[ - C_1(1) - q C_2(1,q) - C_1 p(1) - q C_2 p(1,q) < 0 \]

   [as \( C_1 p(1) = \alpha, \quad C_2 p(1,q) = \alpha \)]

iii) \( f_1 p(p,q) = -[2(C_1 p + q C_2 p) + p(\partial^2 C_1 / \partial p^2)] < 0 \)

   Hence for any \( q \in (0,1) \) there exist a unique \( p(q) \in (0,1) \) such that \( f_1(p(q),q) = 0 \).

Given our assumptions regarding \( f_1(p,q) \), it follows that \( p(q) \) is a continuous function of \( q \).

AII The existence of a unique \( q(p) \) such that \( f_2(p,q(p)) = 0 \)

From equation (3), we have

i) \( f_2(p,0) = ((1-e)(\pi+A_0) + r(\alpha-e))qY > 0 \)

ii) \( f_2(p,1) = ((1-e)(\pi+A_0) + r(\alpha-e))qY - C_2(p,1) - C_2 q(p,1) < 0 \)
   \[ \text{[as \( C_2 q(p,1) = \alpha \)]} \]

iii) \( f_2 q(p,q) = -2C_2 q - q(\partial^2 C_2 / \partial q^2) < 0 \)

   Hence for any \( p \in (0,1) \) there exist a unique \( q(p) \in [0,1] \) such that \( f_2(p,q(p)) = 0 \).

Given our assumptions regarding \( f_2(p,q) \), it follows that \( q(p) \) is a continuous function of \( q \).
III Stability conditions of the point \((p^*, q^*)\)

We have

\[
f_{1q}(p, q) = \beta \theta Y \{(1-e)(\pi + A_0) + r(e-\alpha)\} - qC_{2q}(\cdot) - C_2(\cdot) - pC_{2p}(\cdot)
\]

We are not in a position to provide a sign to the above expression at this stage.

\[
f_{2p}(p, q) = -C_{2p}(p, q) < 0
\]

Slope of \(f_i(p, q) = 0\) in the \(p-q\) plane will be given by

\[
dq/dp = -f_{ip}/f_{iq} \quad \text{for } i = 1, 2
\]

Thus, we have the slope of \(f_2(p, q) = 0\) in the \(p-q\) plane to be negative whereas the slope of \(f_1(p, q) = 0\) could not be determined at this stage.

We also assume that the solution \((p^*, q^*)\) to

\[
f_1(p, q) = 0
\]
\[
f_2(p, q) = 0
\]

is a stable equilibrium point.

Stability here means that if the system is either displaced from \((p^*, q^*)\) or the initial starting position say \((p_0, q_0) \neq (p^*, q^*)\), then the system on its own will adjust and move towards \((p^*, q^*)\).

As mentioned in the appendix (Chapter IV), stability here will require that at \((p^*, q^*)\), the slope of \(f_1 = 0\) to be steeper than \(f_2 = 0\).

Hence, from the slope condition of stability, we can have the following two cases:
Case (a) : If both $f_1() = 0$ and $f_2 = 0$ are negatively sloped, then,

(i) $f_{1q} < 0$
and (ii) $f_{1p}f_{2q} - f_{1q}f_{2q} > 0$

Case (b) : $f_1() = 0$ have a positive slope and $f_2() = 0$ have a negative slope then

(i) $f_{1q} > 0$
and (ii) $f_{1p}f_{2q} + f_{1q}f_{2p} < 0$

AIV The existence of a unique $p(q)$ such that

$\phi(p(q), q) = 0$

From equation (2a), we have

i) $\phi_1(0, q) = [\pi(1-r)(1-\beta(1-e)) + \beta q((1-e)(\pi+A_o) + r e)] \theta Y > 0$

ii) $\phi_1(1, q) = \pi(1-r)(1-\alpha(1,q) - \beta(1-e(q))) \theta Y$

$+ [\beta q((1-e)(\pi+A_o)+r e) + (1-\pi(1-r+\beta qr)) \alpha_p(1,q)] \theta Y$

$- C_1(1) - q C_2(1,q) - C_{1p}(1,q) - q C_{2p}(1,q) < 0$

[as $C_{1p}(1) = \alpha$, $C_{2p}(1,q) = \alpha$]

i) $\phi_1(p, q) = -2\pi \theta Y (1-r+\beta qr) \alpha_p - 2C_{1q} - q C_{2p} - p(\frac{\delta^2 C_1}{\delta p^2}) < 0$

Hence for any $q \in [0,1]$ there exist a unique $p(q) \in [0,1]$ such that $\phi_1(p(q), q) = 0$.

Given our assumptions regarding $\phi_1(p,q)$, it follows that $p(q)$ is a continuous function of $q$. 

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The existence of a unique \( q(p) \) such that \( \phi(p, q(p)) = 0 \)

From equation (3a) we have

i) \( \phi_2(p, 0) = p\theta Y[\beta (1-\alpha) (\pi + A_o) + (1-pn(1-r+\beta qr))\alpha q(p, 0)] > 0 \)

ii) \( \phi_2(p, 1) = -c_2(p, 1) + \beta \theta n(1-\alpha) Y + \theta Y(1-pn(1-r+\beta r))\alpha q(p, 1) \\
\quad - \theta Y A_o e_q(p, 1) - c_2 q(p, 1) < 0 \)

iii) \( \phi_2 q = -[2c_2 q + \beta \theta Y(2(1-r)A_o) e_q + \pi r \alpha q] + \pi c_2 q q < 0 \)

Hence for any \( p \in [0, 1] \) there exist a unique \( q(p) \in [0, 1] \) such that \( \phi_2(p, q(p)) = 0 \).

Given our assumptions regarding \( \phi_2(p, q) \), it follows that \( q(p) \) is a continuous function of \( p \).

AVI Stability conditions of \( (p^*, q^*) \)

We also have

\[
\phi_{1q} = \theta Y[\pi (1-r) (\beta e_q - \alpha q) + \beta (\pi + A_o) (1-e q e_q) \\
\quad + \beta r \pi (e-\alpha + q(e_q - \alpha q) - p \alpha q) ] - c_2 q^2 - c_2 p - q^2 c_2 q
\]

\[
\phi_{2p} = c_{2p} - \pi \theta Y(\beta r \alpha p + (1-r+\beta qr)\alpha q) < 0
\]

Slope of \( \phi_k(p, q) = 0 \) in the \( p-q \) plane

\[
dq/dp = -\phi_{kp} / \phi_{kq} \quad \text{for} \ k = 1, 2
\]

Thus, we have the slope of \( \phi_2(p, q) = 0 \) in the \( p-q \) plane to be negative whereas the slope of \( \phi_1(p, q) = 0 \) could
not be determined at this stage.

Here also, the stability condition of \((p^*, q^*)\) provides us two possible situations, similar to that of case AIII.
APPENDIX III

The net effect on ER of a change in a parameter, accompanied by corresponding adjustments in p and q so that the system retains its optimality.

The expected net revenue of the government, ER was given by

\[ ER = \theta \alpha()Y + p[(1-r)\pi(1-\alpha)Y - C_1 - qC_2 + \beta q A_0 \theta (1-e)Y - \beta \pi \Theta Y((1-r)(1-e) - q(1-e+r(e-a)))] \]  

(A1)

In the previous chapter, we saw that a corrupt auditor who accepts a positive bribe, never exposes anything beyond the self-declared level of income by the TP. Hence, in the above expression, we can use the conclusion of \( \alpha^*(p,q) = e^*(p,q) \) as we are considering a bribe situation.

Using \( \alpha = e \), we have

\[ ER = \theta \alpha Y + p\theta(1-\alpha)Y(1-r)Y + \beta q(A_0 + \pi) - p(C_1 + qC_2) \]  

(A2)

Now, to see the effect of a change in \( \theta \) on ER.

A change in \( \theta \) will have a direct impact on ER; moreover, there are indirect effects through changes in p and q because of change in \( \theta \).

We may write

\[ \frac{d \ ER}{d \theta} = \frac{\partial \ ER}{\partial \theta} + \frac{\partial \ ER}{\partial p} \frac{dp}{d \theta} + \frac{\partial \ ER}{\partial q} \frac{dq}{d \theta} \]  

(A3)

But at the optimum \((p^*, q^*)\), we have from the first order condition for maximization that

\[ \frac{\partial \ ER}{\partial p} = 0, \quad \frac{\partial \ ER}{\partial q} = 0 \]

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Hence,
\[ \frac{\partial \text{ER}}{\partial \theta} = \frac{\partial \text{ER}}{\partial \pi} = \alpha \gamma + p(1-\alpha) \{ \pi (1-r) (1-\beta) + \beta q (A_0 + \pi) \} \]

Since the above expression is positive, it makes sense to resort to a policy of tax hike accompanied by a more intensive audit.

Again,
\[ \frac{\partial \text{ER}}{\partial \pi} = p \theta (1-\alpha) \gamma \{ (1-r) (1-\beta) + \beta q \} > 0 \]

Thus, the government can resort to a raise in penalty rate, which of course needs to be accompanied by an audit increase or decrease depending on the prevailing circumstances.

Now,
\[ \frac{\partial \text{ER}}{\partial \pi} = -p \theta (1-\alpha) \gamma \pi (1-\beta) < 0 \]

Here, irrespective of the prevailing circumstances an increase in tax incentive rate results in a revenue loss to the government. Hence, the government is advised against a raise in tax incentive rate.

\[ \frac{\partial \text{ER}}{\partial \beta} = p \theta (1-\alpha) \gamma [q (A_0 + \pi) - \pi (1-r)] > 0 \]

according as \( q > \frac{\pi (1-r)}{A_0 + \pi} \)

Here, we have a very interesting result at hand. If the super auditing probability \( q \) is greater than some value \( q_0 = \frac{\pi (1-r)}{A_0 + \pi} \), then an increase in corruption level leads to a bigger tax revenue collection. This may be explained in the following terms.

In a situation of a high probability of super audit, i.e. the case of \( q > q_0 \), the fine and super-fine being collected from the TP and TA respectively might overshadow the leakage of tax revenue due to the presence of corrupt auditors, resulting an increase in tax revenue collections.