ON TAX EVADERS AND CORRUPT AUDITORS

The existence of corruption has been recognized world wide, though it may be varying in degree and form [Tanzi (1982), Acharya et.al.(1985), Kabra (1986), Lui (1986), Chu (1990)]. The phenomenon has created a havoc in the functioning of the system wherever it has been rampant. The governments, in such situations, seem to have been awaken by the need for containing this problem and bringing it down to at least a level which do not threaten the normal working of the system. The alarming fact is that the level of corruption seems to be steadfastly growing. Though corruption seems to have crept into every nook and corner of the system, it will not be an exaggeration to point out that activities relating to tax collection is one prominent place where it exists most rampantly and explicitly. Having said the above, it could be a very useful exercise to study how the presence of corruption affects the general functioning of a tax system. Also, unless the presence of corruption is given due recognition, any study of a tax system may turn
out to be a study of something which is far from reality and hence, not very useful in the present context.

For the tax system under our study, the players involved in a tax game may be broadly listed out as (i) the Taxpayers (TPs) (ii) the Tax auditors (TAs) and (iii) the supervising authority/the government/the superauditors (SAs). On the surface, some of the moves/behaviors of them may look very counter-intuitive but as we shall see that these moves fall into their logical places if the corruption factor is accounted for. In this chapter, an attempt has been made to delve into and get an insight into the workings of these agents in their corrupt environment.

Though a vast literature has come up since the seminal paper of Allingham and Sandmo [1972] dealing with tax evasion problem, very few of them have touched upon the problem in the context of a corrupt set-up. The behavioral aspect of the auditors have been almost completely neglected in these studies. For instance, in the Goswami et.al. [1990] model, the authors solve the fixed threat Nash bargaining problem to determine an equilibrium bribe, $B^*$. They have dealt with a zero-one situation where the TP either discloses his full income or do not declare at all. The corrupt TA, on the other hand, exposes the full income or does not expose anything for a side-payment. The above situation is not quite so in real world.\(^1\) Also, after the
determination of $B^*$, they proceed to study the behavioral pattern of the TP and the government. In Virmani [1987], after determining the necessary conditions for a bargaining solutions to a Nash Co-operative Bargain, the author analyses the behavioral pattern of the TP and the government, using the determined conditions. Virmani's main concern is the designing of an incentive system for the Tax collectors which will maximize the government's net revenue, given the tax system and societal institutions affecting corruption. Lui [1986], in a dynamic over-lapping generations model, tries to answer the question of whether and when a corrupt official should accept a given bribe. Also, the changes in the corruption level at the face of some different schemes are brought out. We in this chapter, will look into the behavioral aspects of the TP as well as that of TA, using a model of corrupt set-up. Studying the behavioral patterns of the two agents simultaneously within a general framework, brings out the interdependence of their actions. It is also felt that the analysis of tax evasion problem becomes much more complete in such an approach as the success or the failure of any tax policy depends on the actions and reactions of both the agents. Also, once we know the reasons/forces responsible for certain type of behavior, we might be able to suggest measures to contain such forces so that they start behaving in a
directed/desirable fashion.

In the next section, a corrupt tax-system model has been constructed. Using this model, we hope to study the working of such a system vis-a-vis the course of action/plan of a typical TP or a TA in that corrupt atmosphere. We begin our analysis by determining an equilibrium bribe, \( B^* \) or a bribe rule negotiated between the TP and the TA. How this equilibrium will be affected by variations in some selected parameters, is brought out in our comparative statics exercise. Once the bribe or the bribe rule has been determined, each of the agents, taking account of this can maximize his expected net income. Thus, in the next section, the optimum level of income declaration, \( \alpha^* \) of a typical TP and of exposure, \( e^* \) of a corrupt TA are determined simultaneously. Thus, a Nash Equilibrium pair \(^2\) \( (\alpha^*, e^*) \) is determined. Each of the two agents is assumed to have worked with a common probability of superaudit, \( q \) to reach a consensus regarding the value of \( q \) is significant for it plays a role in determining the magnitudes of the bribe, \( B^* \). The value, of course, is a conjectured one derived by either party or both from various sources. Keeping this specific \( q \)-value in mind, the TP and the TA negotiates for the bribe or the bribe rule. We then proceed to analyze the effect of changes in circumstances within the system on \( (\alpha^*, e^*) \) in our comparative statics exercise.
Next, we take an extension of the above model. Here, it is assumed that each of the agents, on his own or by persuasion of the other, believe that the probability of superaudit is some specific function of the exposure level. In fact, for our analysis in this section, \( q \) has been taken to be a decreasing convex function of \( e \). Our analysis remain valid even for the case when \( q \) is a linearly decreasing function of \( e \).

Finally, we end the chapter with a summary of our findings and certain inferences from them.

THE MODEL

It is a three tier model, consisting of the three sets of players namely TPs, TAs and SAs. A TA audits the tax returns of TPs to check whether they have filed their true incomes or not. The SAs, consisting of a team of incorruptible officials, look into the activities of the TAs. The SAs being honest by assumption have been clubbed together along with the government to form the top tier in the model. Each TP and/or TA tries to maximize his/her expected net income, given the tax policy of the government.

The model is specified by the following assumptions:

A1 : Auditing and superauditing always reveal the true income of the TP. But after auditing, a corrupt TA need not necessarily reveal the true income of the TP.
He may underexpose the amount of tax evaded in consideration for a bribe. Superauditing is done to detect such happenings. It may be noted that in our model, the possibility of an underexposure being an act of error and omission committed by the TA is ruled out.

A 2: The TP, with a true/actual income $Y$, declares $\alpha Y$ ($0 \leq \alpha \leq 1$) as his income. $Y$ is a private information known only to the TP till he is audited. A TA after auditing reports the TP's income as $eY$ ($\alpha \leq e \leq 1$). We shall call 'e' as the exposure level. Once superauditing takes place, the true income of the TP comes out in official record. Thus, the officially known income of the TP at various stages can be put down as

- $\alpha Y$: no audit or before auditing
- $eY$: audit and no superaudit or audit and before superauditing
- $Y$: audit and superaudit.

A3: The probability of audit, $p$ is exogenously given and known to everyone whereas the probability of superaudit, $q$ has been kept a secret as a matter of policy by the government. Hence the players use an expected or conjectural value of $q$ to optimize their actions/strategies. It is possible that the TA from the departmental records and some other sources gets to know the values of $q$ for few preceding years. Let $\bar{q}$ be
the average value of \( q \), taken over these years. Then, the expected or the conjectural value of the \( q \) for the current year will be a value, obtained after interpolating some recent policy statements of the government to \( \bar{q} \). The two parties are assumed to have reached a consensus\(^5\) regarding the current possible value of \( q \), which is being used in the exercise. This is essential for the deal to be struck.

A4 : A proportional tax rate, \( \theta \) is imposed on the income of the TP.

A5 : A proportional penalty rate \( \pi \) is imposed on the tax evaded. \( \pi \) is assumed to be greater than unity [otherwise it will not have any deterring effect]. Thus, penalty required to pay by the TP after auditing is \( \pi \theta(e-\alpha)Y \) and in case of superauditing, \( \pi \theta(1-\alpha)Y \). It should be noted that in case of superauditing, this penalty is over what the TP has paid as bribe to the TA.

A6 : There are costs incurred by the TP in concealing income. It is assumed that the cost incurred, \( C \) is given by

\[
C = C(Y_H), \quad C' > 0, \quad C'' > 0 \text{ with } \\
C'(0) = 0 \text{ and } \lim_{Y \to Y_H} C'(Y) = +\infty
\]

where \( Y_H = (1-\alpha)Y \) is the hidden income.

A7 : The government allows a fraction, \( r \) of the additional
revenue collected (in the form of penalty) from the TP to be kept with the TA as a reward for his effort. Of course, this reward is over the actual salary being paid to the TA for his regular job. 'r' could be termed as the reward rate. For the exposure level, e the reward due to the TA is \( r \pi \theta (e-\alpha)Y \).

A 8: In case of detection of underexposure, the reward (initially bestowed to the TA before superauditing) is taken back by the SA/government. The TA is also penalized a sum of \( A_0 \) times the underexposed portion of the tax evaded. Thus, the penalty on TA is \( A_0 \theta (1-e)Y \). 'A_0' will be referred as super-penalty rate.

A9: Wage level of the TA is exogenously given as 'W' whereas the corruption level, proxied here by the proportion of the corrupt TAs is taken to be a constant, \( \beta \).

DETERMINING THE BRIBE \( B^* \)

The question of bribe enters at the stage when a TA detects a case of understatement of income during the auditing of a TP. If we call those tax evaders who on being detected, would pay the penalty due on them rather than indulge in bribing as honest tax evaders, then in our system the set of honest tax-evaders is assumed to be an empty one. Our TP, on being caught, always offers a bribe, \( B \) to his TA in order to mitigate the quantum of penalty due on him for tax
evasion. The TA on the other hand, has got the option of either (i) accepting the bribe and agreeing to under-expose or not expose at all or (ii) not accepting the bribe and exposing fully.

The second option will come to the forefront in case the TA concerned, happens to be an honest person or even in case of a corrupt auditor, who finds that taking bribe will not be worthwhile in that particular case (a case of failure to strike a beneficial deal). In fact, it is quite clear that bargaining can start only after a tax evader comes face to face with a corrupt auditor. Consider the situation where a corrupt TA audits a TP, who had understated his actual income, Y as \( \alpha Y \). Suppose a bargain is struck for an exposure level, e and a bribe, \( B \).

During the bargain, it is assumed that both the parties have taken account of the two possible situations which may arise to each of them after the exchange of bribe namely the superaudit case and no superaudit case. The expected net gains to each of the party from the bargain may be written down, now.

For the tax evader, in the superaudit case, he pays a penalty of \( \pi \theta (1-\alpha) Y \) and the bribe, \( B \). In case of no bribe and full exposure, he would have needed to pay only the penalty of \( \pi \theta (1-\alpha) Y \). In the no super audit case, he pays a penalty of \( \pi \theta (e-\alpha) Y \) and the bribe \( B \) whereas in case of no
bribe and full exposure, the penalty due on him is \( n\theta(1-\alpha)Y \).

Hence, the expected net gain to the TP from the bargain is (remember \( q \) is the agreed probability of superaudit).

\[
(1-q)\{n\theta(1-\alpha)Y -n\theta(e-\alpha)Y -B\} +q\{n\theta(1-\alpha)Y -n\theta(1-\alpha)Y -B\} = (1-q)n\theta(1-e)Y -B \tag{1}
\]

The corrupt TA, in the no superaudit case, receives the bribe \( B \) and the reward for the partial exposure, \( n\theta(e-\alpha)Y \). In the superaudit case, he keeps the bribe, \( B \), but is charged with a penalty of \( A\theta(1-e)Y \) for the under-exposure. Of course, if he had exposed fully, he would have been entitled to a reward of \( n\theta(1-\alpha)Y \). Thus,

the expected net gain to the TA, from the bargain is

\[
(1-q)\{B +n\theta(e-\alpha)Y -n\theta(1-\alpha)Y\} +q\{B -A\theta(1-e)Y -n\theta(1-\alpha)Y\} = (1-q)(B -n\theta(1-e)Y) +q(B -A\theta(1-e)Y +n(1-\alpha)\theta Y) = B -(1-e)\theta Y\{(1-q)n\pi +qA\} -q\pi(1-\alpha)\theta Y \tag{2}
\]

Each of the parties will wish to maximize the expected net gain to himself from the ultimate bargain. We shall assume that both of them are equally skilled in bargaining.

In a two-person fixed threat situation, the Nash Bargaining solution gives the bribe, \( B \) that maximizes the product of the gains from the negotiation. The Nash Bargaining solution for our problem will be the solution to the following:

Maximize \( L \) with respect to \( B \), where

\[
L = \{(1-q)n\theta(1-e)Y -B\}(B -(1-e)\theta Y(qA +n(1-q)\pi) -q\pi(1-\alpha)\theta Y)
\]
First order condition for maximization gives
\[ \frac{\partial L}{\partial B} = \]
\[ - [B - (1-e)\theta Y(qA_0 + (1-q)r\pi) - q\pi(1-\alpha)\theta Y] + (1-q)\pi\theta(1-e)Y - B = 0 \]
\[ \Rightarrow 2B = (1-q)\pi\theta(1-e)Y + (1-e)\theta Y(qA_0 + (1-q)r\pi) + q\pi(1-\alpha)\theta Y \]
\[ \Rightarrow B^* = \frac{((1-q)(1-e)\pi(1+r) + q[A_0(1-e) + r\pi(1-\alpha)])\theta Y}{2} \]  
(3)

The expected net gain to the TP after the bargain,
\[ = (1-q)\pi\theta(1-e)Y - [(1-q)(1-e)\pi(1+r) + q[A_0(1-e) + r\pi(1-\alpha)]\theta Y/2 \]
\[ = (\theta Y/2)[(1-q)(1-e)\pi(1-r) - q[A_0(1-e) + r\pi(1-\alpha)]] \]

Only if the resultant expected net gain is positive it will be worthwhile for the TP to strike the deal of bribe, \( B^* \). If the expectant net gain is negative, the negotiation will break down and hence in such cases, \( B^* \) as given by (3) will carry no meaning.

Thus, \( B^* \) will be meaningful only in case of positive expected net gain. Hence for a meaningful \( B^* \) (determined in (3)), we must have
\[ q < \frac{\pi(1-e)(1-r)}{\pi(1-e)(1-r) + A_0(1-e) + r\pi(1-\alpha)} \]
or\[ q < \frac{1}{1 + [A_0/\pi(1-r)] + [r(1-\alpha)/(1-e)(1-r)]} \]  
(4)

For the TA also, acceptance of the bribe \( B^* \) will provide him the same expected net gain as that of the TP i.e.
\[ [\pi(1-e)(1-r) - q[A_0(1-e) + r\pi(1-\alpha) + \pi(1-e)(1-\alpha)]]\theta Y/2 \]

Since, our objective is to study the situation where the transaction of bribe is possible and has taken place, we
shall assume that inequality (4) is satisfied. The above assumption will imply that the two parties, for a given pair \((a, e)\) can arrive at an amicable and negotiated bribe, given by \(B^*\) of (3).

Or alternatively we may say that we are considering those cases for which the expected or conjectured \(q\) is less than some value, \(q_0\) where

\[
q_0 = \frac{1}{1 + \frac{A_o}{\pi(1-r)} + \frac{r(1-a)}{(1-e)(1-r)}}
\]

**COMPARATIVE STATICS**

Using expression (3) we can obtain the following comparative statics results to bring out the effect of various parameters on \(B^*\).

\[
\frac{\partial B^*}{\partial \alpha} = -qY < 0
\]
i.e., a higher \(\alpha\) will mean a smaller tax evasion. This, in turn, will imply that in such a case the TP will be liable to a smaller penalty even if he is exposed fully. Hence, in such a case, the TP will be willing and agreeing to pay only a smaller bribe as now the negotiation is about doing away a smaller penalty. The new negotiated bribe, hence will turn out to be smaller than the one, settled earlier.

\[
\frac{\partial B^*}{\partial e} = \frac{-Y}{2} \left\{(1-q)\pi(1+r) + qA_o\right\} < 0
\]

Thus, a higher \(e\) will imply a larger penalty for the TP whereas for the TA, it will mean a bigger reward and a
smaller superpenalty (in case of superaudit). All these factors reinforce on each other to bring about a lower negotiated bribe.

\[ \frac{\partial B^*}{\partial A_0} = -\frac{\theta Y}{2} q(1-e) > 0 \]

Here, a higher \( A_0 \) will increase the likely cost of taking bribe. Hence, the TA will demand for a higher bribe.

Here,

\[ \frac{\partial B^*}{\partial r} = \frac{\pi \theta Y}{2} \{(1-q)(1-e)+(1-\alpha)\} > 0. \]

With increase in reward rate, \( r \), the amount of reward sacrificed by the TA while accepting the bribe increases. Hence the TA hikes up the level of bribe acceptable to him.

With increase in \( \theta \) or \( \pi \), the quantum of penalty due on the TP for the same level of \( e \) is increased. Hence even at the same \( e \), the relief provided to the TP is larger now. The result is that the TP is ready to pay a bigger bribe and TA will also ask for a bigger \( B \) as in the latter case, the opportunity cost for him has also been raised. Again from (3), both \( \frac{\partial B^*}{\partial \theta} \) and \( \frac{\partial B^*}{\partial \pi} \) is found to be positive.

Now,

\[ \frac{\partial B^*}{\partial q} = \frac{\theta Y}{2} \left[ A_0(1-e) + r\pi(1-\alpha) -(1-e)\pi(1+r) \right] \]

\[ = (\theta Y/2)\{(1-e)(A_0-\pi) + r\pi(e-\alpha)\} \]

Clearly, if \( A_0 \geq \pi \), then \( \frac{\partial B^*}{\partial q} > 0 \). In case \( A_0 < \pi \), then, \( \frac{\partial B^*}{\partial q} > 0 \) according as \( r\pi(e-\alpha) > (1-e)(\pi-A_0) \)
An increase in $q$ affects both the parties. In a superaudit case, the TP needs to pay an additional penalty given by $m\theta(1-e)Y$ over the bribe already paid. For his TA the loss is $m\theta(e-\alpha)Y$ (the reward already bestowed to him) plus the super penalty of $A_0\theta(1-e)Y$.

So, a change in $q$ brings about a different picture of cost involved to the two parties in the bribing process. If the increase in cost to the TP is higher than that to TA, a smaller bribe will result and vice-versa i.e. a higher cost to the TA will invite a bigger bribe.

**DETERMINATION OF THE NASH EQUILIBRIUM ($\alpha^*, e^*$)**

So far, a bribe rule, $B^*$ has been determined which maximizes the expected net gain to the TP as well as the TA in the ultimate bargain for a given pair $(\alpha,e)$. Our aim will now be to determine a pair $(\alpha^*, e^*)$ using the above bribe rule where $\alpha^*$ is the best response for the TP, to the choice $e^*$ of the TA and $e^*$ is the best response for the TA, given $\alpha^*$ as the choice of the TP. Precisely, our aim is to find mappings given by $\alpha = \alpha(e), \ e = e(\alpha)$ where $\alpha(e), \ e(\alpha)$ are the best responses of the TP and TA respectively, given the exposure and declaration levels as $e$ and $\alpha$.

To calculate the expected net income, $EY$, of the TP we, first, note the possible situations in which the TP might land up [Refer to the tree-diagram in the appendix I].
In the first instance, we have two possible situations (a) the TP is audited and (b) the TP is not audited.

Now two possible sub-cases in case (a): (a1) the TP might indulge in bribing and (a2) a case of no bribe. Further within the case (a1) two possible situations: (a11) his TA may be superaudited and (a12) no superaudit. Thus in all there are four possible situations namely a11, a12, a2 and b in which our TP might finally land up. Denoting the expected net income of the TP at the four possible situations by \(EY_1, EY_2, EY_3\) and \(EY_4\) respectively, the expected net income of the TP may be written down as

\[
EY = p\{\beta (qEY_1 + (1-q)EY_2) + (1-\beta)EY_3\} + (1-p)EY_4
\]

\[
= p\{\beta (qEY_1 + (1-q)(EY_1 + (1-e)\pi \theta Y)) + (1-\beta)(EY_1 + B^*)\}
\]

\[
+ (1-p)(EY_1 + B^* + \pi \theta (1-\alpha)Y)
\]

\[
= p\{\beta (EY_1 + (1-q)(1-e)\pi \theta Y) + (1-\beta)(EY_1 + B^*)\}
\]

\[
+ (1-p)(EY_1 + B^* + \pi \theta (1-\alpha)Y)
\]

\[
= p[EY_1 + B^* + \beta((1-q)(1-e)\pi \theta Y - B^*)] + (1-p)(EY_1 + B^* + \pi \theta (1-\alpha)Y)
\]

\[
= EY_1 + B^* + \pi \theta (1-\alpha)Y + p[\beta((1-q)(1-e)\pi \theta Y - B^*) - \pi \theta (1-\alpha)Y]
\]

But \(EY_1 = Y - \theta \alpha Y - B^* - \pi \theta (1-\alpha)Y - C(Y_H)\), \(Y_H = (1-\alpha)Y\).

Hence \(EY = Y - \theta \alpha Y - C(Y_H) - p\beta B^* + p\pi \theta Y[\beta(1-q)(1-e) - (1-\alpha)]\) \(5a\)

The expected income of a corrupt auditor after the acceptance of the bribe, \(B^*\) will be

\[
EA = (1-q)(W + B^* + r\pi \theta (e-\alpha)Y) + q(W + B^* - A_0 \theta (1-e)Y)
\]

\[
= W + B^* + ((1-q)r\pi(e-\alpha) - qA_0(1-e))\theta Y \quad (5b)
\]
The TP and the TA will try to maximize \( EY \) and \( EA \) respectively by choice of \( \alpha \) and \( e \).

First order conditions for maximization yields

\[
\frac{\partial EY}{\partial \alpha} = -\theta Y + C'(Y_H)Y - \rho \beta + \rho \theta Y = 0
\]

\[
\Rightarrow \quad -\theta Y + C'(Y_H)Y + \rho \theta q r Y/2 + \rho \theta Y = 0
\]

\[
\Rightarrow \quad C'(Y_H) - \theta(1 - \rho \pi (1 + \beta qr/2)) = 0 \quad (6)
\]

But in a situation where \( \rho \pi (1 + \beta qr/2) \geq 1 \) we have

\[
\frac{\partial EY}{\partial \alpha} > 0 \quad \text{for all} \quad \alpha \in (0,1) \quad \text{and hence in such a case} \quad \alpha^* = 1
\]

The policy implication of the above result is that if the government sets up its tax policy in such a way that the above inequality is satisfied then all the TPs in their own self-interests shall declare their true income.

But we are more interested in studying the situation where a TP conceals a part of his income. Such a situation will be possible only if \( \rho \pi (1 + \beta qr/2) < 1 \). Now onwards, we consider the situations where \( \rho \pi (1 + \beta qr/2) < 1 \).

The equation (6) may therefore be written as \( C'(Y_H) = h \)

where \( h \) is a positive number and equal to \( \theta(1 - \rho \pi (1 + \beta qr/2)) \).

We also know that \( C'(Y_H) \) is a monotonically increasing function of the concealed income, with

\[
\lim_{\alpha \to 1} C'(Y_H) = 0
\]

and

\[
\lim_{\alpha \to 0} C'(Y_H) = \infty
\]

Hence, we can state that there exist a \( Y_H^*, \quad 0 < Y_H^* < Y \) such that \( C'(Y_H^*) = h \) or there exist some \( \alpha^* \) such that \( (1 - \alpha^*)Y = Y_H^* \) where \( 0 < \alpha^* < 1 \).
BEA

Now, \[
\frac{\partial EA}{\partial e} = \frac{\partial B^*}{\partial e} = -r\theta Y - q\theta Y(r\pi - A^e)
\]

\[
= -\left(\frac{\theta Y}{2}\right)\{(1-q)\pi(1+r) + qA^e\} + \theta Y\{r\pi(1-q) - qA^e\}
\]

\[
= \left(\frac{\theta Y}{2}\right)\{q\{A^e + \pi(1-r)\} - \pi(1-r)\} \tag{8}
\]

Since we are considering a bribe situation, our \(q\) is less than \(1/\{1 + (A^e/\pi(1-r)) + (r(1-\pi)/(1-e)(1-r))\}\) [Refer to equation (4)]. This, in turn implies that our \(q\) is less than \(1/\{1 + (A^e/\pi(1-r))\}\) and thus we have

\[
\frac{\partial EA}{\partial e} < 0 \quad \text{for all } e \in [\pi, 1].
\]

Hence \(e^* = \alpha^*\)

Thus, the Nash equilibrium solution to our maximizing problem is \((\alpha^*, \alpha^*)\) where \(\alpha^*\) solves the equation

\[
C'(Y_H) = \theta(1 - p\pi(1+\beta qr/2))
\]

Hence, a corrupt TA in a bribe taking situation, never exposes anything beyond the self-declared income of the TP in our set-up.

Comparative Statics

Using equation (6) we can obtain the following results.

\[
\frac{\partial \alpha^*}{\partial \theta} = \frac{1 - p\pi(1+\beta qr/2)}{-C''(Y_H)} \quad < 0
\]

i.e. ceteris paribus, with a tax hike, the TP will lower its income declaration level to be in its optimum place.

Let us look at equation (7) again. We had

\[
C'(Y_H) = \theta(1 - p\pi(1+\beta qr/2))
\]
Now if, ceteris paribus, there is an increase in $\theta$, then the value of right hand side (RHS) is increased. Hence, to balance the increase in RHS, we must have an increase in $C'(Y_H)$. We know that an increase in $Y_H$ (which means a decrease in $\alpha$) will result in higher value of $C'(Y_H)$. The value of the RHS could have been maintained at the initial level in spite of an increase in $\theta$ if there was a corresponding increase in value of $p(1 + \beta qr/2)$ to neutralise the increase. In that case, there was no need of $\alpha$ changing.

Thus $\alpha^*$ can be maintained even when there is a tax hike, if it is accompanied by changes in other parameters such that the equality (8a) remains valid or such that the value of $\theta(1 - p(1 + \beta qr/2))$ does not get changed i.e. changes in the system to take place such that

$$(1 - p(1 + \beta qr/2))\partial \theta - \theta \partial (p(1 + \beta qr/2)) = 0$$

$$\frac{\partial \alpha^*}{\partial \pi} = \frac{p (1 + \beta qr/2)}{C''(Y_H)} Y > 0$$

Thus, a larger penalty will make the TP declare a higher level of income.

$$\frac{\partial \alpha^*}{\partial \beta} = \frac{pmqr\theta}{2C''(Y_H)} Y > 0$$

i.e. with increase in proportion of corrupt auditors the tax evader needs to increase his compliance level to maintain optimality. This result is
quite counter-intuitive and slightly difficult to comprehend at the very outset. The above result may be explained in the following way: The chance of a TP, getting audited by a corrupt auditor would be enhanced with increase in $\beta$. This, in turn, would imply that the probability of a bribe getting exchanged between the TP and his TA has increased. In the context of the prevailing bribe rule and the cost function of concealing income, it pays the TP to increase the compliance rate with increase in $\beta$. But we should not miss the point that even at $\beta = 1$, we may still have $\alpha^* < 1$ provided $pm(1+qr/2) < 1$. In the latter case, $\alpha^*$ will be provided by the solution of

$$C'(1-\alpha) = \theta{1-pm(1+qr/2)}$$

In fact an increase in any or some of $p$, $\pi$, $r$, $q$ or $\beta$ raises the value of $pm(1+ (r\beta q/2))$ which might lead its value to be greater than or equal to unity. In such a case, as stated earlier, we shall have $\alpha^* = 1$.

It can also easily be shown that an increase in either $p$ and/or $q$ results in higher $\alpha^*$. It is so because a higher $p$ and/or $q$ would make the cost of evasion to go up, hence the TP starts declaring greater income.

Clearly, $\partial \alpha^*/\partial r = pm\beta q/2C''(Y_H) > 0$.

With increase in $r$, the equilibrium bribe, $B^*$ goes up. This rise in $B^*$ can be checked if the TP starts declaring a higher level of income and thus the sign of
$\delta \alpha^* / \delta r$ obtained here is consistent with the rational behavior of the TP in our set-up.

An Extension with Endogenous Probability of Superaudit:

Here, instead of the expected or conjectured $q$ used in the earlier section, we assume that $q$ is an endogenous variable, its value dependent on the exposure level. Here, we will be talking about a TA, who fears that the probability of superauditing is negatively related to his exposure level. It is also assumed that the TP also believes that this fear is genuine. The structure/form of the model remains the same except for this modification in the nature of superaudit probability. For the TP as well as the TA, the relevant value of the probability of superaudit will be given by:

$q = q(e)$, with $q'(e) < 0$, $q''(e) > 0$. Here, $e$ is the negotiated exposure level. Obviously, $\alpha \leq e \leq 1$ and $q$ is assumed to have the following values at corner points.

$q(\alpha) = 1-\alpha$, $q(1) = 0$; $q'(\alpha) = -\infty$, $q'(1) = 0$
In fact, the following q-function fulfills our specifications:

$$q(e) = 1 - \alpha - \{(1-\alpha)^2 - (1-e)^2\}^{1/2}$$

for all $\alpha, 0 \leq \alpha \leq 1$ and $\alpha = e = 1$

[In the above equation, $\alpha$ is a parameter]

The above q-function takes into account the following:

1. The scope or the range for exposure definitely depends on the compliance level of the TP as the exposure range is given by $[\alpha, 1]$. For low (high) $\alpha$, if there is no exposure, meaning $e = \alpha$, then the TA surely will believe that the probability of superaudit will be high (low). Irrespective of initial value of $\alpha$, for high values of $e$, the value of $q$ will be low. This consideration has been taken care of by assuming $q(\alpha) = 1 - \alpha$ and $\lim_{e \to 0} q(e) = 0$ and also the convexity of the function.

2. The TA believes that the marginal gain in terms of lower value of $q$ is much greater at lower end of the exposure range i.e. values of $e$ closer to $\alpha$ than at higher values of $e$. The value of $q$ falls very rapidly initially and then the fall slows down towards $e = 1$. This is accounted by the convexity of the function.

The expression for the expected net income of the TP, $E_Y$ and that of TA, $E_A$ will remain the same as that of
earlier one, except the fact that now, \( q \) is a decreasing convex function of \( e \) as stated above.

Now, \( EY = Y - \theta \alpha Y - C(Y_H) - \beta(1-e)(1-q(e)) \)

\[
EA = W + (\theta / 2) [(1-q(e))(1+r(1-e) - q(A_o(1-e) - \pi(1-a))] + \pi\theta Y(e-a)(1-q(e))
\]

Again, the first order condition for maximization provides

\[
\frac{\delta EY}{\delta \alpha} = C'(Y_H) - \theta(1-\pi(1+\beta q(e)r/2)) = 0 \quad (10)
\]

and

\[
\frac{\delta EA}{\delta e} = 0
\]

=> \( (\theta / 2)[(1-q)(2\pi-\pi(1+r)+qA_o)-q'(){(1-e)(\pi+A_o)+\pi(1-\pi)}]=0 \)

=> \( (\theta / 2)[q(e)(A_o+\pi(1-r)) - \pi(1-r)] \)

\( -q'(e){(1-e)(\pi+A_o)+\pi(1-\pi)} = 0 \quad (11) \)

We may write down equation (10) and (11) in the following form:

\[
f_1(\alpha,e) = 0 \quad (A)
\]

\[
f_2(\alpha,e) = 0 \quad (B)
\]

As we argue in the appendix, equation (B) provides that for any \( \alpha \in [0,1] \) there exist a unique \( e(\alpha) \in [\alpha,1] \) and equation (A) provides for any \( e(\alpha) \in [\alpha,1] \) their exist a unique \( \alpha(e(\alpha)) \in [0,1] \).

Now consider the composite function \( \alpha^*e: [0,1] \rightarrow [0,1] \)

Then by Brouwer's fixed point theorem there exists a fixed point \( \alpha^* \in [0,1] \) i.e. \( \alpha^*e(\alpha^*) = \alpha^* \)

Define \( e^* = e(\alpha^*) \)

Then, \( (\alpha^*,e^*) \) is a unique solution to (A) and (B).
Comparative Statics

Ceteris paribus, the effect of change in any parameter say $\lambda$ on $a^*$ and $e^*$ may be determined in the following way:

We have

$$f_{1a} \frac{da}{d\lambda} + f_{1e} \frac{de}{d\lambda} = -f_{1\lambda} \frac{d\lambda}{d\lambda}$$

$$f_{2a} \frac{da}{d\lambda} + f_{2e} \frac{de}{d\lambda} = -f_{2\lambda} \frac{d\lambda}{d\lambda}$$

and hence

$$\frac{da}{d\lambda} = \frac{(f_{1a} f_{2e} - f_{1e} f_{2a})}{(f_{1a} f_{2e} - f_{1e} f_{2a})}$$

$$\frac{de}{d\lambda} = \frac{(f_{1a} f_{2e} - f_{1e} f_{2a})}{(f_{1a} f_{2e} - f_{1e} f_{2a})}$$

It has been shown in the appendix that $f_{1a}, f_{1e}, f_{2a}, f_{2e}$ are all negative and $(f_{1a} f_{2e} - f_{1e} f_{2a}) > 0$.

We shall use the following expressions to obtain the effect of any parameter, $\lambda$ on $a^*$ and $e^*$.

$$\frac{da}{d\lambda} = \frac{(-f_{1a} f_{2e} + f_{1e} f_{2a})}{(f_{1a} f_{2e} - f_{1e} f_{2a})}$$

$$\frac{de}{d\lambda} = \frac{(-f_{1a} f_{2e} + f_{1e} f_{2a})}{(f_{1a} f_{2e} - f_{1e} f_{2a})}$$

i) Change in $\theta$:

We have $f_{1\theta} = -(1-p\pi(1+{\betaq}(e/2))) \ < 0$; $f_{2\theta} = 0$

Clearly, here, $\frac{da}{d\theta}$ is negative and $\frac{de}{d\theta}$ is positive. Thus in our system, a hike in $\theta$, would result in a lower $a^*$ and a higher $e^*$.

ii) Change in $\pi$:

$$f_{1\pi} = p\theta(1+({\betaq}(e/2)) \ > 0$$

and

$$f_{2\pi} = -(\theta\pi/2)[(1-q)(1+r)+q'(1+r)(1-e)+r(1-\alpha)]$$

$$+ r\theta(1-q-q'(e-\alpha))$$

$$= -(\theta\pi/2)[(1-q)(1-r)+q'(1-e)+r(e-\alpha+2(1-\alpha))]$$

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Clearly, 
\[ f_{2\pi} > 0 \] according as \[ (1-q)(1-r) < -q'(1-e) + r(e-\alpha + 2(1-\alpha)) \]

In the situation of \[ f_{2\pi} \leq 0 \], we shall have \[ \delta \alpha^*/\delta \pi > 0 \] whereas in case of \[ f_{2\pi} > 0 \], then
\[ \delta \alpha^*/\delta \pi < 0 \] according as \[ f_{1e} f_{2\pi} < f_{1\pi} f_{2\pi} \]

Also, if \[ f_{2\pi} \leq 0 \], we have \[ \delta e^*/\delta \pi < 0 \] and if \[ f_{2\pi} > 0 \], then
\[ \delta e^*/\delta \pi < 0 \] according as \[ f_{1\pi} f_{2\alpha} < f_{1\alpha} f_{2\pi} \]

Here, we may note that in the situation of \[ f_{2\pi} \leq 0 \], an increase in \( \pi \) results in a higher \( \alpha^* \) and a lower \( e^* \).

iii) Change in \( \beta \):

Here, \[ f_{1\beta} = p\theta q(e)/2 > 0 \]; \[ f_{2\beta} = 0 \]

Hence, \[ \delta \alpha^*/\delta \beta > 0 \] and \[ \delta e^*/\delta \beta < 0 \], here.

Thus we find that if our system experience an increase in the number of corrupt officials, the TPs start declaring higher level of income whereas the TAs lower their exposure levels.

iv) Change in \( r \):

Here, \[ f_{1r} = p\theta q(e)/2 > 0 \] and
\[ f_{2r} = (\theta Y/2)[-\pi(1-q) - q'(1-e) + \pi(1-\alpha)] + \pi \theta Y\{1-q-q'(e-\alpha)\} \]
\[ = (\pi \theta Y/2)(1-q-2q'\pi(1-\alpha)) > 0 \]

Hence \[ \delta \alpha^*/\delta r > 0 \] according as \[ f_{1r} f_{2e} < f_{1e} f_{2r} \]
and \[ \delta e^*/\delta r > 0 \] according as \[ f_{1\alpha} f_{2r} > f_{1r} f_{2\alpha} \].
v) Change in $p$:

Here, $f_{1p} = \pi \theta (1 + (\beta q(e)r/2)) > 0; \quad f_{2p} = 0$

Hence, $\partial \alpha^* / \partial p > 0$ and $\partial e^* / \partial p < 0$, in our tax system. Thus an increase in $p$ results in a higher $\alpha^*$ and a lower $e^*$.

vi) Change in $A_0$:

Here, $f_{1A_0} = 0; \quad f_{2A_0} = (\Theta Y/2) \{q' - q'(1-e)\} > 0$

Hence, $\partial \alpha^* / \partial A_0 < 0$ and $\partial e^* / \partial A_0 > 0$ in our tax system. Thus, a rise in $A_0$ results in lowering of $\alpha^*$ and enhancement of $e^*$.

vii) An upward shift in q-function:

To examine the effect of an upward shift in q-function on $\alpha^*$ and $e^*$ we use the following technique as q itself is a function of $e$.

We may write $q(e)$ as $q(e) = - \int^e_{1} q'(\tau) \delta \tau \quad [q(1) = 0]$

$$= \int^e_{1} q'(\tau) \delta \tau$$

Replace $q'(\tau)$ by $q'(\tau) + \rho$ in the above to get

$$\bar{q}(e) = \int^e_{1} (q'(\tau) + \rho) \delta \tau$$

Substituting $q'(e)$ by $q'(e) + \rho$ and $q(e)$ by $\bar{q}(e)$ in equation (A) and (B), we obtain

$$f_1(\alpha, e) = C' \left( Y_H \right) - \theta \{1 - p\pi (1 + \beta (q - \rho (1-e))(r/2)) \} = 0 \quad (A')$$

$$f_2(\alpha, e) = (\Theta Y/2) \{A_0(q(e) - \rho (1-e)) - \pi (1-q+\rho (1-e))(1+r)$$

$$- (q' + \rho) \{\pi (1+r)(1-e) + A_0 (1-e) + \pi (1-\alpha) \} \}$$

$$+ \pi \theta \{1-q+\rho (1-e) - (q'(e) + \rho)(e-\alpha) \} = 0 \quad (B')$$
Now the required result will be obtained by taking up $\rho$ as the parameter concerned.

$$\frac{\partial \alpha^*}{\partial \rho} = \frac{(-f_1 \rho f_2 e + f_1 e f_2 \rho)}{H}$$

Here, $f_1 \rho = -\theta \rho \pi (1-e) r / 2 < 0$

$$f_2 \rho = \left(\frac{\theta \gamma}{2}\right) \left[-A_0 (1-e) - \pi (1-e)(1+r) - \pi (1+r)(1-e) - A_0 (1-e) - r \pi (1-\alpha)\right]$$

$$+ r \pi \theta Y (1-e) - (e-\alpha)$$

Therefore, $\frac{\partial \alpha^*}{\partial \rho} > 0$ according as $f_1 e f_2 \rho < f_1 \rho f_2 e$

Similarly, $\frac{\partial e^*}{\partial \rho} > 0$ according as $f_1 \rho f_2 e > f_1 e f_2 \rho$

Thus, at the present stage we are unable to determine the direction of change in $\alpha^*$ or $e^*$ with an upward shift in $q$-function. Thus, an increase in vigilance over the TA's activities does not necessarily provide the intuitive results of higher $\alpha$ and $e$ in a straightforward manner.

**SUMMARY AND CONCLUSION**

We find that the possibility of a deal being struck i.e. a bribe being exchanged between a tax evader and a corrupt TA, given $\alpha$ and $e$, depends on the the anticipated value of $q$. Only if $q$ is less than

$$\frac{1}{1 + \left(\frac{A_0}{\pi (1-r)} + r (1-\alpha)/(1-e)(1-r)\right)},$$

could the parties agree to a meaningful bribe, $B^*$. 151
Assuming that a bribe, agreeable to both the parties could be struck, using the bribe rule the Nash equilibrium position \((a^*, e^*)\) has been obtained where \(a^*\) and \(e^*\) are the optimal declaration and exposure level of the TP and TA, respectively. In the comparative statics to study the effect on \(B^*\), the results obtained were quite in line to our intuition. The final effect on \(B^*\) depended on the relative cost/benefit to the two parties from the respective changes in the parameters. It was found that if the resultant cost (benefit) to TA from the change in parameter is greater (less) than that to TP, the amount of the bribe, \(B^*\) would increase (decrease) and vice-versa.

The existence of a meaningful bribe situation, itself, implies that \(a^*\) should be less than unity. It was found that a corrupt TA who accept a positive bribe will never expose anything beyond what the TP had, himself, declared. In fact, the Nash equilibrium position obtained in the model of exogenous \(q\), is \((a^*, a^*)\) where \(0 < a^* < 1\).

In the comparative statics exercise, \(a^*\) was found falling with tax rate, \(\theta\). Even in Goswami et.al.(1991), a similar result was obtained. In their model, there exist a truth revealing audit probability \(p^*\) at which the TPs declare their true incomes whereas below it, zero income was declared. In case of a tax hike, \(p^*\) is raised which implies that TPs, at the margin, will stop declaring their income
after the tax hike, unless it was accompanied by an increase in audit probability. Thus, in their model also, a tax hike might result in zero declaration of income by those TPs who were earlier declaring their true income.

A rise in penalty resulted in a higher value of $\alpha^*$ in our model. In Goswami et al., $p^*$ is lowered in case of a more severe penalty. Taking up Goswami model, let the prevailing probability of audit be $p_o$ (less than $p^*$). In that case, every one cheats. Now, with a raise in penalty, let $p^*$ take up a new value $p_o^* = p_o$. In such a case, everyone declares true income. The effect of an increase in severity of penalty on $\alpha^*$ is similar in ours and Goswami’s model.

We had a very counter-intuitive result in regard to effect of change is corruption level, $\beta$ on $\alpha^*$. $\alpha^*$ was found to be increasing in $\beta$. The logic behind this result, was explained in the text. This result was quite in contrast to Goswami's where $p^*$ increases with $\beta$. In Goswami et al., the truth revealing audit was raised with increase in $\beta$. The implication of the above is that increase in $\beta$ will encourage cheating unless auditing becomes more intensive to counteract the effect of a greater $\beta$.

An increase in either $p$ or $q$ raises the level of $\alpha^*$ as tax evasion becomes costlier in the new situation. Also a rise in $r$ raises $\alpha^*$. When there is a rise in $r$, the TA
starts demanding a higher bribe and to counter-act this, the TP increase $\alpha$, resulting in a higher $\alpha^*$. In an extension of the model where the superaudit probability, $q$ is taken to be a decreasing convex function of exposure level $e$, we determine a Nash equilibrium position $(\alpha^*, e^*)$. In the comparative statics exercise, a hike in $\theta$ was found to induce a lower $\alpha^*$ and a higher $e^*$. In most of the theoretical models dealing with corruption free system, wherever the effect of $\theta$ on $\alpha$ could be determined a hike in $\theta$ was found to induce an increase in $\alpha$, quite in contrast to empirical evidences [This was mentioned in our introductory chapter]. But here we have a result of lower $\alpha$ with $\theta$ which is quite in line with the empirical evidences available. The undoubtful deterring power of $\pi$ on TP becomes a suspect, here, though in situations of non-positive $f_{2\pi}$, $\alpha^*$ was found to increase with $\pi$ but at the similar situation, $e^*$ decreases with $\pi$. An increase in the number of corrupt auditors is found to induce a TP to raise his compliance level whereas $e^*$ was found to decrease. We could not obtain any straight forward result of the effect of reward rate on $\alpha^*$ or $e^*$. As stated in the text, the actual effect of $r$ on either $\alpha^*$ or $e^*$ will depend on the prevailing circumstances. An increase in frequency of audit seem to instill fear into TP's mind, resulting in a higher $\alpha^*$, but a negative aspect of this
line of action is the resultant lower $e^*$. Unlike the case of change in penalty rate, we obtain a clear-cut result in case of superpenalty rate $A_0$ here. $\alpha^*$ is found to decrease with $A_0$ whereas $e^*$ is enhanced. Lastly, we find that an increase in vigilance over the TA's activities need not necessarily result in higher $\alpha$ and $e$. 
Notes

1. It is more often that TPs declare a portion of their income though understating their true income. Similar is the case with the corrupt auditors. For a corrupt auditor, the options open to him is not necessarily limited to full exposure or zero exposure. Partial exposure is possible if it is in his interest.

2. The pair \((a^*, e^*)\) represents a Nash equilibrium point in the sense that given \(e^*\), \(a^*\) is the best response of the TP and given \(a^*\), \(e^*\) is the best response of the TA.

3. The declared income \(\alpha Y\) of the TP could consists of two components: taxable and avoidance/tax shelter income. The latter income is that portion of his income which gets preferential tax treatment (either zero or lower tax rates for it). Thus, tax avoidance in contrast to tax evasion is the legal reduction in tax liabilities that occurs by taking advantage of various provisions of income tax codes. Our present concern is with the tax evasion issue, not denying the fact that a very significant leakage from expected tax revenue takes place through schemes of tax avoidance.

4. The presence of auditors who are not morally upright has been recognized here. It is feared that these corrupt auditors might use the information regarding 'p' to further their own interest in case it is known only to them and SAs. Also trying to keep 'p' a secret compared to that of q will be a much more difficult proposition considering the level of ethics of the TAs and the fact that the body consisting of TAs is much
larger than that of SAs. Clearly a larger body might allow more possible leakage points.

5. Clearly, the bribe negotiation is a co-operative game between the TP and his TA. The two individuals are supposedly sharing all information available and working towards the best interest of both of them as that strategy turns out to be the optimal for each party, as well.

6. In the model neither the wage level has been linked to any work effort nor the TA strives for some specific level of income. Hence $W$ does not have any functional role, here.

7. For an honest TA, his income after auditing our representative tax evader will always be given by $W + r\pi \theta (1-\alpha)Y$ where $W$ is his salary and $r\pi \theta (1-\alpha)Y$ is the reward for booking the tax evader.
APPENDIX I

Tree diagram showing the possible situations of our TP

Our TP

a: gets audited

a1: successfully bribes his TA

a11: The TA is superaudited

[Situation 1]

a12: is not superaudited

[Situation 2]

b: is not audited

[Situation 4]

a2: unsuccessful in bribing

[Situation 3]
APPENDIX II

AI. The existence of a unique \( \alpha(e) \):

i) \( f_1(0,e) = C'(Y) - \theta(1-p\pi(1+\beta q(e)r/2)) > 0 \) [as \( C'(Y) = \alpha \)]

ii) \( f_1(1,e) = - \theta(1-p\pi(1+\beta q(e)r/2)) < 0 \)

iii) \( f_{1\alpha}(\alpha,e) = -C''(Y_H) Y < 0 \)

(i), (ii), and (iii) show that \( \exists \) a unique \( \alpha(e) \in [0,1] \) for each \( e \in [0,1] \) satisfying \( f_1(\alpha,e) = 0 \). Given our assumptions regarding \( f_1(\alpha,e) \), it follows that \( \alpha(e) \) is a continuous function of \( e \).

AII. The existence of a unique \( \epsilon(\alpha) \):

a) \( f_2(\alpha,\alpha) = \left[ q(\alpha)(A_0 + \pi(1-r)) - \pi(1-r) - q'(\alpha)((1-\alpha)(A_0 + \pi)\theta Y/2 \right. \)

The above expression is clearly positive as \( q'(\alpha) = -\alpha \)

b) \( f_2(\alpha,1) = -\pi(1-r)\theta Y/2 < 0 \)

c) \( f_{2e}(\alpha,e) = (\theta Y/2)[2q'(A_0 + \pi(1-r)) - q''((1-e)(A_0 + \pi) + r\pi(e-\alpha))] < 0 \)

Thus, (a), (b) and (c) show that there exist a unique \( \epsilon(\alpha) \in [\alpha,1] \) for each \( \alpha \in [0,1] \), satisfying \( f_2(\alpha,e) = 0 \)

Given our assumptions regarding \( f_2(\alpha,e) \), it follows that \( \epsilon(\alpha) \) is a continuous function of \( \alpha \).
Alii. Nash stability conditions: [Rejev Dixit (1986)]

Also we have 

\[ f_{1e} = \theta \eta \beta q'(e) e / 2 < 0 \]

and 

\[ f_{2\alpha} = \theta \eta q'(e) e / 2 < 0 \]

Slope of \( f_1(\alpha, e) = 0 \) in \( \alpha \)-e plane

\[ \frac{de}{d\alpha} = -\frac{f_{1\alpha}}{f_{1e}} < 0 \]

Similarly, Slope of \( f_2(\alpha, e) = 0 \) in \( \alpha \)-e plane

\[ \frac{de}{d\alpha} = -\frac{f_{2\alpha}}{f_{2e}} < 0 \]

Thus, we find that both \( f_1(\alpha, e) = 0 \) and \( f_2(\alpha, e) = 0 \) are negatively sloped in \( \alpha \)-e plane.

Now, two possible cases arise:

Case (i) where slope of \( f_1(\alpha, e) = 0 \) is steeper than \( f_2(\alpha, e) = 0 \) at \( (\alpha^*, e^*) \).

[The direction of the arrows shows the adjustment process in the system]

It is quite clear from the figure that in this case, if the system is displaced from \( (\alpha^*, e^*) \) to some \((\alpha_0, e_0) \neq (\alpha^*, e^*)\) the system adjusts on its own and returns to \( (\alpha^*, e^*) \). Thus, \( (\alpha^*, e^*) \) is a stable equilibrium point, in this case.
Case (ii) where the slope of \( f_2(\alpha, e) = 0 \) is steeper than \( f_1(\alpha, e) = 0 \) at \((\alpha^*, e^*)\).

[ The direction of the arrows shows the movement of the system ]

In this case, the system, on being disturbed slightly from the equilibrium position \((\alpha^*, e^*)\), tends to move further away from it. Hence it is a case of unstable equilibrium.

Thus, we can state that stability of \((\alpha^*, e^*)\) requires that absolute value of the slope of \( f_1(\alpha, e) = 0 \) to be greater than the absolute value of the slope of \( f_2(\alpha, e) = 0 \) at \((\alpha^*, e^*)\) i.e. the curve \( f_1(\alpha, e) = 0 \) to be more steeper than \( f_2(\alpha, e) = 0 \) at \((\alpha^*, e^*)\) which provides \( \frac{f_{1\alpha}}{f_{1e}} > \frac{f_{2\alpha}}{f_{2e}} \) or \( f_{1\alpha}f_{2e} - f_{2\alpha}f_{1e} > 0 \).