These days, the government has to perform a variety of welfare functions besides its indulgence in developmental activities. Over and above these, no government will ever compromise on the issue of its external and internal security for want of resources. It is evident that the fund required by the government for the above mentioned purposes must indeed be quite huge. The government tries to mobilize a major portion of its fund and resource requirements through collection of taxes in various forms from its citizens. Hence it is very essential from the government's point of view that it succeeds in its task of collecting the lawfully due taxes from its citizens. In this chapter, with the help of a simple model, we shall try to study the various ways by which a revenue maximizing government may attain its goal. In our analysis, the tax collecting authority (TA) and the government are considered to be one and the same body and hence the two terms are inter-changeably used. In the previous chapter, we studied
the net income maximizing behavior of a taxpayer (TP). Here we are picking up the story from the other side of the same coin, that of a government trying to maximize its expected net revenue in the face of behavior studied in the earlier chapter.

We shall assume that our typical TP tries to attain the optimal position at any given circumstances. Here, this means maximizing his expected net income in the tax situations considered.

The attempt of the government to mobilize resources through tax collections need not necessarily run a smooth course. It may not be able to accomplish its objective of garnering up an earmarked sum of tax revenue. Various forces, for numerous reasons and in different forms may come into play to thwart this attempt. Many authors, as can be seen from our introductory chapter, are of different opinions regarding the approaches being followed (and should have been followed) by the government to attain its goal. Through some simple exercises we shall try to determine the optimal ways of utilization of certain enforcement measures in its collection process. In our analysis, we have tried to tackle the problem for some specific tax situations.

Clearly, while studying the revenue maximizing problem of a government, one must include the revenue from both the indirect as well as direct taxes. It is quite true that the analysis will be quite inadequate and incomplete if only one
of them is taken up. But in our model, due to its own limitations and also by our choice, we will concentrate only on direct taxes, in fact the taxes on assessable income. We will be dealing with the problem of a government, trying to maximize its expected net revenue, obtainable from income taxes and penalty collected from the defaulters.

Clearly, there are many factors which influence the ease with which taxes can be collected as well as the actual amount of the tax collection. Some of them may be more or less under the direct supervision of the government while some may be indirectly maneuverable by it to suit its interests. There may still be others which are completely out of its hand. In this chapter, while determining the optimal audit structure which maximizes the government's expected net revenue, we shall study the influence of the tax rate and penalty schedule on the tax audit size. Thus, we are basically dealing with three factors namely tax rate, audit frequency and the penalty schedule. For sure, all of them definitely have some role in the tax collection exercise. It can also be noted that the government can (or do) exercise at least some influence over the choice of the nature and size of each of these three factors within any system.

Theoretically, the government can fix up any tax rate. But in reality, this range is quite narrow because of the various constraints in operation (the most powerful of them
being the political one). Probability of audit reflects the efficiency of the tax administration in tracking down tax evaders. It also speaks about the attitude of the government towards such illegal activities. Tightening of tax administration and also devolving more powers and resources to them for this purpose along with a greater effort from the administration side itself, will result in higher probability of audit and detection. Though the form of penalty system to be followed, seems to be more or less completely in the government's hand but in reality in most cases, it is mainly determined either politically and/or by traditional forces. Whatever be the determining forces it should be kept in mind that the severity of penalty should be kept lawfully proportionate to the gravity of the crime. Of course, the judgment regarding what is a lawful proportion, can vary depending on the situation, place and/or time. With the help of a simple model, we shall deal with the tax revenue maximization problem of the government. In fact we determine the optimal level of audit which maximizes the tax revenue of the government. Then by undertaking comparative statics exercises we try to see the effect of change in tax or penalty rate on optimal audit rate.

The model with which we shall be analyzing the issue at hand has been laid out in section 1. Here, the assumptions and the objectives in the model have been stated out. In
section 2, the exercise of the government to maximize its expected net tax revenue is worked out. For convenience, from now onwards, we shall use ER for the expression, expected net tax revenue. For our analysis the whole TP population has been divided into a finite number of economic classes. Clearly the ER of the government will be the aggregation of its ER over the economic classes which constitute the system. Taking up a micro-approach towards the solution of the problem, we try to determine the unknown audit frequency for each of the classes which will maximize the ER from that particular class. Auditing is undertaken only in those classes where on the average, the TPs seem to have underreported. If we allow the optimal audit policy for the system to be the amalgamation of the optimal audits for the individual economic classes, then we have achieved our objective. We next proceed to the comparative statics exercises. But as has been argued time and again that study of macro-issues through micro-approach may lead to wrong inferences, our attempt to blow up our micro-results into a macro-one may prove to be wrong. Even under this cloud, we may rightfully claim that the analytical insights obtained through this method are too significant to be ignored.

In our above study, the government was assumed to be free from any kind of economic or political constraints in its course of action. In the next section the search for the optimal audit policy has been undertaken, subject to a
resource-constraint. The analysis in this case, has been worked out for a two-class economy. In the similar vein, the study could have been extended to a multi-class economy, though the computations would have become much more bulky and cumbersome. Finally in the concluding section, the main results have been summarized and policy implications have been pointed out.

THE MODEL

The model involves just two parties - (a) the TPs and (b) the government or the TA. The clubbing together of the government and the TA allows the further assumption that if a tax evader is audited, he is certainly detected, prosecuted and penalized. The penalty rate, \( \pi \) is taken to be a positive function of the tax evaded sum. The government incurs certain costs in the process of tax collection (inclusive of auditing costs). Also, we assume that the costs incurred in tax-collection other than the auditing costs to be a fixed amount. Hence, in our analysis, we just take into account the auditing costs, normalizing the other costs involved to be zero.

The audit cost incurred in auditing \( n \) TPs of the \( i \)th class, \( C_i \) is given by

\[
C_i = C_i(n) \quad \text{with} \quad \frac{\partial C_i}{\partial n}, \quad \frac{\partial^2 C_i}{\partial n^2} > 0
\]

The whole population of TPs has been divided into \( k \) number of classes. The individual economic capacity of the
TPs is the sole criterion considered in this classification. Further, all TPs within each class are assumed to be ex-ante indistinguishable. In the model, \( N_i \) and \( n_i \) represent the total number of TPs of the \( i \)th class and number of audit cases in it, respectively.

Consider a TP from the \( i \)th class. We assume that the number of values which the income of the TPs can take, is finite. Let a possible income value in the \( i \)th class be \( Y_{ij} \) where \( j=1,2,...,t \). Clearly, \( t \leq N_i \). We further assume that \( Y_{ij} \) occurs with a known relative frequency \( f_j/N_i \) where \( N_i = \sum f_j \).

The mean income level of the \( i \)th class, \( Y_i \) will be given by:

\[
Y_i = \frac{\sum f_j Y_{ij}}{N_i}
\]

The classes have been indexed in such a way that \( Y_1 < Y_2 < ... < Y_k \).

If \( \alpha_{im} Y_{im} \) is the income declared by the \( m \)th TP of the \( i \)th class whose true income is \( Y_{im} \), then \( \bar{Y}_i \) given by

\[
\bar{Y}_i = \frac{\sum_{m=1}^{N_i} \alpha_{im} Y_{im}}{N_i}
\]

is the average income declared of the \( i \)th class. The expression \( \hat{\alpha}_i = (\bar{Y}_i/Y_i) \), then may be interpreted as the mean compliance level of a TP from the \( i \)th class.

As a matter of policy, the government decides not to audit the class for which \( \hat{\alpha} \geq 1 \). For those classes for which \( \hat{\alpha} < 1 \), we shall determine the optimum audit i.e. the number of audits which will maximize the net revenue collection.
from that particular class. The calculated $\alpha$ is used as the compliance level of a representative TP of the concerned class in our exercise.

The relevant tax rate for the $i$th class is given by $\theta_i$. Clearly, if $\theta_i < \theta_j$ for $i < j$, then the tax system is progressive.

Finally, the objective of the government is to find the optimum audit rate which will maximize its ER. It is assumed that our government does not face any kind of constraints (whether it be in the form of political, economic or social) in the pursuance of its tax policy. Over and above everything, we shall assume that the government possesses the necessary political will to reach its goal.

The government while framing an audit policy for any specific system needs a prior knowledge of the behavioral patterns of the TPs in the relevant tax situations. This knowledge may possibly be constructed in tentative or concrete form from their past behavioral patterns, studies in human psychology and various other sources which are somehow concerned with this area. We can certainly say that there must be numerous factors which in unison and individually shape up the behavioral pattern of a TP in any specific tax situation. Out of those many factors, we have considered just two, namely tax rate $\theta$ and the probability of audit, $p$. For our TP, the compliance level $\alpha$, is taken
to be a function of θ and p. Before proceeding any further, let us first delve into the possible nature of the function \( \alpha = \alpha(p, \theta) \). To start with, it must be quite clear from introductory chapter that no conclusive evidence or agreement has emerged amongst the scholars regarding the relationship between \( \alpha \) and \( \theta \) in the literature till date. Some people have strongly argued that \( \frac{\delta \alpha}{\delta \theta} \) is negative in sign i.e. \( \alpha \) and \( \theta \) are inversely related. Their argument proceeds along two lines. One, as the tax rate is increased, the tax burden is increased and people are not inclined or willing to pay huge sums in taxes. Second, higher the rate of tax, the more attractive it is to evade it and the more worthwhile is it to accept the risk associated with such tax evasion.

Direct Taxes Inquiry Committee (DTEC, 1971) with regard to high rates of taxation said, "we are convinced that high marginal rate of taxation is a powerful contributory factor towards evasion in as much as they make fruits of evasion so attractive that a less scrupulous person would consider the incidental risks worth taking". ¹

In its report, the DTEC also mentions "when the marginal rate of taxation is as high as 97.75%, the net profit on concealment can be as high as 4,300% of the after-tax income. The implication of 97.75% income-tax is that it is more profitable at a certain level of income to evade tax on Rs.30/- than to earn honestly Rs.1000/-".² In
contrast to the above reports, Yitzhaki (1974), in his study found that $\frac{\partial \alpha}{\partial \theta}$ is positive. Also, in Graetz et al. model (1984, 1986), $\frac{\partial \alpha}{\partial \theta}$ was found to be positive. Kabra, (1982, p95) after an empirical study of the Indian income-tax rates for the non-corporate sector, sums up his findings as "in view of the evidence indicating growing absolute as well as relative size of the income tax evasion by non-corporate sector even in response towards lower tax rates there is no basis for the hypothesis relating to lower tax-rates with reduced tax evasion".

In fact in our previous chapter, the sign of $\frac{\partial \alpha}{\partial \theta}$ was found to be influenced by the accompanying circumstances. Here, we are considering a TP who is in a random audit regime with the prevailing penalty rate being a positive function of the deficient tax or the tax evaded, given by $D = (1-\alpha) \theta Y$. From our previous chapter, for such a situation, we have $\frac{\partial \alpha}{\partial \theta}$ to be positive. Hence, here we take $\alpha$ to be positively related to $\theta$. Now, regarding the relationship between $\alpha$ and $p$, it seems very natural and obvious to assume positive relationship. With increase in $p$, the incidental risk associated with tax evasion is increased even at low tax rates. As probability of getting audited becomes almost one, no rational person will consider it worthwhile to evade tax at any level of tax rate. Dasgupta (1974, p.875) writes, "tax evasion is a propensity born of greed. Like any other propensity, it is restrained only in so far as person having
recourse to it has to pay a price. A tax evader runs the risk of being detected and punished. This risk is the measure of the price that he derives from non-payment of his dues. He would not consider it worthwhile taking the risk if it were higher than the amount he is called upon to pay. A

This assumption totally agrees with the results in our previous chapter i.e. irrespective of the tax rate, a person declares zero income in a situation of zero audit probability and his true income if the audit probability is one.

The expected net tax revenue of the government may be written down as

\[
ER = \sum_{i=1}^{K} R_i
\]

Where \( R_i \) stands for the expected net revenue from the \( i \)th class given by

\[
R_i = N_i \theta_i \alpha_i Y_i + n_i [\pi (D_i) D_i - \bar{C}_i (n_i) ]
\]

(1)

where \( n_i \) is the number of TPs audited in \( i \)th class and \( D = (1-\alpha) \theta Y \) is the mean deficient tax or the evaded tax and \( \bar{C}_i (n_i) \) is the average audit cost incurred in auditing \( n_i \) TPs of \( i \)th class [More about \( \bar{C}_i \) in the appendix ]

2 THE ANALYSIS

As mentioned earlier, our approach will be to maximize each of those \( R_i \) (\( i = 1, 2, \ldots k \)) with \( \alpha_i < 1 \). Henceforth, we
shall be working with the term \( R_i \). In fact for the sake of convenience, we shall do away with the subscript 'i' for the time being but at the same time remember that \( R \) represents the expected revenue of some class (for which \( \alpha < 1 \) and not of the whole system. Thus, instead of expression (1), we shall be working with

\[
R = N\theta \alpha Y + n[\pi(D) - \bar{C}(n)]
\]  

(1a)

The first order condition for maximization of \( R \) with respect to \( n \) provides

\[
\frac{\partial R}{\partial n} = N\theta Y \frac{\partial \alpha}{\partial n} + \pi(D) \bar{C}(n) - n[\{\pi() + \pi'(n)\} \theta Y(\partial \alpha/\partial n) + \bar{C}'(n)]
\]

\[
= \theta Y(\partial \alpha/\partial n)\{N-n(\pi()+\pi'(n))\}+\pi()-C(n)+nC'(n) = 0
\]

(2)

Now, the second order condition for maximization requires that \( \frac{\partial^2 R}{\partial n^2} \) be negative. Here,

\[
\frac{\partial^2 R}{\partial n^2} = -2\theta Y(\partial \alpha/\partial n)\{\pi() + \pi'(n) - n\theta Y(\partial \alpha/\partial n)\pi'(n)\} - 2\bar{C}'(n) - n\bar{C}''(n)
\]

(3)

The above expression is clearly negative if

\[
\pi() + \pi'(n) - n\theta Y(\partial \alpha/\partial n)\pi'(n) \geq 0 \text{ which is true.}^5
\]

Thus, any solution, \( n^* \) of expression (2) will be a revenue maximizing audit.

To examine the existence of a solution \( n^* \) of (2), we proceed as follows:

We have

\[
\frac{\partial R}{\partial n} (\text{at } n = 0) = N\theta Y(\partial \alpha/\partial n) \bigg|_{n=0} + \pi(\theta Y) \theta Y > 0
\]

and

\[
\frac{\partial R}{\partial n} (\text{at } n = N) = N\theta Y(\partial \alpha/\partial n) \bigg|_{n=N} - \bar{C}(N) - N(\partial \bar{C}/\partial n) \bigg|_{n=N}
\]

\[
= N\theta Y \frac{\partial \alpha}{\partial n} \bigg|_{n=N} - \lim_{n \to N} \{\bar{C}(n)(1+\epsilon \bar{C}:n)\}
\]
where \( c_{C:n} \) is the audit elasticity of average cost. For our \( C(n) \) function, we have \( \lim_{n \to N} c_{C:n} \) as infinity.

Since \( \lim_{n \to N} \partial \alpha / \partial n \) is finite, \( N \Theta Y \partial \alpha / \partial n \bigg|_{n=N} \) is a finite number. Thus, \( (\partial R / \partial n) \) (at \( n = N \)) < 0.

The fact that \( \partial R / \partial n \) exists and is continuous for \( n \) in \([0,N]\) and also that its value is positive and negative at the two end points, respectively guarantees the existence of at least one point of maximum [see the appendix provided at the end of chapter II]. In case, if there is more than one such point, then comparing the values of \( R \) at all these points and taking the value of \( n \) which provides the greatest \( R \), we get the optimum audit size, \( n^* \) and hence the optimum probability of audit, \( p^* \) which is the ratio of \( n^* \) to \( N \).

**Comparative Statics**

Now, we shall study, ceteris paribus, the effect of the some of the parameters, which together determine the optimal audit size, on the optimality condition.

Implicit differentiation of \( R_n = 0 \) allows us to write

\[
\partial n^*/\partial \theta = (\partial R_n/\partial \theta) / -(\partial R_n/\partial n)
\]

We have already determined that \( \partial R_n / \partial n \) is negative (equation (3)). Hence the sign of \( \partial n^*/\partial \theta \) will be identical to that of \( \partial R_n / \partial \theta \).

Now, \( \partial R_n / \partial \theta \)

\[
= Y(\partial \alpha / \partial n)(N-n(\pi+Dn'))-n\Theta Y(\partial \alpha / \partial n)(dp/\partial \theta)+\pi'(1-\alpha-\Theta(\partial \alpha/\partial \Theta))Y
\]

+ D(dp/\partial \theta) +\pi Y(1-\alpha-\Theta(\partial \alpha/\partial \Theta))
\[
\theta Y(d\pi/d\theta)(1-\alpha-n(\partial \alpha/\partial n)) + \pi Y(1-\tilde{\alpha}(\theta))
\]
\[
+ Y(\partial \alpha/\partial n) \{ N-n(\pi+n'(D+\theta(1-\tilde{\alpha}(\theta))) Y \}
\]
\[
[ \text{here, } \tilde{\alpha}(\theta) = \alpha(\theta)(1+c_a(\theta) ) ]
\]
\[
= \theta Y(d\pi/d\theta)(1-\tilde{\alpha}(n)) + \pi Y(1-\tilde{\alpha}(\theta)) + Y[N-n(\pi+n'(1-\alpha+1-\tilde{\alpha}(\theta))) \theta Y] \partial \alpha/\partial n
\]

It can be shown that \(d\pi/d\theta\) is non-negative.6 Hence from the above expression, we can say that if

\[
[N-n(\pi+n'(1-\alpha+1-\tilde{\alpha}(\theta))) \theta Y] \geq 0 \text{ then } \partial R_n/\partial \theta > 0,
\]
and in this case \(\partial n^*/\partial \theta > 0 \) (4a)

The meaning of \(\partial n^*/\partial \theta > 0\) is that with a hike in tax rate, one needs to intensify auditing (i.e. increase the number of audits) to regain a position of optimality within the system.

But if the third term of (4) is negative then the sign of \(\partial R_n/\partial \theta\) will depend on the relative strength of all the terms.

In the above context, it will be more meaningful to examine whether the above exercise of a tax-hike to be supported by a corresponding increasing in audit is worth its while. It will be so only if the new optimal situation (after the changes) proves itself to be superior to the earlier one in the sense of higher expected net revenue. It can easily be shown that the new optimal position commands a greater expected net revenue then the earlier one.7

Within the above model, let us see whether any substantial difference is created in the situation on the introduction of an additional characteristic to the behavior
pattern of our TP. Many people have argued in favor of lower taxes for according to them, marginal high tax rates produce strong disincentive to productive effort and high earnings, thus narrowing down the tax base and hence, ultimately lowering the tax collection. The above arguments have been made on the premise that the division of time between labor and leisure is influenced by the tax rate. Higher tax rate discourages people to work and instead enhances their preference for leisure. The above characteristic in the TP’s tax behavior may be incorporated in our model by including the function \( Y = Y(\theta) \) with \( Y'(\theta) < 0 \).

With this new element introduced in the model, let us determine the effect of change in \( \theta \) on \( n^* \). Again, the sign of \( \frac{\partial n^*}{\partial \theta} \) will be identical to that of \( \frac{\partial R_n}{\partial \theta} \).

Now, \( \frac{\partial R_n}{\partial \theta} = \theta Y \left( \frac{\partial \pi}{\partial \theta} \right)(1-\alpha) + \pi Y(1-\alpha) + \theta \left( \frac{\partial Y}{\partial n} \right) \{ N - n(\pi + 2Dn') \} + Y \left( \frac{\partial n}{\partial n} \right) \{ N - n(\pi + \pi'(1-\alpha + 1-\alpha \theta)) \} + (1-\alpha) \theta \pi \)

Here, \( \frac{\partial \pi}{\partial \theta} = (\frac{\partial \pi}{\partial D}) \{(1-\alpha) Y - \theta (Y \left( \frac{\partial \alpha}{\partial n} \right) - (1-\alpha) Y \left( \frac{\partial Y}{\partial \theta} \right)) \}

Thus, the sign of \( \frac{\partial \pi}{\partial \theta} \) depends on the relative strength of the two opposing forces which comes to play on \( D \) from a unit change in \( \theta \): a positive force of magnitude \( (1-\alpha) Y \) which pushes up the value of \( D \) and a negative force of magnitude \( \theta \{ Y \left( \frac{\partial \alpha}{\partial n} \right) - (1-\alpha) Y \left( \frac{\partial Y}{\partial \theta} \right) \} \) which pulls down the value of \( D \). Depending on whether the positive force is stronger (or weaker) than the negative force, the sign of \( \frac{\partial \pi}{\partial \theta} \) will be positive or negative, respectively.
Assuming that the positive force is stronger, then \( \frac{\partial R_n}{\partial \theta} \) will be positive if

\[
N = n(n + 2Dn')^8
\]

In such a situation we have

\[
\frac{\partial n^*}{\partial \theta} > 0
\]

For the other cases, we can not at this stage say anything as the sign of \( \frac{\partial R_n}{\partial \theta} \) will then depend on the relative values of the positive and negative terms.

We notice that with the increase in complexity of the system under study, it becomes more difficult to obtain clear cut results. Anyway, at the moment without the consideration of sign of \( \frac{\partial n^*}{\partial \theta} \), let us examine whether an increase in tax rate can create a potentially superior optimal position. The question is, *Does an increase in the tax rate along with a corresponding adjustment in the audit frequency take the system to a potentially higher expected net income position?* It can be shown that the answer is in the affirmative.

Now let us see the effect on \( n^* \) if there is a proportional hike in penalty rate at all levels of \( D \). This can be studied by replacing \( \pi(D) \) and \( \pi'(D) \) by \( \pi(D) + \xi D \) and \( \pi'(D) + \xi \) respectively in the expression \( R_n = 0 \) [as we have done in the previous chapter, to analyze similar questions]. Implicit differentiation of the transformed equation with respect to \( \xi \) and taking limit as \( \xi \) tends to zero, we get the effect of change in penalty rate function on \( n^* \).
The transformed equation will be
\[ R_n = \theta Y[N - n\{\pi + \xi + (\pi' + \xi)D\}] \left( \partial \alpha / \partial n \right) + (\pi + \xi)D - (C + n\bar{C}) = 0 \] (5)

Now,
\[ \lim_{\xi \to 0} \frac{\partial n^*}{\partial \xi} = \lim_{\xi \to 0} \frac{\partial R_n / \partial \xi}{\partial R_n / \partial n} = \frac{\partial R_n / \partial \xi}{\partial R_n / \partial n} \]

Thus,
\[ \frac{\partial R_n / \partial \xi}{\partial R_n / \partial n} = D\{D - 2n\theta Y(\partial \alpha / \partial n)\} \]

Thus,
\[ \frac{\partial R_n / \partial \xi}{\partial R_n / \partial n} > 0 \quad \text{according as} \quad 1 - \alpha > 2n\partial \alpha / \partial n \]
\[ \text{or} \quad (1 - \alpha) / \alpha > 2(\partial \alpha / \alpha) / (\partial n / n) = 2c_{\alpha:n} \]

i.e. \( \frac{\partial R_n / \partial \xi}{\partial R_n / \partial n} \) is positive or negative according as the relative degree of dishonesty, \((1 - \alpha) / \alpha\) is greater or less than twice the audit elasticity of compliance.

From the above, it can be concluded that the government should not indulge blindly in a trade-off of audit frequency for penalty rate or vice-versa in its tax policy since such a policy may not yield the maximum potential net revenue. As a matter of fact, in a situation where the relative degree of dishonesty is greater than double the audit elasticity of compliance, a rise in penalty rate needs to be supplemented by greater audit intensity for the system to regain its optimality.

Again, from the expression \( R_n = 0 \), we can get
\[ \frac{\partial n^* / \partial Y}{\partial R_n / \partial n} = \frac{(\partial R_n / \partial Y)}{(-\partial R_n / \partial n)} \]
Now,
\[ \frac{\partial R_n}{\partial Y} = \theta (\partial \alpha / \partial n) \{ N - n(\pi + D\pi') \} - 2nD\theta \pi'(\partial \alpha / \partial n) + (1 - \alpha) \theta (\pi + D\pi') \]
\[ = \theta (\pi + D\pi') \{ 1 - n(\partial \alpha / \partial n) \} + \theta (\partial \alpha / \partial n) (N - 2nD\pi') \]
\[ = \theta (\pi + D\pi') (1 - \bar{\alpha}(n)) + \theta (\partial \alpha / \partial n) (N - 2nD\pi') \] (6)

Clearly the above expression is positive if
\[ N \geq 2nD\pi'() \]
or \[ 1 \geq 2\pi(n/N)(\partial \pi/\pi)/(\partial D/D) \]
or \[ (\varepsilon_{\pi:D})^{-1} \geq 2\pi \] (6a)
i.e. if twice the effective penalty is less than or equal to the inverse of the elasticity of penalty rate then \( \partial n^*/\partial Y \) is certainly positive.

The implication of the above result is that in the above tax situation, with increase in \( Y \), \( n^* \) needs to be raised.

Clearly increase in \( Y \), here is equivalent to climbing up the economic ladder or stepping into a richer economic class. Thus, we find that in our tax situation if (6a) is satisfied, the optimal audit rates are higher for richer classes. The above finding suggests that the government should include more intensive auditing for richer classes in its tax policy.

This finding which shows the dependence of the optimal audit of the representative economic class on the average income level of the class gives an economic justification to our approach of seeking optimal audits for individual economic classes.
The assumption of absence of any kind of political and resource constraint considered in the previous section is rather unrealistic and restrictive. The political feasibility of applying the same yard-stick to all classes in regard to enforcement of tax-policy and in fact, in policy frame-work looks quite bleak. Usually, power equations are uneven and the policy and its enforcement always tend to favor the more powerful class. The working of a real economy is a very complex process. We will not attempt to incorporate the political constraints in our model, though they may be playing a very significant role in the design and implementation of any tax policy.

Indeed it would be a very difficult task to formulate them into appropriate mathematical equations so that they may satisfactorily be incorporated into the model. But in this section, we shall at least incorporate the resource constraint (which almost every government faces) in our model. As mentioned in the beginning of the chapter, the governments, these days, have a lot of immediate tasks (inclusive of the developmental/welfare project under its scheme of action) in hand. Hence it can not divert an unlimited resource from its treasury for tax collection and other allied activities. We shall give due recognition to the above fact in the present section. Here, we assume that there is only a limited resource earmarked for the TA and
its activities and hence study our earlier analysis in the context of the TA/government facing a resource constraint.

**Assumption 9:** The TA/government faces a resource constraint in regard to its tax collection and allied activities. It has only a limited resource, $R_A$, earmarked for this purpose. Hence, we must have

$$\sum_{i=1}^{k} n_i c_i(n_i) \leq R_A$$

For notational convenience, write the constraint in the form

$$R_A = \sum_{i=1}^{k} n_i c_i(n_i) = g(n_1, n_2, \ldots, n_k)$$

Thus, the problem at hand is that of a government desiring to maximize its expected net revenue under a given resource constraint.

The mathematical formulation of the above problem may be written down as

$$\text{Maximize } ER = \sum_{i=1}^{k} R_i;$$

where $R_i = \theta_i x_i + n_i [\pi (1-\alpha_i) \theta_i y_i - c_i(n_i)]$

subject to $\sum_{i=1}^{k} n_i c_i(n_i) \leq R_A$

Form the Lagrangian function

$$L = \sum_{i=1}^{k} R_i + \mu (\sum_{i=1}^{k} n_i c_i(n_i) - R_A)$$

where $\mu \neq 0$ is an undetermined Lagrange multiplier.

The necessary condition for the constrained maximization of $ER$ is:

$$\frac{\partial L}{\partial n_i} = \frac{\partial R_i}{\partial n_i} - \mu \left( c_i(n_i) + n_i c'_i(n_i) \right) = 0$$

for $i = 1, 2, \ldots, k$ \hspace{1cm} (7a)

$$\frac{\partial L}{\partial \mu} = R_A - \sum_{i=1}^{k} n_i c_i(n_i) = 0$$ \hspace{1cm} (7b)

Thus, we have $k+1$ equations in $k+1$ unknowns namely $n_i$
(i=1,2..k) and $\mu$. Solving the above system of equations we can determine the stationary values of the Lagrangian function $L$.

The second order sufficient condition requires that all the bordered principal minors starting with $|\overline{H}_2|$ must alternate in sign starting with positive. 10

For computational convenience, we shall work out our resource constraint optimizing problem for a two-class economy.

Consider a 2-class economy, subscript 1 for the poor class and subscript 2 for the rich class. We shall also assume that $\pi_2 > \pi_1$.

The expected revenue of the two class-economy may be written down as

$$R = \sum_{i=1}^{2} R_i; \quad R_i = N_i \theta_i \alpha_i Y_i + n_i \{ \pi(D_i) D_i - C_i(n_i) \}$$

The resource constraint is

$$R_A = \sum_{i=1}^{2} n_i \overline{C}(n_i)$$

To maximize $R$ subject to the resource constraint, form the Lagrangian function

$$L = \sum_{i=1}^{2} R_i + \mu(R_A - \sum_{i=1}^{2} n_i \overline{C}(n_i))$$

$$= \sum_{i=1}^{2} N_i \theta_i \alpha_i Y_i + n_i \{ \pi(D_i) D_i - C_i(n_i) \} + \mu (R_A - \sum_{i=1}^{2} n_i C(n_i)) \quad (8)$$
The necessary first order condition provides
\[
\frac{\partial L}{\partial n_i} = (N_i - n_i(\pi() + D_i\pi'(\cdot)))\theta_i Y_i(\partial \alpha_1 / \partial n_i) \\
+ \pi()D_i - (1+\mu)(\overline{c}_i(\cdot) + n_i\overline{c}_i'(\cdot)) = 0 \tag{9a}
\]
for \(i = 1, 2\)

\[
\frac{\partial L}{\partial \mu} = R_A - \sum_{i=1}^{2} n_i \overline{c}(n_i) = 0 \tag{9b}
\]

Solving the system for \(n_1\) and \(n_2\), we obtain the required optimal audits \(n_1^*\) and \(n_2^*\) of the two classes, respectively.

It can easily be shown that the sufficiency condition for maximization is satisfied, here [Refer appendix II]

From the necessary condition, we obtain
\[
\frac{\{N_1-n_1(\pi_1+D_1\pi_1')\}\theta_1 Y_1(\partial \alpha_1 / \partial n_1) + \pi_1 D_1}{\overline{c}_1(n_1) + n_1\overline{c}_1'(n_1)} = 1 + \mu
\]

or
\[
\frac{\{N_2-n_2(\pi_2+D_2\pi_2')\}\theta_2 Y_2(\partial \alpha_2 / \partial n_2) + \pi_2 D_2}{\overline{c}_2(n_2) + n_2\overline{c}_2'(n_2)}
\]

Denoting the left hand side (LHS) of the above equation by \(S\), we can write it as
\[
S(n_1, n_2, \lambda) = 0 \text{ where } \lambda \text{ represents the parameters involved in the above equation.}
\]
Denote the budget constraint by \( R(n_1, n_2) = 0 \).

**Comparative Statics**

For any parameter \( \lambda \) the comparative statics results with respect to \( n_1 \) and \( n_2 \) may obtained from the following expressions.

\[
\frac{dn_1}{d\lambda} = \frac{-R_{n2}S_\lambda}{(S_{n1}R_{n2} - S_{n2}R_{n1})}
\]

\[
\frac{dn_2}{d\lambda} = \frac{R_{n1}S_\lambda}{(S_{n1}R_{n2} - S_{n2}R_{n1})}
\]

where \( S_{n1} = \frac{\partial S}{\partial n_1} \), \( R_{n1} = \frac{\partial R}{\partial n_1} \) etc.

We have

\[
\frac{\partial S}{\partial n_1} = -2(\pi_1 + D_1\pi_1' - n_1\pi_1'\theta_1\gamma_1(\frac{\partial \alpha_1}{\partial n_1})\theta_1\gamma_1(\frac{\partial \alpha_1}{\partial n_1})(\bar{C}_2 + n_2\bar{C}_2')
\]

\[-(2\bar{C}_1' + n_1\bar{C}_1')(N_2 - n_2(\pi_2 + D_2\pi_2'))\theta_2\gamma_2(\frac{\partial \alpha_2}{\partial n_2}) + D_2\pi_2]
\]

By equation (3) and (9a), it is seen that \( S_{n1} < 0 \),

Now from the form of \( S \)-expression, it can easily be seen that \( \frac{\partial S}{\partial n_2} = S_{n2} \) is positive.

Also, \( R_{n1} = \frac{\partial R}{\partial n_1} \), \( i = 1, 2 \) is negative.

Hence, \( S_{n1}R_{n2} - S_{n2}R_{n1} \) is positive. Now, it is quite clear that the sign of \( \frac{\partial n_1}{\partial \lambda} \) will be equal to that of \( S_\lambda \) and of \( \frac{\partial n_2}{\partial \lambda} \) to that of \(-S_\lambda\), for any parameter \( \lambda \). Thus in fact we find that the effect of any parameter on \( n_1 \) and \( n_2 \) are opposite in direction i.e. if any increase (decrease) of any parameter requires enhancing (lowering) of \( n_1 \) for optimality then the system shall require a lowering (enhancing) of \( n_2 \) simultaneously as well.

Now,

\[
\frac{dn_1}{d\theta_1} = \frac{-R_{n2}\theta_1}{(S_{n1}R_{n2} - S_{n2}R_{n1})}
\]

Clearly, the sign of above expression will be same to
that of $S_{\theta_1}$.

Now, $S_{\theta_1} = \partial S / \partial \theta_1$

$$= \{(N_1 - n_1 (\pi_1 + D_1 \pi_1')) - 2n_1 \pi_1' (\partial D_1 / \partial \theta_1) \theta_1 \}Y_1 (\partial \alpha_1 / \partial n_1)$$

$$+ D_1 \pi_1' (\partial D_1 / \partial \theta_1) + n_1 (\partial D_1 / \partial \theta_1) (\bar{C}_2 + n_2 \bar{C}_2')$$

[Here, $\partial D_1 / \partial \theta_1 = \{1 - \alpha_1 - \theta_1 (\partial \alpha_1 / \partial \theta_1) \}Y_1 = \{1 - \tilde{\alpha}_1 (\theta_1) \}Y_1$
and surely it is non-negative as $\tilde{\alpha}_1 (\theta_1) \leq 1$.]

The sign of $\partial S / \partial \theta_1$ will be determined by the term

$$\{N_1 - 2n_1 \pi_1' (\partial \alpha_1 / \partial \theta_1) \}Y_1 \partial \alpha_1 / \partial n_1$$

$$+(\pi_1 + D_1 \pi_1') \{ (\partial \alpha_1 / \partial \theta_1) - n_1 Y_1 \partial \alpha_1 / \partial n_1 \}$$

$$= \{N_1 - 2n_1 (\partial \alpha_1 / \partial \theta_1) \}Y_1 \partial \alpha_1 / \partial n_1$$

$$+ (\pi_1 + D_1 \pi_1') \{1 - \alpha_1 - \theta_1 (\partial \alpha_1 / \partial \theta_1) - n_1 (\partial \alpha_1 / \partial n_1) \}Y_1$$

$$= N_1 (1 - 2p_1 \pi_1 \varepsilon_1 : \theta) Y_1 \partial \alpha_1 / \partial n_1$$

$$+(\pi_1 + D_1 \pi_1') Y_1 \{1 - \alpha_1 (1 + \varepsilon \alpha_1 : \theta_1 + \varepsilon \alpha_1 : n_1) \}$$

$$= N_1 (1 - 2p_1 \pi_1 \varepsilon_1 : \theta) Y_1 \partial \alpha_1 / \partial n_1$$

$$+(\pi_1 + D_1 \pi_1') Y_1 \{1 - \tilde{\alpha}_1 (n_1, \theta_1) \}$$

Clearly $\partial S / \partial \theta_1 > 0$ if $2p_1 \pi_1 \leq (\varepsilon \pi_1 : \theta_1)^{-1}$

[as $\tilde{\alpha}_1 (n_1, \theta_1) \leq 1$]

In such a tax situation, we have $\partial n_1^* / \partial \theta_1 > 0$ (11)

Let us denote a tax situation where twice the effective penalty is less than the inverse of tax elasticity of
penalty by TS-I. Now, we can say that if TS-I prevails in class I, a hike in $\theta_1$ must be accompanied by increase in number of audits in the concerned class (i.e. class-I) and lowering of audit in class-II, for the economy to get back to optimal position.

But if $N_1$ was less than $2n_1^*\pi_1'()\theta_1\partial D_1/\partial \theta_1$ in the initial optimal position, then the sign of $\partial n_1^*/\partial \theta_1$ will depend on relative values of the two terms in the expression $\partial S/\partial \theta_1$ and of course the sign of $\partial n_2^*/\partial \theta_1$ will be just the opposite of $\partial n_1^*/\partial \theta_1$ in all circumstances.

In the same vein, we can work out the effect of change in $\theta_2$ on $n_1^*$. We have

$\partial n_1^*/\partial \theta_2 = -Rn_2S_\theta_2/(S_n1Rn2 - Sn2Rn1)$

Again from the S-expression, it is seen that $S_\theta_2$ will be negative if $2p_2\pi_2 < (\pi_2:\theta_2)^{-1}$ and so will be $\partial n_1^*/\partial \theta_2$.

Thus, if the prevailing tax situation in class II is TS-I, then a hike in $\theta_2$ must be accompanied by lowering of audit in class I and a simultaneous increase in number of audits in class II for the system to regain its optimality. If the tax situation prevailing in class II is something other than TS-I then again similar statement regarding the effect of $\theta_2$ on $n_1$ and $n_2$ can be made as done in earlier case of $\theta_1$.

To examine the effect of a proportionate upward shift in penalty rate $\pi_1$ (at every level of $D_i$) on $n_i^*$ transform the expression $S = 0$ by replacing the terms $\pi_1()$ and $\pi_1'()$ by
\( \pi_i() + \xi D_i \) and \( \pi_i'(()) + \xi \). Now partial differentiation of the transformed equation and taking limit as \( \xi \) tends to zero provides the following:

For convenience, use the notation: For \( i = 1,2 \)

\[
Z_i = \{ N_i - n_i (\pi_i + \xi D_i + D_i (\pi'_i + \xi'_i)) \} \theta_i Y_i (\partial \alpha_i / \partial n_i) (C_i + n_i C_i')
\]

Then, the transformed \( S=0 \) takes the form

\[
Z_1 (C_2 + n_2 C_2') - Z_2 (C_1 + n_1 C_1') = 0
\]

Now, limit \( \partial n_1^*/\partial \xi_1 \) = \( \lim_{\xi_1 \to 0} \frac{-R_{21} S_{11}}{R_{11} R_{21} - R_{11} S_{22}} \)

We have \( (\partial S / \partial \xi_1) \) = \( \left\{ -2 n_1 D_1 \theta_1 Y_1 (\partial \alpha_1 / \partial n_1) + D_1 \right\} (C_2 + n_2 C_2') \)

\[
= D_1 \left\{ -2 n_1 \theta_1 Y_1 (\partial \alpha_1 / \partial n_1) + D_1 \right\} (C_2 + n_2 C_2')
\]

The sign of the above is positive or negative according as \( (1 - \alpha_1) / \alpha_1 \) is greater or less than \( 2(n_1 / \alpha_1)(\partial \alpha_1 / \partial n_1) \) or \( (1 - \alpha_1) / \alpha_1 \) is greater or less than \( 2c_1 \alpha_1 : n_1 \)

Now, since \( 1 - \alpha_1 \) may be looked upon as a measure of dishonesty and \( \alpha_1 \) of honesty, hence the expression \( (1 - \alpha_1) / \alpha_1 \) may be seen as a measure of relative dishonesty. Thus depending on whether the relative dishonesty is greater or less than twice the audit elasticity of compliance in that concerned class, we shall have \( \partial n_1^*/\partial \xi_1 \) as positive or negative. Here a positive \( \partial n_1^*/\partial \xi_1 \) will imply that a proportionate upward shift in \( \pi_1 \) (at every level of \( D_1 \)) will require an increase in number of audits in class I and
consequently a lower audit in class II for the system to be in optimality. Similar interpretations for other situations.

Denote the tax situation where the relative dishonesty of the representative TP of the class under study is greater (less) than twice the audit elasticity of compliance prevailing in that same class by TS-II (TS-III).

Now, we can reframe our earlier findings as "if the prevailing tax situation in class-I is of TS-II form, then upward shift in penalty rate should be accompanied by an increase in number of audit in class I and a decrease in $n_2^*$ for the system to be in optimal". But in case the prevailing tax situation in Class I is of TS-III form we shall require a decrease in $n_1^*$ and an increase in $n_2^*$ for optimality.

We could study the effect of an upward shift in $\pi_2$ on $n_1^*$ and $n_2^*$ as we have done in the case of $\pi_1$. In fact we shall find that under the similar prevailing circumstances in the system, the effect of an upward (downward) shift in $\pi_2$ on $n_1^*$ and $n_2^*$ will be just the opposite of the effect of an upward (downward) shift in $\pi_1$.

Summary and conclusion
In the model, the whole tax population was divided into a finite number of economic classes, the division solely on the basis of the economic capacity of each individual TP.
As a policy, the government decides not to audit the class whose estimated $\alpha$ or $\hat{\alpha}$ is greater than or equal to unity. Since our approach to seeking the optimal audit policy for the government was piece-meal, getting an optimal audit for any representative class was equivalent to finding our solution. The above exercise was carried out in section 2 for a situation where no political or economic constraint exist for the authority. We were successful in our venture. The optimum audit policy may be written down as

$$ n^* = \{ n_1^*, n_2^*, \ldots, n_k^* \} $$

where

$$ n_i^* = \begin{cases} n_i^*; & 0 < n_i^* < N_i \text{ for } \hat{\alpha}_i < 1 \\ 0 & \text{otherwise.} \end{cases} $$

In the comparative statics, the sign of $\partial n^*/\partial \theta$ is found to be ambiguous. Even bringing in an additional character to TP's economic behavior did not resolve the ambiguity. Also, we find that a trade-off between penalty rate and audit frequency with regard to optimality for the government does not necessarily exist in all situations. In fact, in a situation where the relative degree of dishonesty is greater than twice the audit elasticity of compliance a higher penalty rate requires a higher audit frequency for optimality. Another important finding is that the probability of audit should increase with the economic status of the class concerned.
In section 3, the search for the optimum audit is worked out for a two-class economy in a resource constraint situation. Here again, an optimal audit policy comprising of \( n^* = (n_1^*, n_2^*) \) is determined. In the comparative statics exercise, we could find the effect on \( n_i^* \) of changes in tax parameters within its own class as well outside it. We find that if the prevailing tax situation in class - I is of TS-I form, then \( n_1^* \) increases with \( \theta_1 \) whereas \( n_2^* \) decreases with \( \theta_1 \). We had defined a tax situation TS-I as the one where the inverse of the audit elasticity of compliance prevalent in the concerned class is at least as great as twice the effective penalty rate for the same class. In a similar tax situation existing in class-II, a hike in \( \theta_2 \) will require a smaller \( n_1^* \) but a larger \( n_2^* \).

A similar analysis to examine the effect of change in the slope of penalty rate function, \( \pi_i \) on \( n_i^* \) (i = 1,2) is also performed. Here, we study the required adjustments in \( n_i^* \) to attain optimality after an increase in the slope of \( \pi_i \)-function (equivalent to an enhancement in penalty rate by a proportion of the tax evaded sum, \( D_i \) at every level of \( D_i \)) has taken place. We find that if the prevailing tax situation in class I is of TS-II form (described in text) then with increase in the slope of \( \pi_1 \)-function, \( n_1^* \) increases whereas \( n_2^* \) decreases. If the prevailing tax situation in class-I is of TS-III form, increase in slope of \( \pi_1 \) - function needs a lower \( n_1^* \) and a higher \( n_2^* \) for
optimality. Now, going to the prevailing tax situation in class-II, if it is of TS-II form, $n_1^*$ decreases with $\pi_2$ whereas $n_2^*$ increases with $\pi_2$. In case the tax situation in class-II of the TS-III form, with $\pi_2$, $n_1^*$ increases whereas $n_2^*$ decreases.

Thus, taking up a 2-class economy facing a resource constraint, we could determine the optimal audit for the two classes and also the effect of change in any tax parameter concerning a class on the optimal audit of its own class and for the other class as well. Here we found that a change in tax situation of class-I or class-II have an impact on both $n_1^*$ & $n_2^*$. Thus, an isolated correction of audit policy in a single class for a change in tax situation in any one of the class may not provide the optimal position for the system as a whole. Since a new tax situation in a small corner of the system brings up new equation in the system, correction or reform is called for the whole system.
Notes


2. Ibid., p.65

3. In chapter II, we had situations where $\theta$ did not at all affect the level of $\alpha$ and also situations where $\theta$ has a positive effect on $\alpha$. We did not however, find any situation where decrease in $\theta$ resulting in greater compliance i.e higher $\alpha$.

4. Balbir Singh (1973) also stressed the significant role of higher value of $p$ in checking tax evasion. He said, "Lowering the rate is no guarantee for eliminating tax evasion. Whatever the tax rates, they will have to be implemented effectively."

5. $D = -n\theta Y(\partial \alpha / \partial n)$

$$= \theta Y(1 - \alpha - n\partial \alpha / \partial n)$$

$$= \theta Y(1 - \alpha (1 + (\partial \alpha / \partial n)(n/\alpha)))$$

$$= \theta Y(1 - \alpha (1 + \varepsilon_{\alpha:n})) = \theta Y(1 - \tilde{\alpha}(n)) \geq 0$$

where $\varepsilon_{\alpha:n}$ is audit elasticity of compliance or the ratio of proportionate change in $\alpha$ to proportionate change in $n$. $\tilde{\alpha}(n) = \alpha(1 + \varepsilon_{\alpha:n})$ represents the new compliance level after a change in $n$ and can at most equal to unity.

6. $d\pi/d\theta = (\partial \pi / \partial D)(1 - \alpha - \theta(\partial \alpha / \partial \theta))Y = \pi'(1 - \tilde{\alpha}(\theta))Y$

Clearly, $d\pi/d\theta \geq 0$
7. The change in expected net revenue due to a simultaneous change in $\theta$ and $n$ is given by

$$dR = \frac{\partial R}{\partial \theta}d\theta + \frac{\partial R}{\partial n}dn$$

But, we know that the changes in $\theta$ and $n$ in our context take place in such a manner that the first order condition for maximization remains satisfied even in the new situation i.e. we still have $\frac{\partial R}{\partial n} = 0$.

Hence the overall change in $R$, here is

$$dR = \{NaY + n(1-\alpha)Yn(D)\}d\theta$$

Where $\tilde{n}(D)$ is the value of $\pi(D)$ in the new situation given by $\tilde{n}(D) = \pi(D)(1 + \varepsilon_{\pi:D})$; $\varepsilon_{\pi:D}$ is the elasticity of penalty rate. Clearly the $dR$ is positive

8. $N \geq n(\pi + 2D\pi')$

$$\Rightarrow N > n\{\pi + \pi'(1-\alpha) + 1-\tilde{\alpha}(\theta)\}$$

as $2D = 2(1-\alpha(\theta)) + \{2 - (\alpha(\theta) + \tilde{\alpha}(\theta))\}Y$

So, in this case all the terms in $\frac{\partial R}{\partial n}$ are non-negative with at least one term strictly positive.

9. Here, $dR = (\partial R/\partial \theta)d\theta + (\partial R/\partial n)dn$.

The change in $\theta$ is presumed to be accompanied by an adjustment in $n$ so that the new $n$ is the optimal audit in the changed circumstances that is to say $\partial R/\partial n = 0$ at the new $n$. In that case we have

$$dR = (\partial R/\partial \theta)d\theta$$

$$= [NaY(1+Y\theta) + n(1-\alpha)\{(\pi + Dn')(\theta Y + \theta) + Y\tilde{n}(D)\}d\theta$$

$$= \{Na + n(1-\alpha)\tilde{n}(D)\}Y(\theta)d\theta$$

where $\tilde{Y}(\theta) = Y(1 + \varepsilon_{Y:\theta})$; $\tilde{n}(D) = \pi(D)(1 + \varepsilon_{\pi:D})$

Though $\tilde{Y}(\theta)$ is less than $Y(\theta)$ as per our assumption,
dR is clearly positive. Thus, the new situation represents a position of potentially higher net expected revenue.

10. The relevant bordered Hessian determinant is

\[
|\bar{H}| = \begin{vmatrix}
L_{11} & L_{12} & \cdots & L_{1k} & g_{n1} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
L_{k1} & L_{k2} & \cdots & L_{kk} & g_{nk} \\
g_{n1} & g_{n2} & \cdots & g_{nk} & 0
\end{vmatrix}
\]

and its successive bordered principal minors are

\[
|\bar{H}_2| = \begin{vmatrix}
L_{11} & L_{12} & g_{n1} \\
L_{21} & L_{22} & g_{n2} \\
g_{n1} & g_{n2} & 0
\end{vmatrix},
\]

\[
|\bar{H}_3| = \begin{vmatrix}
L_{11} & L_{12} & L_{13} & g_{n1} \\
L_{21} & L_{22} & L_{23} & g_{n2} \\
L_{31} & L_{32} & L_{33} & g_{n3} \\
g_{n1} & g_{n2} & g_{n3} & 0
\end{vmatrix},
\]

etc.

11. We have

\[
S_{n1}(dn_1/d\lambda) + S_{n2}(dn_2/d\lambda) = -S_{\lambda}
\]

\[
R_{n1}(dn_1/d\lambda) + R_{n2}(dn_2/d\lambda) = 0
\]

and hence the expression of \((dn_1/d\lambda)\) and \((dn_2/d\lambda)\).
Appendix I

We have Total Cost = variable cost + fixed cost

Let the average VC, involved in auditing n TPs be h(n),

where h(n) =

\[\begin{align*}
&f(n), f' < 0, f'' > 0, n < a \\
f(a), a \leq n \leq b \\
g(n), g' > 0, g'' > 0, n > b
\end{align*}\]

Also, g(b) = f(a)

Though AVC has been taken as a function of n, the number of TPs audited, for the sake of analytical convenience and without much loss of generality, let us assume h(n) to be a continuous function of n where n is real number (Fig. 1).

Now, \(C(n) = ATC = h(n) + FC/n\)

Clearly, h(n) is differentiable for all n, except may be at n = a or n = b.

Now, for \(\delta > 0\), we have

\[h'(a) = \lim_{\delta \to 0} \frac{f(a+\delta)-f(a)}{\delta} = 0 = \lim_{\delta \to 0} \frac{f(a-\delta)-f(a)}{-\delta}\]
\[
h'(b) = \lim_{\delta \to 0} \frac{g(b+\delta) - f(b)}{\delta} = 0 = \lim_{\delta \to 0} \frac{f(b-\delta) - f(b)}{-\delta}
\]

Hence, \(h'(n)\) exists for all \(n\). Also, \(FC/n\) is differentiable for all \(n\). Therefore, \(\overline{C}'(n)\) exists for all \(n\).

In fact, \(\overline{C}'(n) = h'(n) - FC/n^2\)

Now, \(\overline{C}'(n) > 0\), if \(h'(n) > FC/n^2 > 0\)

\[h'(n) = f'(n) < 0 \text{ if } n < a \Rightarrow \overline{C}'( ) < 0 \text{ if } n < a\]
\[= 0 \text{ if } a \leq n \leq b \Rightarrow \overline{C}'( ) < 0 \text{ if } a \leq n \leq b.\]

Thus, the only possible range where \(\overline{C}(n) > 0\) is when \(n > b\).

In fact when \(n > b\),

\[\overline{C}'(n) = g'(n) - (FC/n^2)\]

Let \(n_m\) be a point such that

\[n_m = \{ n : \overline{C}'(n) = 0 \text{ and } \overline{C}'(x) < 0 \text{ for all } x < n \}\]

In fact, the above \(n_m\) is the point of least average cost (ATC) of auditing.

Notationally \(n_m = \{ n : \text{minimum } \overline{C}(n) \}\)

Clearly, \(n_m > b\)

Our contention will be that the government in our modified model) while trying to maximize its net revenue will audit at least \(n_m\) TPs.

Proof: The expected revenue of the government when it audits \(n_i\) persons in each \(i\)th class is

\[R = \sum N_i \theta_i \alpha_i Y_i + \sum n_i \pi (1-\alpha_i) Y_i\]
Now, $\delta R/\delta n_i = \pi(1-\alpha_i)Y_i > 0$ for all $i$.

Thus, the government can enhance its revenue with increasing audit. The only restricting factor which could come before it, is the auditing cost incurred. Hence, in the region, where $C(n)$ is continuously decreasing, the net revenue (which is revenue minus cost) will keep on increasing. Therefore, a revenue maximizing government will audit at least $n_m$ persons. What the government will need to decide is whether to audit just $n_m$ or more, the latter depending on whether the resultant enhancement in revenue from more audit, overweighs the increasing cost incurred in auditing.
APPENDIX 2

To check the sufficiency condition, first let us write down the relevant bordered Hessian determinant

\[
|\bar{H}| = \begin{vmatrix} L_{11} & L_{12} & g_{n1} \\ L_{21} & L_{22} & g_{n2} \\ g_{n1} & g_{n2} & 0 \end{vmatrix}
\]

where

\[ L_{ij} = \frac{\partial^2 L}{\partial n_i \partial n_j} \quad \text{for } i, j = 1, 2; \]

\[ g_{ni} = \frac{\partial g(n_1, n_2)}{\partial n_i} \quad \text{where } i = 1, 2 \]

and the resource constraint being

\[ R_A = \sum n_i \bar{c}_i(n_i) = g(n_1, n_2) \]

The sufficient condition for maximization requires \(|\bar{H}|\) to be positive.

Now,

\[
L_{ii} = -2[\pi_1 + D_i \pi_i' - n_i \pi_i' \theta_i \gamma_i (\partial \alpha_i / \partial n_i)] \theta_i \gamma_i \partial \alpha_i / \partial n_i
\]

\[-(1+\mu)(2\bar{c}_i' + n_i\bar{c}_i'') \quad \text{for } i = 1, 2\]

which from expression (3), has been proved to be positive.

\[ g_{ni} = \bar{c}_i + n_i \bar{c}_i', \quad i = 1, 2 \]

Also \[ L_{ij} = 0 \quad \text{for } i \neq j \]
Thus,

\[
|\bar{H}| = \begin{vmatrix}
L_{11} & 0 & g_{n1} \\
0 & L_{22} & g_{n2} \\
g_{n1} & g_{n2} & 0
\end{vmatrix}
\]

\[= -g_{n1}^2 L_{22} - g_{n2}^2 L_{11}\]

Since \(L_{11}, L_{22}\) are negative, we have \(|\bar{H}| > 0\).