TAX EVASION UNDER ALTERNATIVE PENALTY SCHEMES: AN ANALYTICAL STUDY

1 INTRODUCTION

When a taxpayer (TP) is faced with the task of declaring his income for the purpose of computing his tax returns, he may not necessarily state his true income. The choices before him are whether to cheat or not and if cheat, then by how much and so on. Clearly, all these are choices under uncertainty. These are so, because of the fact that even if he cheats there is a possibility that he might not be caught in the act. Hence the theoretical analyses concerning the problem of tax evasion have been worked out within the framework of an individual's decision behavior under uncertainty. The approach commonly used, has been one which may be termed the expected utility approach (Allingham and Sandmo (1972), Yitzhaki (1974), Kolm (1973), McCaleb (1976), PenCavel (1979), Christiansen (1980), Koskela (1983), Russell and Rickard (1987) etc.). In these studies, the utility function of the taxpayer considered is of concave
nature along with the assumption that the TP's behavioral conforms to the Von-Neumann Morgenstern axioms for behavior under uncertainty. In this chapter, with the help of a very simple theoretical model we try to study the tax evasion behavior of a TP who tries to maximize his expected net income. In fact, we pick up the Srinivasan (1973) model with a Yitzhaki penalty schedule (i.e. of penalty being charged on evaded tax sum). We have already stated out the reasons for our choice of this approach in the introductory chapter.

An attempt has been made to obtain an insight into the behavior patterns of the TP under three different audit schemes combined with various penalty schedules. A proportional rate of tax has been considered through out our analysis. One should not miss the policy implications of each specific behavioral pattern of the TP under varying tax situations though the study does not directly deal with this aspect. The analysis could have been extended by taking different tax-schedules and combining them with the various penalty schemes under consideration. Limiting to just one specific tax schedule has enabled the study to concentrate on the different behavioral pattern of the TP on the face of alternative penalty schedules under given specific audit rules.

The scope and purpose of this chapter is a very
limited one. Here, the optimal income to be declared in the income tax returns vis-a-vis the tax rate, penalty schedule, the actual income level and the probability of detection have been analyzed. In the comparative statics, the manner in which the optimal income declaration will vary with regard to other tax parameters have been brought out clearly.

The lay-out of the chapter is as follows. In the next section, the model to be used for our analysis is specified. This section has been further divided into three sub-sections, each one covering a specific audit regime. First one deals with a random audit rule, in which each TP faces a fixed, predetermined probability of audit (p) regardless of his or her report. The second subsection, discusses the case when p has been made a function of α (the proportion of income actually declared). In the third one, p is taken to be a function of the income level, Y. We must note that the relevant value of p which molds or is responsible for the behavioral pattern of the TP is the one he believes (or conjectures). It may be far from the actual value prevailing. In each of the sub-sections our analysis starts with the simplest of the cases where the TP finds himself amidst a situation in which all the tax parameters are exogenously given. Thus the TP, under the given circumstances has to simply maximize his expected net income
by his choice of \( \alpha \). Gradually, we endogenise some of the parameters in the model, taking the analysis closer to reality. In each case, the interplay between the parameters and the optimum proportion of income declaration is brought out.

The results obtained in this chapter can be had at a glance from the three tables, provided at the end.

Finally, in the concluding section, the major findings are pointed out.

2 THE MODEL

Consider a TP with an income of \( Y \) who declares his income as \( aY \) \((0 \leq a \leq 1)\). Thus \( a \) is a proportion of true income actually declared by the TP. Let \( \theta \) (where \( 0 < \theta < 1 \)) be the given proportionate rate of tax i.e. the TP pays taxes at rate \( \theta \) on every rupee of income that is declared. Let \( p \) be the given probability of audit. We shall assume that in every audit case, the true situation gets revealed. It is in this sense that no distinction has been made between the two terms: probability of audit and probability of detection. Indeed, \( p \) could depend on many factors like the income stated, the available resources in the hand of the tax authority for this purpose, the true income level of the TP, the efficiency of the judicial/tax system, etc. By taking up the three different audit regimes stated earlier in our ana-
alysis, we have tried to cover at least some of the possible situations. Obviously in a study of this nature, it is almost impossible to be completely exhaustive. Nonetheless taking up these simplified versions, one obtains significant analytical results regarding a TP’s behavior pattern.

If the under-reporting of income by a TP is detected, he is made to pay a penalty of \( n > 1 \) on each rupee of evaded tax.

In case of no audit, the income of the TP, \( Y_n \) is \((1 - \alpha \theta)Y\) whereas in case of audit the income of the TP, \( Y_c \) is \( \{1 - \alpha \theta - n \theta (1 - \alpha)\}Y\). The condition of \( n > 1 \) lands a tax evader who is caught, in a worse position than an honest TP of same income level.

2.1 Random audit model where 'p' is the exogenously given probability of audit

Now, the expected net income of the TP can be written down as

\[
EY = p\{Y - \theta \alpha Y - n \theta (1 - \alpha)Y\} + (1 - p)(Y - \theta \alpha Y)
= \{1 - \theta \alpha - p \theta n (1 - \alpha)\}Y
\]  

(1)

Case A1: Here we assume that except for the decision variable \( \alpha \), the rest of the tax variables involved in the expression \( EY \) are exogenously determined.

From equation 1, we have

\[
EY \text{ (at } \alpha = 1 \text{ i.e. at true income declaration)} = (1 - \theta)Y > 0
\]

and

\[
EY \text{ (at } \alpha = 0 \text{ i.e. at zero income declaration)} = \{1 - p \theta \pi\}Y
\]
If the penalty rate be of such a magnitude that for the exogenously given 'p' and 'θ', πθp = 1. In that case, we have EY (at α = 0) < 0.

Thus, in such a situation since EY (at α = 1) is greater than EY (at α = 0), no rational person will try to hide whole of his income. It is another matter that π of such a magnitude may not be politically feasible.

Allowing the TP to maximize his expected net income, let us determine the optimal level of declaration α*.

Differentiating EY with respect to α, we have

\[ \frac{\partial EY}{\partial \alpha} = (pθπ - θ)Y \]  

Clearly \( \frac{\partial EY}{\partial \alpha} \geq 0 \) for all \( \alpha \) if \( pπ \geq 1 \).

Now consider the case when \( pπ > 1 \). Here, \( \frac{\partial EY}{\partial \alpha} > 0 \) for all \( \alpha \). In such a situation, we find that EY is a strictly increasing function of \( \alpha \) in the range \([0,1]\) and hence the TP will maximize his EY when he declares his income fully i.e. \( \alpha^* = 1 \).

If \( pπ = 1 \), then \( \frac{\partial EY}{\partial \alpha} = 0 \).

Here, EY is independent of \( \alpha \) and hence the TP will maximize EY at any value of \( \alpha \).

But in case \( pπ < 1 \), then \( \frac{\partial EY}{\partial \alpha} < 0 \) for all \( \alpha \) and hence, here \( \alpha^* = 0 \).

Note that the expression \( pπ \) can be looked upon as the expected or effective penalty rate. Thus,

Proposition 1: Suppose our TP is confronted with a tax situation where \( p \) and \( π \) are the exogenously determined
probability of audit and penalty rate respectively. In such a situation, if the effective penalty rate is greater than unity, then irrespective of the prevailing tax rate, the TP will act honestly. But wherever the effective penalty is less than unity then the TP will declare zero income.

Case B1:
Instead of assuming a flat penalty rate, let \( \pi \) be a continuous decreasing convex function of \( \alpha \) i.e. \( \pi = \pi(\alpha) \) with \( \pi_\alpha < 0, \pi_{\alpha\alpha} > 0 \). Here the magnitude of the penalty is being made dependent on the level of honesty of the TP.

In this case, equation 1 takes the following form:

\[
EY = (1 - \theta \alpha - p\theta(1-\alpha)\pi(\alpha))Y
\]

(2)

Here, 
\[
EY(\text{at } \alpha=0) = (1 - p\theta \pi(0))Y
\]

and 
\[
EY(\text{at } \alpha=1) = (1 - \theta)Y
\]

We shall normalize our penalty schedule in the following way: for any given \( p \) and \( \theta \), let the effective penalty rate be such that (i) at \( \alpha=0 \), \( p\pi(0) \geq 1/\theta \), (thus making \( EY \) non-positive here) and (ii) for honest persons irrespective of value of \( p \) the effective penalty rate is zero i.e. \( p\pi(1)=0 \) or \( \pi(1) = 0 \).

The first order condition for maximization provides

\[
E_\alpha = \partial EY/\partial \alpha = [-1 + p\{\pi() - (1-\alpha)\pi'()\}]\theta Y = 0
\]

(2a)

The second order condition requires that

\[
E_{\alpha\alpha} = \frac{\partial^2 EY}{\partial \alpha^2} = (2\pi'() - \pi''() (1-\alpha))p\theta Y \text{ be negative}
\]
Since $\pi' < 0$, $\pi'' > 0$, clearly $E_{\alpha\alpha} < 0$

Now, $E_{\alpha}$ (at $\alpha = 0$) = $-1 + \theta \{\pi(0) - \pi'(0)\} > 0$
and $E_{\alpha}$ (at $\alpha = 1$) = $-\theta Y < 0$

The fact that $E_{\alpha}$ exists and is continuous in $[0,1]$ and also that $E_{\alpha}$ ($\alpha = 0$) > 0 and $E_{\alpha}$ ($\alpha = 1$) < 0 guarantees the existence of at least one point of maximum [see the appendix provided at the end for the proof]. In case, if there is more than one such point, then comparing the values of $EY$ at all these points and taking the value of $\alpha$ which provides the greatest $EY$, we get the required $\alpha^*$. Thus,

Proposition 2: Consider a tax situation where $p$ and $\theta$ are the exogenously determined probability of audit and tax rate respectively, with an endogenously determined penalty rate which is a continuous decreasing convex function of the compliance rate. If the penalty rate be such that the expected income at $\alpha=0$ is non-positive, then there exists an interior $\alpha^*$ which maximizes $EY$ of the TP.

Note that $E_{\alpha\alpha}(\alpha^*) < 0$. However, we shall assume $\alpha^*$ to be a position of regular maximum for $EY(\alpha)$ i.e. $E_{\alpha\alpha}(\alpha^*) < 0$.

We shall now work out the effect of variation in the parameters $p$, $\theta$ & $\pi$ on $\alpha^*$.

Using equation (2a) we can write down

$$\frac{\partial \alpha^*}{\partial \theta} = \frac{E_{\alpha\theta}}{E_{\alpha\alpha}} = 0$$

i.e. $\alpha^*$ is independent of the level of $\theta$
Also, \[ \frac{\partial \alpha}{\partial p} = \frac{\pi(\alpha) - \pi'(\alpha)(1-\alpha)}{-E_{\alpha}} > 0 \]

Thus, here an increase in \( p \) raises the compliance level, \( \alpha^* \).

It can also be seen that in the above tax situation, we have \( \frac{\partial \alpha^*}{\partial Y} = 0 \) i.e. the compliance rate of the TP is independent of his income level.

To see what happens to \( \alpha^* \) with change in convexity of \( \pi \) function, we shall use the following method.

We can write \[ \pi(\alpha) = \int_1^\alpha \pi'(Y) \, dY \]

Replacing \( \pi'(Y) \) by \( \pi'(Y) + \xi \) where \( \xi \) is a small positive number in the above expression, we have

\[ \tilde{\pi}(\alpha) = \int_1^\alpha (\pi'(Y) + \xi) \, dY \]

\[ = \pi(\alpha) - (1-\alpha)\xi \]

or \[ \tilde{\pi}(\alpha) - \pi(\alpha) = -(1-\alpha)\xi < 0 \text{ for all } \alpha < 1. \]

Thus, we see that flattening of the slope of the \( \pi \) function in absolute terms results in lowering down of the \( \pi \) function; what it means is that at all levels of \( \alpha \) in particular for \( \alpha^* \), the new penalty is lower than the earlier one.

Let us now transform the equation (2a) by bringing in \( \tilde{\pi}(\alpha) \) and \( \pi'(\alpha) + \xi \) in place of \( \pi(\alpha) \) and \( \pi'(\alpha) \). Then we get.

\[ E_{\alpha} = [-1 + p\{\pi(\alpha) - (1-\alpha)\xi - (1-\alpha)(\pi'(\alpha) + \xi)\}] \Theta Y = 0 \]
Now, differentiating the above with respect to $\xi$ and taking limit as $\xi \to 0$, we have

$$\frac{\partial \alpha^*}{\partial \xi} = -2p(1-\alpha)/-E_{\alpha^-} < 0$$

Thus, we find that if the tax authority becomes soft towards tax evaders, the compliance level of the TP goes down.

Case C1

Now, let us consider another situation where the penalty rate, $\pi$ is a function of $\alpha$ as well as of the actual income level, $Y$ of the TP i.e.,

$$\pi = \pi(\alpha,Y) \quad \text{with } \pi_{\alpha} < 0, \pi_Y > 0, \pi_{ij} = 0; \ (i,j = \alpha, Y)$$

Also, we shall assume that $\pi(1,Y) = 0$ i.e. for an honest TP, irrespective of his income level, the effective penalty rate is zero. This is so because an honest TP will not be chargeable of any sort of penalty under any circumstances.

The rationale behind our penalty function is the following: Let there be two tax cheats A and B each of whom declares $\alpha$ proportion of their respective incomes $Y_i$ and $Y_j$ where $Y_i > Y_j$. Then clearly the government will be deprived of a much larger revenue from the act of A than that of B. Also, rich persons are conspicuous in their acts. For these reasons, the penalty has been made more severe for the rich TP and hence the assumption of $\pi_Y > 0$. Again, one who is more honest, should be penalized less and hence the other assumption of $\pi_{\alpha} < 0$. 

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Here, the equation (1) takes the following form:

\[ E_Y = \{1 - \theta x - p\theta \pi(\alpha, Y)(1-\alpha)\} Y \] (3)

Again for given \( p \) and \( \theta \), let the value of \( \pi(0, Y) \) be \((p\theta)^{-1}\) so that \( E_Y \) at \( \alpha = 0 \) is zero.

As in the case B1 be where \( \pi = \pi(\alpha) \), the existence of an optimal \( \alpha^* \in (0,1) \) can be shown here, too.

The corresponding first order condition, here is

\[ E_\alpha = [-1 + p\{-\pi(\alpha, Y) - (1-\alpha)\pi_\alpha(\alpha, Y)\}] Y = 0 \] (3a)

Also, the second order condition requirement of \( E_{\alpha\alpha} \) being negative is clearly satisfied as

\[ E_{\alpha\alpha} = 2p\pi_\alpha(\alpha, Y) Y \] and \( E_\alpha() \) is negative.

The effects on \( \alpha^* \) of individual variations in parameters namely \( p \), \( \theta \) and a change in convexity of the \( \pi \) function will be the same as that in case B. For the effect of \( Y \) on \( \alpha^* \), from (3a), we can have

\[ \partial \alpha^*/\partial Y = -p\pi_Y(\alpha, Y)/E_{\alpha\alpha} > 0 \]

Thus, we find that in such a tax situation, a richer TP or a TP when his income level rises, will tend to declare a greater proportion of his actual income.

Case D1

Now, let us consider the case where the penalty rate, \( \pi \) is an increasingly increasing function of the size of the evaded tax i.e.

\[ \pi = \pi(D) ; \pi'(\cdot), \pi''(\cdot) > 0 \quad \text{where } D = (1-\alpha)\theta Y \text{ is} \]

the amount of tax evaded sum or tax deficient.
Here, the equation (1) takes the form

\[ E_Y = \{1- \theta \alpha - p\theta (1-\alpha)\pi(D)\}Y \]  

\[ \text{(4)} \]

The first order condition for maximization of \( E_Y \) provides

\[ E_\alpha = -\theta + p\theta \{ \pi(D) + (1-\alpha)\pi'()\theta Y \} = 0 \]

or

\[ -1 + p\{ \pi(\theta Y) + (1-\alpha)\pi'(\theta Y)\theta Y \} = 0 \]  

\[ \text{(4a)} \]

Now, \( E_\alpha \) (at \( \alpha = 0 \)) = \[-1 + p\{ \pi(\theta Y) + (1-\alpha)\pi'(\theta Y)\theta Y \}\] \( \theta \)

Thus we see that if the effective penalty rate for a TP who declares zero income is equal to unity, we have \( E_\alpha \) (at \( \alpha = 0 \)) to be positive.

What we actually require for \( E_\alpha \) (at \( \alpha = 0 \)) to be positive is that \( p\{ \pi(\theta Y) + \pi'(\theta Y)\theta Y \} > 1 \).

Now, \( E_\alpha \) (at \( \alpha = 1 \)) = \(-\theta Y < 0 \)

Thus, in such situations, the existence of \( \alpha^* \in (0,1) \) is ensured.

**Comparative statics**

Total differentiation of equation (4a) provides us the following results:

Since \( \partial \alpha^*/\partial \theta = -E_{\alpha \theta} / E_{\alpha \alpha} \) and

\[ E_{\alpha \theta} = p\{ \pi'() + (1-\alpha)\pi''()\theta Y \}(1-\alpha)Y + p(1-\alpha)\pi'(\theta Y)Y > 0 \]

and hence, here \( \partial \alpha^*/\partial \theta > 0 \).

Thus, under the tax situation considered, the TP will raise his compliance level with increase in tax rate.

Again from equation (4a), we have

\[ \partial \alpha^*/\partial p = \{ \pi() + (1-\alpha)\pi'()\theta Y \}/-E_{\alpha \alpha} > 0 \]
Thus as expected, a higher probability of audit persuades the TP to raise his compliance level.

Now, to see the effect of income level on compliance rate, we have.

\[ E_{\alpha Y} = p\{p'() + (1-\alpha)\pi''()\theta Y\}\theta + p(1-\alpha)p'()\theta \]
\[ = p\{2p'() + (1-\alpha)\pi''()\theta Y\}(1-\alpha)\theta > 0 \]

Since \( \partial \alpha / \partial Y = -E_{\alpha Y} / E_{\alpha\alpha} \) is positive here, we conclude that in the tax system under consideration, a richer TP will tend to declare a higher proportion of his income.

To check the effect of on upward movement in \( \pi \)-function with respect to \( D \) on \( \alpha^* \). We can write

\[ \pi(D) = \int \pi'(z)dz \quad [D = (1-\alpha)\theta Y] \]

Replace the integral term \( \pi'(z) \) by \( \pi'(z) + \xi \) to get the new \( \pi \)-function. What in effect, we are doing is that rotating the initial \( \pi \)-function anti-clockwise with origin as the pivot point. This way, the penalty rate for honest TPs has been maintained at zero while the penalty rate at other evasion levels have been raised. We thus have

\[ \tilde{\pi}(D) = \int \{\pi'(z) + \xi\}dz \]
\[ = \pi(D) + \xi D \]

Now, transforming equation (4a) by replacing \( \pi(D) \) and \( \pi'(D) \) by \( \tilde{\pi}(D) \) and \( \pi'(D) + \xi \) respectively, we get implicit differentiation of the transformed equation and taking the limit as \( \xi \to 0 \), we obtain

\[ \partial \alpha^* / \partial \xi = (pD + (1-\alpha)\theta Y)/ - E_{\alpha\alpha} > 0 \]

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Thus if there is an upward swing of the $\pi$-function in the manner described above, the compliance level of the TP will also rise.

2.2 The Audit Regime where Probability of Audit is a function of the Compliance Level.

Till now, we have been considering the case when the probability of detection, $p$, is exogenously given. In fact, the relevant value of $p$, when the TP sets out to optimize his course of action will be the one which he believes and perceives as its value. Let the TP believe that the chance of his getting audited is negatively related to his compliance rate. In particular, let $p$ be a linear decreasing function of $\alpha$. We shall study the earlier situation in this new light.

For simplicity, let $p = p(\alpha) = (1-k\alpha)$ for all $\alpha \in (0,1)$ and $k$ is any real number such that $0 < k \leq 1$.

Here, we find that as $k$ moves along the real line from zero to one, $p$ moves from one to $1-\alpha$, thus showing that for a particular $\alpha$, a higher $k$ lowers the corresponding value of $p$. Thus, $k$ may be termed as $p$-moderator where $p$ is the probability of audit.

Case A2

For this case the expected income equation (1) takes the form

$$EY = (1 - \theta\alpha - p(\alpha)\theta(1-\alpha)\pi)Y$$

$$= (1 - \theta\alpha - \theta(1-\alpha)(1-\alpha)k\pi)Y$$  (5)
Here, \( \partial EY/\partial \alpha = [-\theta + \theta((1-\alpha k) + k(1-\alpha))\pi]Y \)
\[= [-1 + (1+k-2\alpha k)\pi]\partial Y \] (5a)
Clearly, \( \partial EY/\partial \alpha \) is a continuous function of \( \alpha \).

Now, \( E_\alpha \) (at \( \alpha = 0 \)) = \[[-1 + (1+k)\pi]\partial Y \] and \( E_\alpha \) (at \( \alpha = 1 \)) = \[[-1 + (1-k)\pi]\partial Y \]

Since by assumption, we have \( \pi > 1 \), hence \( E_\alpha \) (at \( \alpha = 0 \)) is positive. But the value of \( E_\alpha \) (at \( \alpha = 1 \)) depends on the specific values of \( \pi \) and \( k \). In case the values of \( \pi \) and \( k \) are such that \( (1-k)\pi < 1 \), then \( E_\alpha \) at \( \alpha = 1 \) is negative. In this case, there exists an interior \( \alpha^* \) i.e. \( \alpha^* \in (0,1) \) where
\[ \alpha^* = (\pi(1+k) - 1)/2\pi k \]

Comparative statics for the case of an interior \( \alpha^* \)

We have \( \partial \alpha^*/\partial \pi = [2\pi k(1+k) - \pi(1+k)-1]2k/(2\pi k)^2 \)
\[= 1/2\pi k^2 > 0 \]
and \( \partial \alpha^*/\partial k = (k\pi - \pi(1+k) -1)/2\pi k^2 \)
\[= -(1+\pi)/2\pi k^2 < 0 \]

In our set-up, increasing \( k \) is in fact equivalent to reducing the probability of audit and we see the consequence of it from the above expression. Thus, reducing \( p \) results in lowering of the compliance rate.

Case B2

Here, the expected income expression of equation (1) will take up the following form.
\[ E_Y = \{1 - \theta \alpha - \theta (1-\alpha)(1-\alpha k)\pi(\alpha)\}Y \] (6)

The first order condition for maximization provides

\[ E_{\alpha} = [-\theta + \theta \{(1-\alpha)\pi(\alpha) + k(1-\alpha)\pi(\alpha) - (1-\alpha)(1-\alpha)\pi'(\alpha)\}]Y = 0 \]

or

\[ -1 + \pi(\alpha)(1+k-2\alpha k) - (1-\alpha)(1-\alpha)\pi'(\alpha) = 0 \] (6a)

Clearly, \( E_{\alpha} \) is a continuous function of \( \alpha \) in \((0,1)\).

Now,

\[ E_{\alpha} (at \alpha = 0) = [-1 + \pi(0)(1+k) - \pi'(0)]\theta Y \]

is positive from the nature of our \( \pi \)-function and \( E_{\alpha} (at \alpha = 1) = -\theta Y \) which is negative.

Thus as in the earlier cases, the existence of an interior optimum point is ensured, here too.

Clearly, the expression \( E_{\alpha\alpha} \) given by

\[ \pi'(\alpha)(1+k-2\alpha k) + (1-\alpha)\{\pi'(\alpha) - (1-\alpha)\pi''(\alpha)\} + k\{(1-\alpha)\pi'(\alpha) - 2\pi(\alpha)\} \]

is negative.

Comparative Statics

Total differentiation of (6a) and working out the effect of individual tax parameters on \( \alpha^* \) in ceteris paribus situations, we obtain

\[ \partial \alpha^*/\partial k = \left[\pi(\alpha)(1-2\alpha) + \alpha(1-\alpha)\pi'(\alpha)\right]/-E_{\alpha\alpha} \]

In case, \( \alpha^* < 1/2 \), then clearly \( \partial \alpha^*/\partial k > 0 \) Thus, in a situation of very low compliance level (where the optimum compliance rate does not exceed half), if the audit probability is lowered, the compliance level tends to improve.

Surprisingly, in our tax system, the compliance rate is independent of the tax rate.

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As in the case of B1, to check the effect of change in convexity of \( \pi \)-function on \( \alpha^* \), transform equation (6a) by replacing \( \pi(\alpha) \) and \( \pi'(\alpha) \) by \( \tilde{\pi}(\alpha) \) and \( \pi'(\alpha) + \xi \) respectively. Here we obtain
\[
E_\alpha = -1 + \{\pi(\alpha) - (1-\alpha)\xi\}(1+k-2ak) - (1-\alpha)(1-ak)(\pi'(\alpha) + \xi) = 0
\]
Differentiating the above with respect to \( \xi \) and taking limit as \( \xi \to 0 \), we have
\[
\frac{\partial \alpha^*}{\partial \xi} = \frac{-[(1-\alpha)(1+k-2ak) + (1-\alpha)(1-ak)]}{E_{\alpha\alpha}} = \frac{(1-\alpha)(2 + k(1-3a))}{E_{\alpha\alpha}}
\]
Now, \( \alpha^* < 1 \) \Rightarrow 1 - 3\alpha > -2
\[
\Rightarrow k(1 - 3\alpha) > -2k > -2 \quad \text{(k being a positive fraction)}
\]
Thus we have shown that \( \frac{\partial \alpha^*}{\partial \xi} \) is negative. We can interpret this result as in case of B1 i.e. ceteris paribus, if the penalty rate is increased, the compliance level also gets enhanced.

Case C2
Here, \( E_Y = \{1 - \theta_\alpha - \theta(1-\alpha)(1-ak)\pi(\alpha,Y)\}Y \) (7)

For this case also the existence of an optimum interior \( \alpha^* \) can be shown similarly to case B2. Here, first order condition for maximization provides.
\[
E_\alpha = -1 + \pi(\alpha,Y)(1+k-2ak) - (1-\alpha)(1-ak)\pi^*_\alpha(\alpha,Y) = 0 \quad (7a)
\]
and also clearly the expression, \( E_{\alpha\alpha} \) given by
\[
\pi^*_\alpha(\alpha,Y)(1+k-2ak) - (1-ak)\{\pi^*_\alpha() - (1-\alpha)\pi^*_{\alpha\alpha]()\} + k\{\pi^*_\alpha() - 2\alpha\pi()\}
\]
is negative.
Comparative Statics

The effects of the parameters $k$, $\theta$ on $\alpha^*$ will be similar to that obtained in case B2.

Now let us check the effect of $Y$ on $\alpha^*$.

From (7a), we have

$$\frac{\partial \alpha^*}{\partial Y} = \frac{\pi_Y(1+k-2ak) - \pi_Y(1-\alpha)(1-ak)\pi_{\alpha Y}(\alpha,Y)}{-E_{\alpha\alpha}} > 0$$

Let us check the effect of changes in slope of $\pi$ with respect to $\alpha$ and $Y$ separately on $\alpha^*$.

We may write

$$\pi(\alpha,Y) = \int_{1}^{\alpha} \pi_{\alpha}(\alpha,Y) d\alpha$$

Changing $\pi_{\alpha}(\alpha,Y)$ to $\pi_{\alpha} + \xi$, we have

$$\tilde{\pi}(\alpha,Y) = \int_{1}^{\alpha} (\pi_{\alpha}(\alpha,Y) + \xi) d\alpha = \pi(\alpha,Y) - (1-\alpha)\xi$$

Also, $\pi(\alpha,Y) = \int_{0}^{Y} \pi_{Y}(\alpha,Y) dY$

Changing $\pi_{Y}(\alpha,Y)$ to $\pi_{Y} + \zeta$, we have

$$\tilde{\pi}(\alpha,Y) = \int_{0}^{Y} (\pi_{Y}(\alpha,Y) + \zeta) dY = \pi(\alpha,Y) + \zeta Y$$

Transform (7a) by replacing $\pi(\alpha,Y)$ with $\tilde{\pi}(\alpha,Y)$ with $\pi_{\alpha}(\alpha,Y)$ and $\pi_{\alpha} + \xi$ to obtain

$$E_{\alpha} = -1 + (\pi(\alpha,Y) - (1-\alpha)\xi)(1+k-2ak) - (1-\alpha)(1-ak)(\pi_{\alpha}(\alpha,Y) + \xi) = 0$$

Differentiating the above with respect to $\xi$ and taking limit as $\xi$ tends to 0 we have

$$\frac{\partial \alpha^*}{\partial \xi} = (1-\alpha)(2 + k(1-3\alpha))/E_{\alpha\alpha} < 0.$$
We can interpret the above results in the following manner. Since \( \pi_\alpha \) is negative, addition of a positive \( \xi \) to \( \pi_\alpha \) will raise the magnitude of \( \pi \) at all levels of \( \alpha \). A TP will find himself being penalized harsher, though nothing has changed regarding his compliance level or his true income. The TP may consider this change in the penalty rate vis-a-vis his compliance level as lowering of the premium on honesty by the government. This is being reflected in the lowering of compliance level with higher \( \xi \).

Again transforming (7a) by replacing \( \pi \) by \( \pi \) and \( \pi_Y \) by \( \pi_Y + \zeta \), we have.

\[
E_\alpha = -1 + (\pi(Y) + \zeta Y)(1+k-2\alpha k) - (1-\alpha)(1-\alpha k)\pi_\alpha(\alpha,Y) = 0
\]

Differentiating the above with respect to \( \xi \) and letting \( \xi \rightarrow 0 \), we have

\[
\partial \alpha^* / \partial \xi = (1+k-2\alpha k)Y/ -E_{\alpha\alpha} > 0
\]

Thus if the penalty rate for the same crime is made more severe, then the TP tends to raise his compliance rate.

Case D2

Here, the equation (1) takes the form

\[
E_Y = (1 - \theta \alpha - \theta(1-\alpha)(1-\alpha k)\pi(D)) \theta Y
\]

where \( D = (1-\alpha)\theta Y \) is the deficient tax.

The first order condition for maximization of \( E_Y \) provides

\[
E_\alpha = -1 + (1+k-2\alpha k)\pi(D) + (1-\alpha)(1-\alpha k)\pi'(D)\theta Y = 0
\] (8a)

Now,

\[
E_\alpha \text{ (at } \alpha = 0) = -1 + (1+k)\pi(\theta Y) + \pi'(\theta Y)\theta Y
\]
This is positive from the nature of our penalty function. Also, 
\[ E_\alpha \text{ (at } \alpha = 1) = -1 \]
As in earlier cases, here also the existence of an interior optimum is ensured.

**Comparative statics**

From equation (8a), we obtain 
\[
\frac{\partial \alpha^*}{\partial \theta} = \pi'(1-\alpha)Y\{1+k-2\alpha k+1-\alpha k\} + (1-\alpha)^2 (1-\alpha k)\pi''(D)\theta Y^2 / -E_{\alpha \alpha} \\
= (1-\alpha) Y[\pi'(2-k(1-3\alpha)) + D(1-\alpha k)\pi''(D)] / -E_{\alpha \alpha}
\]

It can easily be seen that the above expression is positive.

Thus, in this tax-situation, the TP is found to raise his compliance level with a tax hike.

Again from (8a) we obtain 
\[
\frac{\partial \alpha^*}{\partial k} = (1-2\alpha)\pi(D) - \alpha(1-\alpha)\pi'(D)\theta Y / -E_{\alpha \alpha}
\]
Here, we find that in case of \( \alpha^* \geq 1/2 \), \( \frac{\partial \alpha^*}{\partial k} < 0 \).

Increase of \( k \), here means lowering of probability of audit. Thus, if \( \alpha^* \geq 1/2 \), the TP lower his compliance level when probability of audit is being lowered; very much in the expected line.

But if the initial \( \alpha^* < 1/2 \), then in the above expression, the first part of the numerator is positive whereas the second part is negative. Hence, if \( \alpha^* < 1/2 \),
\[
\frac{\partial \alpha^*}{\partial k} > 0 \text{ according as } (1-2\alpha)\pi(D) > \alpha D\pi'(D)
\]
Again from (8a),
\[ \partial \alpha^*/\partial Y = (1-\alpha)\theta \pi'(D)(1+k-2ak+1-ak) + (1-\alpha)^2 \theta^2 (1-\alpha k) \pi''(D)Y/E_{\alpha\alpha} \]
\[ = (1-\alpha)\theta[\pi'(D)(2+k(1-3a)) + (1-\alpha)\theta(1-\alpha k)\pi''(D)Y]/E_{\alpha\alpha} \]

Clearly, the above expression is positive and hence in this tax system, a richer person tends to have a higher compliance level.

To check the effect of an upward movement in \( \pi \)-function on \( \alpha^* \).

Taking the cue from case D1, transform the equation (8a), replacing \( \pi(D) \) and \( \pi'(D) \) by \( \pi(D) + \xi D \) and \( \pi'(D) + \xi \) respectively to obtain

\[ E_{\alpha} = -1 + (1+k-2ak) (\pi(D)+\xi D) + (1-\alpha)(1-ak) (\pi'(D)+\xi)\theta Y = 0 \]

Differentiating the above with respect to \( \xi \) and taking the limit as \( \xi \to 0 \), we have

\[ \partial \alpha^*/\partial \xi = [(1+k-2a)D + (1-\alpha)(1-ak)\theta Y]/E_{\alpha\alpha} \]
\[ = [(1 + k(1-2a))D + (1-\alpha)(1-ak)\theta Y]/E_{\alpha\alpha} \]

Here, since both \( \alpha^* \) and \( k \) are less 1, it can easily be shown that \( 1 + k(1-2\alpha^*) > 0 \),

Hence, we have \( \partial \alpha^*/\partial \xi > 0 \) i.e., an upward shift in \( \pi \)-function leads to raising of the compliance level.

2.3 The audit regime where the audit intensity is directly linked to the income level of the TP.

Usually, the authorities, especially those in the developing
countries are acutely short of resources. They can not afford to investigate even a sizable chunk of the TP population. In this light let us suppose that as a matter of policy, the authority concentrate its investigative activities more on the rich TPs. The authority might have thought of such a line considering the fact that rich TPs are fewer in number and also, the evasion activities of any of them has a much more pronounced effect on the economy than that of a poor TP. We shall precisely assume the above mentioned auditing policy in the analysis of this section. It is also assumed that the TP is aware of it. The TP goes about trying to maximize his expected net income, believing that the government has got some knowledge of his income. For our analysis purpose, the matter of how well-informed the government is of the TP's income is irrelevant. To play safe, the TP sets about his action on the premise that the government possesses a perfect knowledge of his income.

Let the probability of detection, \( p \) as perceived by the TP be given by
\[
p = p(Y) \text{ with } \quad p'(Y) > 0, \quad p''(Y) = 0, \quad p(Y) = 0 \text{ for } Y \leq Y_L.
\]

Where \( Y_L \) is some minimum income earmarked by the authority and those TPs whose income is less or equal to \( Y_L \) are not audited.

Quite clearly, the analysis in this new audit regime will be very much similar to that of random audit regime but
for the fact that here, the compliance level of the TP will be governed by his perceived value of p. The existence problem of an interior $\alpha^*$ and the movement of $\alpha^*$ with regard to the other tax parameters will be along the same line as those obtained in random audit regimes. The only thing worth finding out in this new audit regime is how $\alpha^*$ is governed by the income level $Y$. We shall do this exercise for all the four penalty schedules under consideration in other regimes.

Case A3: For this case, \[ E_\alpha = (p(Y)\theta\pi - \theta)Y \] (9)

This is a case of corner solutions. Depending on value of $Y$, hence $p(Y)$, we may have $\alpha^* = 0$ or 1.

Case B3: Here,

\[ E_\alpha = [-1 + p(Y)(\pi(\alpha) - (1-\alpha)\pi'(\alpha))]\theta Y = 0 \] (10)

Hence,

\[ \frac{\partial \alpha^*}{\partial Y} = \frac{[p'(Y)(\pi(\alpha) - (1-\alpha)\pi'(\alpha))]}{-E_{\alpha\alpha}} \]

\[ = -\frac{p'(Y)}{p(Y)}E_{\alpha\alpha} \]

Thus, we find that a TP with an initial positive compliance level, will raise his compliance level with income.

Case C3: Here,

\[ E_\alpha = [-1 + p(Y)(\pi(\alpha, Y) - (1-\alpha)\pi(\alpha, Y))]\theta Y = 0 \] (11)

Hence,

\[ \frac{\partial \alpha^*}{\partial Y} = \frac{[\{p'(\pi)/p(\pi)\} + p(\pi)\pi'(\alpha, Y)]}{-E_{\alpha\alpha}} > 0 \]

Again we find that a person when he becomes richer will raise his compliance level in this tax system.

Case D3: Here,

\[ E_\alpha = -1 + p(Y)(\pi(D) - (1-\alpha)\pi'(D)\theta Y) = 0 \] (12)
Also, we can write $\delta \alpha^*/\delta Y = -E_{\alpha Y}/E_{\alpha \alpha}$

Now, from (12), we have

$E_{\alpha Y} = \{p'(1)/p(1)\} + p(1-\alpha)\theta\{2\pi'(1) + \pi''(1)D\} > 0$

and $E_{\alpha \alpha} = p(Y)\{2\alpha(1-\alpha)\pi\alpha(1)\theta Y\theta Y < 0$ as $\pi_{\alpha} < 0; \pi_{\alpha \alpha} > 0$

Clearly in this case also, we find that with increase in the income level a TP becomes more tax compliant.

The fact that when the probability of audit is directly linked to the income level of the TP, a rise in the TP's income raises his perceived value of $p$ and in the process, compelling him to raise his compliance level on the course of maximizing his expected income.
TABLE 1

RANDOM AUDIT REGIME: p IS THE GIVEN PROBABILITY OF AUDIT

<table>
<thead>
<tr>
<th>Case</th>
<th>Penalty Condition</th>
<th>Status Movement of the ( \alpha ) with respect to ( \theta ) ( p ) ( \pi ) ( Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Exogenously determined. if (i) ( p \pi \geq 1 ) ( 1 ) ( 0 )</td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>( \pi(\alpha) ) with ( \pi' &lt; 0, \pi'' &gt; 0 ) ( 0 &lt; \alpha &lt; 1 ) zero positive positive zero</td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>( \pi(\alpha, Y) ) with</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \pi_\alpha &lt; 0, \pi_\gamma &gt; 0 ) ( 0 &lt; \alpha &lt; 1 ) zero positive positive positive</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \pi_{ij} &gt; 0 ) for ( i=j ) ( 0 ) for ( i \neq j )</td>
<td></td>
</tr>
<tr>
<td>D1</td>
<td>( \pi(D) ) where ( D = (1-\alpha)\theta Y ) ( 0 &lt; \alpha &lt; 1 ) positive positive positive positive</td>
<td></td>
</tr>
</tbody>
</table>

\( 0 < \alpha < 1 \) zero positive positive positive zero
<table>
<thead>
<tr>
<th>Case</th>
<th>Penalty</th>
<th>Status</th>
<th>Movement of the $\alpha$ with respect to</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>rate, $\pi$</td>
<td>$\theta$</td>
<td>$p$</td>
</tr>
<tr>
<td>A2</td>
<td>Exogenously determined.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(i) $(1-k)^n \geq 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ii) $(1-k)^n &lt; 1$</td>
<td>$0 &lt; \alpha &lt; 1$</td>
<td>zero</td>
</tr>
<tr>
<td>B2</td>
<td>$\pi(\alpha)$ with $\pi &lt; 0$, $\pi'' &gt; 0$</td>
<td>$0 &lt; \alpha &lt; 1$</td>
<td>zero</td>
</tr>
<tr>
<td></td>
<td>if $\alpha \leq 1/2$; ambiguous in other cases</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>$\pi(\alpha,Y)$ with $\alpha &lt; 0$, $\pi_Y &gt; 0$</td>
<td>$0 &lt; \alpha &lt; 1$</td>
<td>zero</td>
</tr>
<tr>
<td></td>
<td>$\pi_{ij} &gt; 0$ for $i=j$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 0$ for $i \neq j$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D2</td>
<td>$\pi(D)$ where $D = (1-\alpha)\theta Y$ with $\pi'$, $\pi'' &gt; 0$</td>
<td>$0 &lt; \alpha &lt; 1$</td>
<td>positive</td>
</tr>
<tr>
<td></td>
<td>if $\alpha \geq 1/2$; ambiguous in other cases</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 3

Audit Regime Where Probability of Audit $p$ is

$$p = p(Y) \text{ with } p' > 0, \quad p'' > 0 \quad \text{and} \quad p(Y) = 0 \quad \text{for} \quad Y < Y_L$$

<table>
<thead>
<tr>
<th>Case</th>
<th>Penalty</th>
<th>Status</th>
<th>Movement of the $\alpha$ with respect to $\theta$</th>
<th>$p$</th>
<th>$\pi$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A3</td>
<td>Exogenously determined.</td>
<td>none-or-all revelation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B3</td>
<td>$\pi(\alpha)$ with $\pi' &lt; 0$, $\pi'' &gt; 0$</td>
<td>$0 &lt; \alpha &lt; 1$</td>
<td>zero</td>
<td>positive</td>
<td>positive</td>
<td>positive</td>
</tr>
<tr>
<td>C3</td>
<td>$\pi(\alpha, Y)$ with $\pi'_\alpha &lt; 0$, $\pi'_Y &gt; 0$</td>
<td>$\pi_{ij} &gt; 0$ for $i=j$</td>
<td>$0 &lt; \alpha &lt; 1$</td>
<td>zero</td>
<td>positive</td>
<td>positive</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\pi_{ij} = 0$ for $i \neq j$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D3</td>
<td>$\pi(D)$ where $D = (1-\alpha)\theta Y$ with $\pi', \pi'' &gt; 0$</td>
<td>$0 &lt; \alpha &lt; 1$</td>
<td>positive</td>
<td>positive</td>
<td>positive</td>
<td>positive</td>
</tr>
</tbody>
</table>
Summary and Conclusion

In all the three audit regimes under consideration we find that in situations of exogenously determined penalty rate, there are probable situations of TPs being compulsively honest due to the lethal combination of some critical penalty rate along with the prevailing audit regime. It is another matter that such situations may neither be politically feasible nor socially acceptable. In other situations i.e. where an endogenous penalty rate has been combined with the prevailing audit regime, the optimum compliance rate $\alpha^*$ is found to be less than unity.

For such cases (where $\alpha^* < 1$), it makes economic sense to study the movement of $\alpha^*$ with respect to each of the tax parameters namely tax rate, probability of audit, penalty rate and the income level of the concerned TP. In our study in all but one case, we find that optimum compliance rate is not responsive to changes in the tax rate. In the case where penalty rate was taken to be an increasing positive function of deficient tax (or the amount of the tax evaded) a tax hike brings about an improvement in tax compliance. This result runs quite contrary to the general belief that a lower tax rate means better compliance. It may be noted here that in a study of an aggregate empirical model of evasion in the U.S., Crane-Nourzad (1987) found marginal tax rates to be positively related to evasion whereas the
average tax rate, negatively related. For the particular situation under consideration in our study, it may be that the higher penalty outstrips the additional tax to be borne by the TP from the tax-hike. The tax authority could not have asked for a better situation than the present one which is a situation of improved tax ethics (and hence of cleaner social environment) along with a much enhanced revenue collection. The enhanced revenue collection is partly from the hike in tax rate and partly from the rise in compliance level.

In regard to effect of $p$ on $a^*$, we obtain an interesting result in the audit regime where TP believes that $p$ is negatively related to $a$. In case B2 and case C2, we find that if the initial optimum compliance level $a^*$ is less than or equal to half, the TP is seen to respond to an increase in audit probability by lowering his compliance level. This is quite in contrary to the standard result found in literature (that of a higher audit inducing the TP to improve his compliance level and become more honest). Again for case D2, we can talk of compliance responding positively to an increase in probability of audit only if $a^* \geq 1/2$.

In all the situations under consideration, the penalty rate is found to be a very effective deterrent to tax evasion i.e to say that an increase in penalty rate, in a
ceteris paribus condition, leads to a higher compliance rate from the TP. It is also seen that a rich TP can be made more responsive to the prevailing tax policy if either the audit probability or the penalty is made a positive function of the income level.

In our model, the possibility of partial detection is excluded. Wadhawan (1992) pointed out that in real world where the tax authorities typically have limited information about the TP (and additional information can only be obtained at some cost), partial detection is a strong possibility. Wadhawan shows that in a world of partial detection, it is possible that under certain conditions, irrespective of magnitude of penalty, it is not optimal for a TP to act honestly. It can easily be shown that incorporation of the possibility of partial detection in our model in the line of Wadhawan would not disturb any of the results obtained here.
Appendix

Theorem: If \( f(x) \) is a function such that (1) \( f'(x) \) exists and is continuous over \( [a,b] \) and (2) \( f'(a) > 0 > f'(b) \), then the maximum of \( f(x) \) occurs at \( x^* \in (a,b) \) such that \( f'(x^*) = 0 \).

Proof: Since \( f(x) \) is continuous over \( [a,b] \), \( f() \) attains its maximum in \( [a,b] \) (a compact set). Let \( x^* \in [a,b] \) be such that \( f(x^*) > f(x) \) for all \( x \) in \( [a,b] \).

Now, \( x^* \neq a \). (because \( f(a+t) > f(a) \), for \( t > 0 \) and small) and \( x^* \neq b \). (because \( f(b-t) > f(b) \), for \( t > 0 \) and small)

Therefore, \( x^* \in (a,b) \)

Also, if \( f'(x^*) \neq 0 \), then for small \( \delta \),

\( (x^* - \delta, x^* + \delta) \subseteq [a,b] \) and there is \( x \in (x^* - \delta, x^* + \delta) \) such that \( f(x) > f(x^*) \). Hence, \( f'(x^*) = 0 \).

Location of \( x^* \)

Define \( M = \{ x \in (a,b) : f'(x) = 0 \} \)

By (1) & (2), \( M \neq \emptyset \). Also \( x \in M \Rightarrow a < x < b \).

Therefore, \( \bar{x}^* = \sup x \) and \( \underline{x}^* = \inf x \)

\( x \in M \)

are well defined.

Now, because \( M \) is closed, \( x^*, \bar{x}^* \in M \)

Clearly, \( x^* < x^* \Rightarrow f'(x^*) > 0 \)

and \( \bar{x}^* > x^* \Rightarrow f'(x^*) < 0 \)

Thus, \( x^* \in (x^*, \bar{x}^*) \supseteq M \)
Therefore, solve \( f'(x) = 0 \), i.e. find the set \( M \) and locate the maximum of \( f(x) \) over \( M \). If \( M \) contains an unique point, then \( x^* = x^* = \bar{x}^* \).