SOME MATHEMATICAL MODELS FOR TRANSFORMATION OF SENTENCES FROM ONE FORM TO OTHER

Formal work in linguistics has both produced and used important mathematical tools. Motivated by a survey of models for context and word meaning, syntactic categories, phrase structure rules and trees, an attempt is being made in this chapter to present some mathematical models for structuring of sentences from active voice to passive voice and interchange of affirmative & negative sentence, which is the form of a transitive verb whose grammatical subject serves as the patient, receiving the action of the verb.

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- Parts of this paper is appeared in online: The Computing Research Repository - CORR 2009, CoRR, as following form-

For this purpose we have parsed all sentences of a corpus and have generated groups for each of them. It has been observed that when we take constituents of the sentences as subgroups, the sequences of phrases form permutation groups. Application of isomorphism property yields permutation mapping between the important subgroups. It has resulted in a model for transformation of sentences from active voice to passive voice. A computer program has been written to enable the software developers to evolve grammar software for sentence transformations.

Mathematical structures have been built to demonstrate various forms of sentence and transformation have been developed to transform of sentences from one form to other and thus verified by taking suitable examples.

4.1 Introduction

The researchers at computer Science Department at Stanford University had worked on development of computer programs for accepting and manipulating transformational grammars corresponding to a version of the theory based on Chomsky’s aspects of the theory of syntax. This project (1968) has made some interesting contributions to linguistic theory, particularly in the area of formal definitions of grammars, lexical insertion and traffic rules for transformations. In an attempt to obtain an effective rule for distinguishing sentences from non-sentences, Bargelli and Lambek (2001) have discussed the Mathematics of sentence structure. The arena of statistical analysis of texts has been applicable in information retrieval and natural language processing in general. In this context, we can cite the research paper of Dunning (1993), who has described the basis of a measure based on likelihood
ratios for the analysis of a text. With the interest on categorical modeling of the cognitive abilities underlying the acquisition, use and understanding of natural language, Michael Moortgat (2002) has addressed two central questions of categorical grammar, the invariants of grammatical composition and the uniformity of the form/meaning correspondence across languages. Karin Muller (2002) has presented a probabilistic context free grammar, which describes the word and syllable structure of German words. Jakoboson (1961) had used mathematical approach in his book to define language and its structure in sentences.

Sentences can be written or spoken in the active or passive voice. In the active voice, the subject of the sentence acts upon something or someone. In the passive voice, the subject is acted upon. Active voice is distinguished from the passive voice by the identity of the actor. As a general rule, active voice is preferred because it meets two of the most important requirements of legal writing: clarity and conciseness. Active voice is clear because it focuses the reader’s attention on the “doer of the action” it is also more concise simply because it usually involves fewer words. The passive voice is a very versatile construction. It is particularly useful when the performer of the action is unknown or irrelevant to the matter of the hand. Scientist ordinarily uses the passive voice to describe natural processes or phenomena under study. In technical and scientific articles, especially in the presentation of experimental methods, researchers use the passive voice as a convenient means of impersonal reporting. The passive voice allows them to avoid calling attention to themselves and to omit reference to any subjective thoughts or biases they might have brought to their work. The effect is to land the article the
air of objectivity. In English as in many other languages, the passive voice is the form of a transitive verb whose grammatical subject serves as the patient, receiving the action of the verb. In transformation from an active-voice clause to an equivalent passive-voice construction, the direct object switch grammatical role. The direct object gets promoted to subject, and the subject demoted to an (optional) complement.

The book of Marcus Kracht (2003) contains an account of studies in language and linguistic theories from a mathematical point of view. Jean mark Gawron (2004) has defined the relations in linguistic aspects. The author has also generated some sets and relations for English Obstruents and has also suggested function approach for voice transformation in his book “Mathematical Linguistics”.

This paper is an answer to the need of the students who use the English language as matter of course, but face difficulty in applying grammatical techniques in syntactic theory, like that of transformation of sentences from active voice to passive voice. Whether mathematics can have any answer to this language phenomenon has been the concern of this paper.

4.2 Modus Operandi

We define passivization of a sentence as-

Passivization: Transitive sentence $\rightarrow$ Sentence.

Passivization: S $\rightarrow$ the passive version of S.

As an illustration, we can cite the following example-

Transitive: John ate the bagel.
The bagel was eaten by John

We have made an attempt to transform a string ‘a’ of active voice words into a string of passive voice words ‘p’. We are dealing it with the help of a mathematical tools based on algebraic properties.

4.3 Structuring of Affirmative Sentences from Active Voice to Passive Voice

The basic sentence structure for the English language form follows a SVO pattern in affirmative, which means that the sentence begins with a subject (S) or something performing an action, followed by a verb (V) or the action, followed by an object (O) something that receives the action. The sentence in passive voice may be treated as the one having OVS pattern.

We can consider a set N defined as –

N = {i, we you, he, she, they, me, us, you, him, her, them, noun}

--------- (4.3.1)

Which can generate a cyclic group of subjects and objects order 13, as demonstrated in following table-

<table>
<thead>
<tr>
<th>I</th>
<th>We</th>
<th>Yo</th>
<th>He</th>
<th>She</th>
<th>They</th>
<th>The</th>
<th>He</th>
<th>Hi</th>
<th>Yo</th>
<th>Us</th>
<th>Me</th>
<th>Noun</th>
</tr>
</thead>
<tbody>
<tr>
<td>a¹</td>
<td>a²</td>
<td>a³</td>
<td>a⁴</td>
<td>a⁵</td>
<td>a⁶</td>
<td>a⁻⁶</td>
<td>a⁻⁵</td>
<td>a⁻⁴</td>
<td>a⁻³</td>
<td>a⁻²</td>
<td>a⁻¹</td>
<td>a⁰ = e</td>
</tr>
</tbody>
</table>

Table (4.3.1) - Group elements for subject/ object
Since \( a^1 * a^{-1} = e \) algebraically, so inverse of ‘i’ can be assumed to be ‘me’ and similarly for others.

Considering the set of articles

\[ A = \{ \text{the, a, an} \} \]

We have formed an algebraic structure \( S \) for \( N \) and \( A \) in which \( N \) is closed with respect to an operation (say connection) with \( A \), as

The – article, Policeman – subject \( \Rightarrow \) the policeman – subject

Taking a set of all verbs according to their forms as

\[ V = \{ v_{11}, v_{12}, v_{13}, v_{14}, v_{21}, v_{22}, v_{23}, v_{24}, v_{31}, v_{32}, v_{33}, v_{34}, v_{11}^{-1}, v_{12}^{-1}, v_{13}^{-1}, v_{14}^{-1}, v_{21}^{-1}, v_{22}^{-1}, v_{23}^{-1}, v_{24}^{-1}, v_{31}^{-1}, v_{32}^{-1}, v_{33}^{-1}, v_{34}^{-1} \} \]

In \( v_{ij} \) \( i \) represents tense and \( j \) represents form of tense.

Forms of actual elements for the verb ‘write’ can be depicted as under-
### Table (4.3.2) – Table for verb elements in verb Group structure.

<table>
<thead>
<tr>
<th>Verb form</th>
<th>Inverse element</th>
</tr>
</thead>
<tbody>
<tr>
<td>v₁₁</td>
<td>Write</td>
</tr>
<tr>
<td>v₁₂</td>
<td>Is, am, are writing</td>
</tr>
<tr>
<td>v₁₃</td>
<td>Have, has written</td>
</tr>
<tr>
<td>v₁₄</td>
<td>Have been, has been written</td>
</tr>
<tr>
<td>v₂₁</td>
<td>Wrote</td>
</tr>
<tr>
<td>v₂₂</td>
<td>Was, were writing</td>
</tr>
<tr>
<td>v₂₃</td>
<td>Had written</td>
</tr>
<tr>
<td>v₂₄</td>
<td>Had been written</td>
</tr>
<tr>
<td>v₃₁</td>
<td>Shall, will Write</td>
</tr>
<tr>
<td>v₃₂</td>
<td>Shall be, will be going</td>
</tr>
<tr>
<td>v₃₃</td>
<td>Shall have, will have written</td>
</tr>
<tr>
<td>v₃₄</td>
<td>Shall have been, will have been written</td>
</tr>
</tbody>
</table>

We can now form an algebraic space for an affirmative sentence with the help of three earlier defined structures S, V and O, where O is same as the group of subjects S.

Let \( a = (S)\#(V)\*(O) \),

\[
\text{(4.3.4)}
\]

Where S stands for subject elements, V for verb elements and O for object elements. \# and * represent respective exterior operations between them., for example in the sentence

“The policeman has caught the thief.”
the structure $a$ can be permuted in 6 ways, which gives a new string of elements. Out of them one string is OVS.

Defining a transformation in-group $S$ as

$$f(a^i) = (a^i)^{-1}$$

we can take a mapping in set $V$

$$g(v_{ij}) = (v_{ij})^{-1}$$

Since four elements of the set $V$, defined by table 4.3.2, have not their inverses and thus it can be said that those elements map into null element. For these elements we form a new set named kernel of mapping, defined as

$$K(g) = \{v_{ij} : g(v_{ij}) = \phi\} = \{v_{14}, v_{24}, v_{32}, v_{34}\}.$$  

Transformation of algebraic structure $a$, defined by (4.3.4) shall be

$$T(a) = T(S_i \# V_{jk} \ast O_p) = (O_p^{-1} \otimes V_{jk} \ast S_i^{-1}) = p$$  

Where $V^\prime$ represents the third form of verb.

Defining two exterior operations $\otimes$ and $\ast$, we can form a new algebraic structure

$$P = (O_p^{-1} \otimes V_{jk} \ast S_i^{-1})$$

which is non commutative under these operations.

In result (1.9), $\otimes$ is defined as
\[ \otimes = A_{jk,p^{-1}} \]  

(4.3.10)

and * is a conjunction operator defined between verb and subject elements.

The values of \( A_{jk,p^{-1}} \) for particular objects and verbs defined in active voice shall be structured as elucidated in the following table-

*Table 4.3.4- Value of \( A_{jk,p^{-1}} \) used in the transformation from active to passive voice.*

<table>
<thead>
<tr>
<th>p ( jk )</th>
<th>1 I</th>
<th>2 We</th>
<th>3 You</th>
<th>4 He</th>
<th>5 She</th>
<th>6 They</th>
<th>7 Noun</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 Am</td>
<td>Are</td>
<td>Are</td>
<td>Is</td>
<td>Is</td>
<td>Are</td>
<td>Is</td>
<td>Are</td>
</tr>
<tr>
<td>12 Am</td>
<td>Are</td>
<td>Are</td>
<td>Is</td>
<td>Is</td>
<td>Are</td>
<td>Is</td>
<td>Is, are</td>
</tr>
<tr>
<td>13 Have</td>
<td>Have</td>
<td>Have</td>
<td>Has</td>
<td>Has</td>
<td>Have</td>
<td>Has, have</td>
<td></td>
</tr>
<tr>
<td>21 Was</td>
<td>Were</td>
<td>Were</td>
<td>Was</td>
<td>Was</td>
<td>Were</td>
<td>Was, were</td>
<td></td>
</tr>
<tr>
<td>22 Was</td>
<td>Were</td>
<td>Were</td>
<td>Was</td>
<td>Was</td>
<td>Were</td>
<td>Was, were</td>
<td></td>
</tr>
<tr>
<td>23 Had</td>
<td>Had</td>
<td>Had</td>
<td>Had</td>
<td>Had</td>
<td>Had</td>
<td>Had</td>
<td></td>
</tr>
<tr>
<td>31 Shall</td>
<td>Will</td>
<td>Will</td>
<td>Will</td>
<td>Will</td>
<td>Will</td>
<td>Will</td>
<td></td>
</tr>
<tr>
<td>33 Shall have</td>
<td>Will have</td>
<td>Will have</td>
<td>Will have</td>
<td>Will have</td>
<td>Will have</td>
<td>Will have</td>
<td></td>
</tr>
</tbody>
</table>
Equation (4.3.8) gives a rule by which active voice can transform into passive voice.

This transformation is an exact rule for simple affirmative sentences. By taking an example we can easily conclude this.

Active:- They are looking the movie.

In this sentence ‘they’ is subject by table 1 its value is \(a^6\), ‘are looking’ is verb by table 2 its value is \(v_{12}\) and the is article and movie is a noun but for we have earlier prove that the combination of article and noun is again in the category of subject/ object. Hence ‘the movie’ is an object. By table (4.3.1) inverse of \(a^6\) is \(a^7\), by table (4.3.2) “is looking” is mapped in being looked and again by table (4.3.1) inverse of ‘the movie’ is ‘the movie’.

From equation (4.3.8), if we put the values in this we have,

\[
T(a) = T(a_6 \# v_{12} * a_{13}) = (a_{13}^{-1} \otimes v_{12} * a_6^{-1}) = (a_{13} \otimes v_{12} * a_7) = p
\]

Now from table 3, \(\otimes\) can be finding.

Again retrieving sentence by giving the values to elements in transformation, we have

Passive:- The movie is being looked by them.

A negative sentence can be braked into three parts S+ V\(^{-}\)+O

Where, \(V^{-}\) is the set of verbs in negative forms.

If we want to transform a negative sentence from active voice to passive voice, first we convert a negative sentence in to positive form then transformed it from active to passive voice, now we convert this affirmative passive sentence in to negative sentence.
4.4 Mathematical structure of negative sentence

Mathematically we can represent a negative sentence as an algebraic space together with two operations ^ and * having any one of the forms, given as under:

\[ i. \quad (S^\sim \, V \, * \, O) \]
\[ ii. \quad (S \, V \, * \, \sim \, O) \]  \hspace{1cm} \text{---------------- (4.4.1)}

where \( ^\sim = B_{p,ij} \)

\( B_{p,ij} \) shall be prearranged as elucidated in the following table:

\textit{Table 4.4.1 - ^ operator used for the construction of negative sentence}

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>I</td>
<td>We</td>
<td>You</td>
<td>He</td>
<td>She</td>
<td>They</td>
<td>Noun</td>
</tr>
<tr>
<td>ij</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>do</td>
<td>do</td>
<td>do</td>
<td>does</td>
<td>does</td>
<td>do</td>
<td>does, do</td>
</tr>
<tr>
<td>12</td>
<td>am</td>
<td>are</td>
<td>are</td>
<td>is</td>
<td>is</td>
<td>are</td>
<td>is, are</td>
</tr>
<tr>
<td>13</td>
<td>have</td>
<td>have</td>
<td>have</td>
<td>has</td>
<td>has</td>
<td>have</td>
<td>has, have</td>
</tr>
<tr>
<td>14</td>
<td>have been</td>
<td>have been</td>
<td>have been</td>
<td>have been</td>
<td>have been</td>
<td>have been</td>
<td>has been/have been</td>
</tr>
<tr>
<td>21</td>
<td>did</td>
<td>did</td>
<td>did</td>
<td>did</td>
<td>did</td>
<td>did</td>
<td>did</td>
</tr>
<tr>
<td>22</td>
<td>was</td>
<td>were</td>
<td>were</td>
<td>was</td>
<td>were</td>
<td>was</td>
<td>was, were</td>
</tr>
<tr>
<td>23</td>
<td>had</td>
<td>had</td>
<td>had</td>
<td>had</td>
<td>had</td>
<td>had</td>
<td>had</td>
</tr>
<tr>
<td>24</td>
<td>had been</td>
<td>had been</td>
<td>had been</td>
<td>had been</td>
<td>had been</td>
<td>had been</td>
<td>had been</td>
</tr>
<tr>
<td>31</td>
<td>shall</td>
<td>will</td>
<td>will</td>
<td>will</td>
<td>Will</td>
<td>will</td>
<td>will</td>
</tr>
<tr>
<td>32</td>
<td>shall be</td>
<td>will be</td>
<td>will be</td>
<td>will be</td>
<td>will be</td>
<td>will be</td>
<td>will be</td>
</tr>
<tr>
<td>33</td>
<td>shall</td>
<td>will</td>
<td>will</td>
<td>will</td>
<td>will</td>
<td>will</td>
<td>will have</td>
</tr>
</tbody>
</table>
Here $S$ the set of subject forms a group, as discussed in section (4.3), $V$ the set of verbs forms a group under *negation* and $O$ the set of others also forms a group under *negation*.

Elements and their inverses of group $V$ has been taken as:

*Table 4.4.2- inverse elements for verb group of negative sentence*

<table>
<thead>
<tr>
<th>Element of $V$ ($v$)</th>
<th>Inverse ($~v$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>go</td>
<td>not go</td>
</tr>
<tr>
<td>goes</td>
<td>not goes</td>
</tr>
<tr>
<td>going</td>
<td>not going</td>
</tr>
<tr>
<td>went</td>
<td>not went</td>
</tr>
<tr>
<td>gone</td>
<td>not gone</td>
</tr>
</tbody>
</table>

In group $O$, antonym ($~o$) of the word $o \in O$, is the inverse of $o$.

As an example, let us consider a word “fast” whose antonym is “slow” but “not fast” is the negation of fast and therefore should be considered as an inverse of fast, but due to the uniqueness of inverse element in a group, both slow and not fast must be considered equivalent.

On an exemplary basis, we can formulate a negative sentence as:

**S-** I (s)

**V-** not found ($~v$)

**O-** the road very bad. (o)
In order to form a negative sentence mathematically, it can be structured as:

\[(s)^{\wedge}(\sim V)^{*}(O)\], that is,

I \wedge not found* the road very bad.

After applying the operations we have

“I did not find the road very bad”

Another example to illustrate the point more clearly, can be given as under:

The knife (s)

V- is (v)

O- blunt (~o)

The knife \( \cdot \) is *~ blunt = the knife is not blunt.

4.5 Interchange of affirmative and negative sentence

The process of interchange can be assumed to take place under following two transformations:

\[T(\text{negative}) = \text{affirmative}\]

\[i. \quad T_1(S_p^{\wedge} \sim V_{lm}^{*} O) = (S_p \cdot V_{lm}^{*} \sim O) \]

\[\hat{i}i. \quad T_2(S_p \cdot V_{ij}^{*} \sim O) = (S_p \cdot V_{ij}^{*} O^{-1}) \]

\[\text{--------- (4.5.1)}\]

To authenticate this transformation rule, let us consider a negative sentence:

“I did not find the road very bad”

In this sentence the verb is in negative form hence it is first type of negative sentence. Hence the mathematical form of this sentence is

\[S_1^{\wedge} \sim V*O, \text{ where}\]

S- I
Applying the transformation $T_1$ we get:

$$(S_1 \cdot V_{21} * \sim O),$$

Antonym of *very bad* is *very good* which is equivalent to *not very bad*, hence $\sim o = \text{not very bad}$.

Also

$S$- I

$V$- found

$\sim O$- the road not very bad

Hence the affirmative sentence is obtained as- “I found the road not very bad”.

Let we take another negative sentence “the knife is not blunt”.

This is the second type of negative sentence in which adjective is in negation. Its algebraic structure can be taken as :

$$(S_p \cdot V_{ij} * \sim O)$$

Where,

$S$- the knife

$V$- is

$\sim O$- not blunt

The sentence in affirmative form can be found after applying the transformation $T_2$, i.e. $(S_p \cdot V_{ij} * O^{-1})$

*Not blunt* is inverse of *blunt* and antonym of *blunt* is *sharp*.

Hence $O = \text{sharp}$.

Hence the affirmative sentence comes out as-

“The knife is sharp.”
By taking the inverse transformation defined by equation (4.4.1), we can convert an affirmative sentence into negative sentence.

The demonstration of the process can be done for following sentence:

“*He came in time*”

This sentence is in the form \((S_4 \cdot V_{21}^* \sim O)\)

Applying the inverse transformation

\[
T_1^{-1}(S_p \cdot V_{ij} \sim O) = (S_p^\sim V_{lm}^*O),
\]

we get:

\[
T_1^{-1}(S_4 \cdot V_{21} \sim O) = (S_4^\sim V_{21}^* O)
\]

\[
T_1^{-1}(\text{He. can. in time}) = (\text{He \sim come* late})
\]

Which gives- “*He did not come late*”
The figurative representation of the model for transformation of sentence is as:

Fig. (4.5.1)- Pictorial depiction of the Transformation of sentence

4.6 Conclusion

Linguists seek to understand the properties of all natural human languages - how they are structured, how and why they vary and change, how they are acquired, and how they are used by people to communicate. Mathematics, on the other hand, is the language of science. Mathematics
abstracts the fundamental issue at the heart of an example, frequently finding connections with other, initially dissimilar, problems.

Therefore Linguistics and mathematics can be studied together quite naturally. They both try to investigate and appreciate patterns and structure for approaching the study in a rigorous and systematic manner.

In this paper we have defined some mathematical models for the sentence transformations, these models can help to understand the grammatical rules for those who face difficulties to use grammar rules, this study also provide a software for the transformation of a sentence from active to passive Voice. One can use this software to convert an affirmative sentence from active voice to passive voice. This software is fully based on the mathematical model suggested by us. A further work can be done to develop new software for all types of sentence transformation.