CHAPTER 4

SINGLE WEIGHT ZERO CROSS-CORRELATION CODES

4.1 INTRODUCTION

Designing of suitable spread sequence codes is very important issue in the research of SAC-OCDMA communication systems. Various optical codes are proposed for use in SAC-OCDMA systems. Development of codes primarily developed upon the code parameters such as code length, code weight and the cross-correlation values.

Some of the codes are designed giving importance to the length of the code word. Such code families are, modified prime sequence codes (Kwong et al 1991), padded modified prime codes (Liu & Taso 2000), new-modified prime codes (Liu et al 2007), double-padded modified prime codes (Karbassian & Ghafouri-Shiraz 2007) and enhanced-modified prime codes (Lalmahomed et al 2010). All these codes have longer code length and weight that lead to lower spectral efficiency, lower throughput, higher power consumption and complex transceivers.

Some of the code development procedure depends on code weight (Zhang et al 2000, Murugesan 2004 and Jau & Lee 2003). But, none of these codes are designed with weight equal to one.

Some other codes are designed mainly to provide better correlation functions and hence to reduce MUI and PIIN. Modified quadratic congruence
codes (Wei et al 2001), modified frequency hopping codes (Wei & Ghafouri 2002), modified double weight codes (Aljunid et al 2004) and extended modified prime sequence codes (Murugesan & Ravichandran 2004) are proposed with ideal IPCC value. PMP codes (Lin et al 2005) have the lower IPCC value (≤1). A new code family namely zero cross-correlation codes is proposed by Anuar et al (2007) but its code length is longer. These are some of the codes found in the literature.

In this chapter, a new family of code is designed called single weight ZCC (SWZCC) codes with shorter code length, zero cross-correlation value, unity code weight and with simple code construction procedure. The performance of the SAC-OCDMA system is analyzed with the proposed code by considering the effect of shot noise and thermal noise. Further, the performance is compared with MQC and PMP codes. Table 4.1 compares the performance parameters of different code families for the number of users (cardinality) K=30.

Table 4.1  Comparison of parameters between different code families for 30 users

<table>
<thead>
<tr>
<th>Type of code Family</th>
<th>Code Weight (w)</th>
<th>Code Length (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OOC</td>
<td>4</td>
<td>364</td>
</tr>
<tr>
<td>Prime code</td>
<td>31</td>
<td>961</td>
</tr>
<tr>
<td>Hadamard code</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>MDW code</td>
<td>4</td>
<td>90</td>
</tr>
<tr>
<td>MFH code</td>
<td>7</td>
<td>42</td>
</tr>
<tr>
<td>ZCC code</td>
<td>4</td>
<td>120</td>
</tr>
<tr>
<td>SWZCC code</td>
<td>1</td>
<td>30</td>
</tr>
</tbody>
</table>
4.2 DESIGNING OF SWZCC CODES

The SWZCC code can be obtained from the identity matrix with size greater than one. In this, the identity matrix is considered as the basic SWZCC code family. The row and column of the identity matrix respectively represents the size and length of the code. Since the number of one’s present in each row of the identity matrix is one, the number of common ones between two code words is zero. Therefore, the identity matrix can be called as zero cross-correlation matrix. The number of users for the given matrix size $M_s$ can be increased by rotating the identity matrix by $M_s - 1$ times. Finally, code spreading technique is used in order to generate the codes with zero cross-correlation value.

The steps involved in the generation of the zero cross-correlation codes are given below.

Step 1:

First generate a basic ZCC identity matrix. For example the basic $3 \times 3$ identity matrix can be represented by

$$D1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 2:

To increase the number of simultaneous users, using time shifting property rotate the identity matrix by one unit. The number of rotations required for the $3 \times 3$ matrix is 2. So, $M_s - 1$ numbers of rotations required for an $M_s \times M_s$ matrix. Here, $M_s$ is the size of SWZCC matrix. The resultant matrix $D2$ is shown below.
In general $D_2$ for the $3 \times 3$ $D_1$ matrix can be represented by

$$D_2 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 \\
\end{bmatrix}$$

and contains 3 numbers of sub matrices where the sub matrices are

$$a_1 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix} \quad a_2 = \begin{bmatrix}
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
\end{bmatrix} \quad a_3 = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 \\
\end{bmatrix}$$

Generally an $M_s \times M_s$ matrix will have an $M_s$ number of sub matrices.

Step 3:

The code size of matrix $D_2$ is increased from three to nine after performing time shifting. But, the number of common one’s occupied by the two codes between the sub matrixes $a$, $b$ and $c$ is equal to one i.e., correlation of the code is one. Hence to reduce the correlation value from one to zero, code spreading technique is used in this matrix ($D_2$). For that first an 9 numbers of $3 \times 3$ zeros matrix is generated, 3 from each sub matrices like $a_{11}$, $a_{12}$, $a_{13}$, $a_{21}$, $a_{22}$, $a_{23}$, $a_{31}$, $a_{32}$ and $a_{33}$.
Finally, arrange the $a_1$ matrix elements as the diagonal elements for the first $3 \times 9$ matrix of $D$. Similarly spread the $a_2$ and $a_3$ matrix elements as the diagonal elements for the second and third $3 \times 9$ matrix of $D$.

$$D_{3 \times 3} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$

In the above matrix number of rows represents code size (cardinality) and columns represents the code length of the code. Thus, a family of codes for 9 users is generated from $3 \times 3$ basic matrix. The length of the code is 9 and the weight is equal to one. In general, from $M_s \times M_s$ matrix, $M_s^2$ number of codes is generated with length $M_s^2$. Since the length of the code and size of the code are equal, the proposed codes are the shorter length codes than the other codes found in the literature. In addition, cross-correlation value of the proposed code is always zero independent of code size. Also, the weight of the code is always equal to one for all users in the code family. This code is named as single weight zero cross-correlation (SWZCC) codes.

The expressions for the cardinality of the code ($K$) and code length ($N$) of the proposed SWZCC code are given by
\[ K = M_s^2 \]  
\[ N = M_s^2 \]

and thus \[ K = N. \]

The important properties of the proposed SWZCC code and the other codes are given in Table 4.2.

**Table 4.2 Comparison of properties of SWZCC codes and other code families**

<table>
<thead>
<tr>
<th>Parameters / Code Families</th>
<th>Cross-Correlation ( \lambda_{x,y} (x \neq y) )</th>
<th>Auto-Correlation ( \lambda_{x,y} (x = y) )</th>
<th>Code Cardinality ( K )</th>
<th>Code Length ( N )</th>
<th>Code Weight ( w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MQC Codes</td>
<td>1</td>
<td>( p+1 ) ((p \text{ is a prime number)})</td>
<td>( p^2 )</td>
<td>( p^2+p )</td>
<td>( p+1 )</td>
</tr>
<tr>
<td>MPS Codes</td>
<td>0-within the group</td>
<td>( P )</td>
<td>( p^2 )</td>
<td>( p^2 )</td>
<td>( p )</td>
</tr>
<tr>
<td></td>
<td>1-between the groups</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EMPS Codes</td>
<td>1-within the group</td>
<td>( p+1 )</td>
<td>( p^2 )</td>
<td>( p^2+p )</td>
<td>( p+1 )</td>
</tr>
<tr>
<td></td>
<td>and between the groups</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PMP Codes</td>
<td>0-within the group</td>
<td>( (p-1)/M )</td>
<td>( M p^2 )</td>
<td>( p^2 )</td>
<td>( (p-1)/M )</td>
</tr>
<tr>
<td></td>
<td>( \leq 1 )-between the groups</td>
<td>((p-1)/M)+1 ) (M is a factor of (p-1))</td>
<td>( M p^2 )</td>
<td>( p^2+p )</td>
<td>( (p-1)/M)+1 )</td>
</tr>
<tr>
<td>SEMP Codes</td>
<td>( \leq 1 ) within the group</td>
<td>((p-1/M)+1 ) (M is a factor of (p-1))</td>
<td>( M p^2 )</td>
<td>( p^2+p )</td>
<td>( (p-1/M)+1 )</td>
</tr>
<tr>
<td></td>
<td>and between the groups</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SWZCC Codes</td>
<td>0</td>
<td>1</td>
<td>( M_s^2 )</td>
<td>( M_s^2 )</td>
<td>1</td>
</tr>
</tbody>
</table>
From the table, it is observed that the proposed SWZCC code has better correlation properties than the other codes. Though the size of SWZCC code is lower compared to PMP and SEMP codes for the given code length and for the given prime number \( p \) or for the matrix size \( M_s \), its zero cross-correlation property has an ability to eliminate the PIIN noise completely and thus will improve the system performance. Since the code weight of the SWZCC code is always one, the number of fiber bragg gratings required to construct the encoder /decoder structure is one. Further, the code family can be designed for any integer number greater than one.

### 4.3 PERFORMANCE ANALYSIS

The bit error rate performance of the system with the proposed SWZCC code is analyzed by taking into consideration of shot noise and thermal noise only. Since the cross-correlation value of the proposed code is always zero, it is obvious that the effects of phase induced intensity noise are completely eliminated. Thus, the expressions for the signal to noise ratio and the bit error rate performance can be written as (Anuar et al 2010)

\[
\text{SNR} = \frac{K^2 \sigma_w^2}{2eB N \sigma_w^2 + 4k_BT \eta B} 
\]

\[
\text{BER} = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{\text{SNR}}{8}} \right) 
\]

where \( \text{erfc} \) - complementary error function

- \( R \) - Responsivity of the photodiode, 0.7495
- \( \eta \) - Quantum efficiency, 0.6
- \( h \) - Planks’ constant, 6.626×10^{-34} Js
e - Electron charge, \(1.602 \times 10^{-19} \text{ C}\)

\(\lambda\) - Operating wavelength, 1550 nm

B - Electrical equivalent noise band-width of the receiver, 80 MHz

\(P_s\) - Effective source power at the receiver in dBm

k - Number of simultaneous users

w - Code weight

\(K_B\) - Boltzmann’s constant, \(1.379 \times 10^{-23}\)

\(R_L\) - Load resistance in ohms 1030Ω

N - Code length

\(T_n\) - Absolute Temperature in degrees Kelvin, 300 K

Figure 4.1 depicts the bit error rate of the proposed SWZCC codes for the various values of \(M_s\) (matrix size). Each and every value of \(M_s\) supports \(M_s^2\) number of users each with code length equal to the code size. Figure 4.1 shows the error performance for \(M_s=7\), \(M_s=9\) and \(M_s=11\) for the corresponding code lengths of 49, 81 and 121 respectively. The graph is plotted for the effective source power value equal to -10dBm. From the graph it is found that the maximum number of simultaneous users supported by the system with the SWZCC code of size \(M_s=11\) is about 110 for the acceptable BER of \(10^{-9}\).
Figure 4.1  Number of simultaneous users versus BER of SWZCC codes for different values of $M_s$

In Figure 4.2, the BER performance is shown for the value of $M_s = 13$ with effective source power at receiver of -6dBm, -8dBm and -10dBm. For $M_s=13$, the code size and code length of SWZCC code is 169. For a fair comparison, the performance of MQC codes with $p=13$ and $P_s = -6$dBm is also shown. The code size and code length of MQC code when $p=13$ is 169 and 182 respectively.

From the figure, it is noticed that, for an increase in the amount of effective source power, the proposed SWZCC code produces better BER. For example, for the effective source power of -10dBm, the bit error rate of the system is approximately $10^{-5}$ for the given number of users, say $K=160$. If the effective source power is -6dBm, then the bit error rate is approximately $10^{-14}$ for the same users. Thus, it is clear that when the effective source power is increased then the performance is also increased. Furthermore, the proposed codes can able to produce much better BER results compared to MQC for the
considered effective source power of -6dBm. For instance, the number of simultaneous users supported by the system for the given bit error rate performance is also increased with respect to effective source power. For $M_s=13$, the number of simultaneous users is increased from 110 to 160 for the given BER of $10^{-9}$ when the source power is increased from -10dBm to -6dBm respectively. But, MQC supports approximately 100 numbers of simultaneous users for the given BER of $10^{-9}$ and for the effective source power of -6dBm.

Figure 4.2 Number of simultaneous users versus BER of MQC and SWZCC codes with various values of effective source power

Figure 4.3 compares the number of simultaneous users and the SNR of MQC codes and SWZCC codes. The SNR is calculated for the various values of effective source power. For the sufficient amount of effective source power (say, for -6dBm), it is obvious that the system with the proposed SWZCC codes provides higher SNR. Since the PIIN is absent in the proposed code, it shows higher SNR than the MQC codes.
Figure 4.3 SNR versus number of simultaneous users of MQC codes and SWZCC codes

Figure 4.4 presents the comparison of BER of PMP codes and SWZCC codes for the prime number \( p = 11 \) and for the matrix size \( M = 19 \) respectively. When \( p = 11 \), PMP code can support either 242 or 605 number of users. The code length of the PMP code is 121. For the given matrix size \( M = 19 \), the code size and the code length of SWZCC code is 361. Though the length of the SWZCC code is higher than the PMP code, the proposed SWZCC codes gives much better performance than the PMP code. This occurs because of the ability of the intensity noise suppression of the SWZCC code. The performance is analyzed for the effective source power of -6dBm, -8dBm and -10dBm. The BER performance of the proposed SWZCC codes is better even for -10dBm compared to the PMP codes. It is clear that, the proposed code provides much better performance than the PMP codes for the effective source power of -6dBm. Further, for the acceptable error rate of \( 10^{-9} \) and for the effective source power of -6dBm, the number of simultaneous users supported by the system with SWZCC codes is approximately 200. But, it is only 100 for the system with PMP code.
Figure 4.4  Number of simultaneous users versus BER of PMP (p=11) codes and SWZCC (Ms=19) codes for the various values of effective source power

Table 4.3 compares the calculated values of the proposed SWZCC codes and the other code families.

Table 4.3  Calculated values of parameters of SWZCC codes and other code families

<table>
<thead>
<tr>
<th>Parameters / Code Family</th>
<th>Prime number p</th>
<th>Code length N</th>
<th>Code weight w</th>
<th>Code Size K</th>
<th>Approximate k for BER of 10^{-9}</th>
</tr>
</thead>
<tbody>
<tr>
<td>MQC codes</td>
<td>p=7</td>
<td>56</td>
<td>8</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>p=11</td>
<td>132</td>
<td>12</td>
<td>121</td>
<td>75</td>
</tr>
<tr>
<td>PMP codes</td>
<td>p=7 M=2</td>
<td>49</td>
<td>3</td>
<td>98</td>
<td>98</td>
</tr>
<tr>
<td></td>
<td>M=3</td>
<td>49</td>
<td>2</td>
<td>147</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>p=11 M=2</td>
<td>121</td>
<td>5</td>
<td>242</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>M=5</td>
<td>121</td>
<td>2</td>
<td>605</td>
<td>110</td>
</tr>
<tr>
<td>SEMP codes</td>
<td>p=7 M=2</td>
<td>56</td>
<td>4</td>
<td>98</td>
<td>98</td>
</tr>
<tr>
<td></td>
<td>M=3</td>
<td>56</td>
<td>3</td>
<td>147</td>
<td>140</td>
</tr>
<tr>
<td></td>
<td>p=11 M=2</td>
<td>132</td>
<td>6</td>
<td>242</td>
<td>105</td>
</tr>
<tr>
<td></td>
<td>M=5</td>
<td>132</td>
<td>3</td>
<td>605</td>
<td>150</td>
</tr>
<tr>
<td>SWZCC codes</td>
<td>M=7</td>
<td>49</td>
<td>1</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>M=11</td>
<td>121</td>
<td>1</td>
<td>121</td>
<td>110</td>
</tr>
</tbody>
</table>
Figure 4.5 shows the variation in BER with the effective source power when the number of simultaneous users is 36. Since the cross-correlation value of the SWZCC code is zero, the code completely eliminates the phase induced intensity noise. Thus, in this system BER analysis is carried out by considering only the shot noise and thermal noise. Error rate is calculated separately for shot noise and thermal noise. For example, when the effective source power is -20dBm, the system with only shot noise produces an error rate of $10^{-14}$. At the same -20dBm point, the system with thermal noise produces much higher error rate say $10^{-3}$. Therefore, it is clear that the effect of thermal noise is dominant than shot noise on the system performance.

![Figure 4.5 BER versus Effective source power of SWZCC codes when k = 36](image)

Figure 4.5  BER versus Effective source power of SWZCC codes when k = 36

Figure 4.6 shows the BER variation with the effective source power when the number of simultaneous users is 64. When the number of active users is increased from 36 to 64, the effect of shot noise on the system
performance is also increased. For the effective source power of -20dBm, the BER produced by the system when only shot noise considered is $10^{-5}$ and the system with only thermal noise is $10^{-1}$. This rate is much higher than the system with 36 users. Thus, when the number of simultaneous users is increased then the effect of shot noise on the system performance is increased.

![Graph showing BER versus Effective source power for SWZCC codes when k = 64](image)

**Figure 4.6** BER versus Effective source power of SWZCC codes when k = 64

Figure 4.7 shows effective source power Versus BER for MQC code and SWZCC code. The graph is plotted for various numbers of simultaneous users. The proposed code shows better performance than the MQC codes. Better performance is achieved because of absence of intensity noise in SWZCC code while it is present in the MQC codes. It is observed that the MQC codes produces a constant bit error rate after reaching a particular level say -20dBm. At the same time SWZCC codes gives better performance. That is for the given number of simultaneous users and for the
sufficient amount of power the system with the proposed codes show better performance than the system with MQC codes.

![Figure 4.7 Effective source power versus BER of MQC codes and SWZCC codes](image)

**Figure 4.7 Effective source power versus BER of MQC codes and SWZCC codes**

### 4.4 SUMMARY

In this study, only simple mathematical rules are used to construct the single weight ZCC codes. Since the proposed code has zero cross-correlation value, the code completely eliminates MUI as well as PIIN. The other advantages includes simple and easy code construction procedure and encoder-decoder design, single weight, reduced code length and the system supports higher number of simultaneous users for the given bit error rate. Further, the code can be generated using simple mat lab programs. Finally the bit error rate performance of the system is much better than the system with MQC code and PMP code.