CHAPTER III

STRONGLY SEMI-CLOSED GRAPH OF A MAPPING

0. INTRODUCTION:

It seems that an extensive study, appeared in the recent past, centered around the concept of graph of a mapping, is a search for conditions under which (1) a certain mapping has a graph of requisite nature and (2) a mapping having a certain graph is of requisite nature. The graph condition of a mapping is linked, intimately, with the separation or covering or some other basic conditions enjoyed by its domain and/or its range spaces. Here, it is more appropriate to mention that a mapping of a space into a compact Hausdorff space is continuous iff $f$ has a closed graph.

Besides these, one may begin to search for a certain type of graph itself to achieve the objectives with which initially motivated. Having an idea of obtaining the characterizations of $H$-closed spaces in [12] and those of $C$-compact spaces in [12a], of course with the graph conditions, the strongly-closed graph is felt to be introduced by Herrington and Long. Also, it is found in [12] that
if the graph of a mapping is strongly-closed, then it is closed also, but not conversely. Further, \( \star \)-closed graph was introduced by Herrington and Long in \([12\ b]\) with which they succeeded in characterizing an \( H \)-closed Urysohn space. \( \Theta \)-subclosed graph has been introduced in \([12\ c]\) with an idea essentially to attain a generalization of 'uniformboundedness principle' from analysis.

In this chapter, a condition with which the graph is termed to be strongly semi-closed is introduced. It is shown that the strongly semi-closed graph is weaker than a strongly-closed graph and is stronger than a semi-closed graph. However, it is independent of a closed graph. Also, it is found that some types of known mappings may fail to have a strongly semi-closed graph and that a mapping having a strongly semi-closed graph need not be a certain type of mappings. Furthermore, some conditions are discussed under which certain types of mappings have strongly semi-closed graphs, and, also, some conditions are investigated under which range or/and domain of a mapping having a strongly semi-closed graph enjoy semi-\( T_1 \) or/and semi-\( T_2 \) properties.

1. TERMINOLOGY:

As usual, a set \( A \) in a space \( X \) is semi-open if
there exists an open set \( O \) in \( X \) such that \( O \subseteq A \subseteq \text{cl} \ O \), where \( \text{cl} \ O \) denotes the closure of \( O \) in \( X \). Every open set is semi-open. The complement of a semi-open set is a semi-closed set. The intersection of all the semi-closed sets containing \( A \) is the semi-closure of \( A \) and is denoted by \( \text{sc} \ A \); also, \( A \subseteq \text{sc} \ A \subseteq \text{cl} \ A \), and \( A \subseteq B \) implies \( \text{sc} \ A \subseteq \text{sc} \ B \). A subset \( M \) of a space \( X \) is a semi-neighbourhood of a point \( x \) of \( X \) if there exists a semi-open set \( A \) in \( X \) such that \( x \in A \subseteq M \).

A subset \( A \) of \( X \) is semi-open iff it is a semi-neighbourhood of each of its points. \( f : X \to Y \) is called semi-continuous [16] if \( f^{-1}(V) \) is semi-open in \( X \) for each open \( V \) in \( Y \), equivalently, iff, for each \( x \in X \) and each neighbourhood \( V \) of \( f(x) \), there exists a semi-neighbourhood \( U \) of \( x \) such that \( f(U) \subseteq V \). \( f : X \to Y \) is said to be irresolute [4] if \( f^{-1}(V) \) is semi-open in \( X \) for each semi-open \( V \) in \( Y \), equivalently, iff for each \( x \in X \) and each semi-neighbourhood \( V \) of \( f(x) \), there exists a semi-neighbourhood \( U \) of \( x \) such that \( f(U) \subseteq V \). Also, \( f : X \to Y \) is irresolute iff, for all \( B \subseteq Y \), \( \text{sc} \ f^{-1}(B) \subseteq f^{-1}(\text{sc} \ B) \). \( f : X \to Y \) is pre-semi-open[4] if \( f(A) \) is semi-open in \( Y \) for each semi-open \( A \) of \( X \). A space \( X \) is semi-T\(_1\) [19] if, for each pair of distinct points \( x, y \) of \( X \), there exists a semi-open set \( A \) containing \( x \) but not \( y \) and a semi-open set \( B \) containing \( y \) but not \( x \). A space \( X \) is semi-T\(_2\) if, to each pair of distinct points \( x, y \) of \( X \), there exists disjoint semi-open sets \( U \) and \( V \) containing \( x \) and \( y \), respec-
-tively, equivalently, iff, for each pair of distinct points \( x, y \) of \( X \), there exists a semi-open set \( V \) containing \( y \) such that \( x \not\in \text{scl } V \). Also, every semi-\( T_2 \) space is semi-\( T_1 \)[19].

By a semi-clopen set, we shall mean a set which is both semi-open and semi-closed.

2. MAPPINGS WITH STRONGLY SEMI-CLOSED GRAPHS

If \( f : X \rightarrow Y \), then the subset \( G(f) = \{ (x, f(x)) : x \in X \} \) of the product space \( X \times Y \) is known to be the graph of \( f \).

**DEFINITION 2.1:** The mapping \( f : X \rightarrow Y \) has a strongly semi-closed graph if for each \( (x, y) \not\in G(f) \), there exist semi-open sets \( U \) and \( V \) containing \( x \) and \( y \), respectively, such that \( (U \times \text{scl } V) \cap G(f) = \emptyset \).

A simple but useful characterization of mappings with strongly semi-closed graphs is given below:

**THEOREM 2.1:** The mapping \( f : X \rightarrow Y \) has a strongly semi-closed graph iff, for each \( x \in X \) and \( y \in Y \) such that \( y \neq f(x) \), there exist semi-open sets \( U \) and \( V \) containing \( x \) and \( y \), respectively, such that \( f(U) \cap \text{scl } V = \emptyset \).

**PROOF:** Immediately follows from Definition 2.1.
The mapping \( f : x \rightarrow y \) has a strongly-closed graph [12] iff, for each \((x, y) \notin G(f)\), there exist open sets \( U \) and \( V \) containing \( x \) and \( y \), respectively, such that 
\[(U \times \text{cl} \ V) \cap G(f) = \emptyset. \]
Also, \( f : x \rightarrow y \) has a semi-closed graph [Chapter II and also [7j] if, for each \((x, y) \notin G(f)\), there exist semi-open sets \( U \) and \( V \) containing \( x \) and \( y \), respectively, such that 
\[(U \times V) \cap G(f) = \emptyset. \]

Evidently, every strongly-closed graph is strongly semi-closed and every strongly semi-closed graph is semi-closed, but the converses are not true, in general. It may be seen by the following examples.

**EXAMPLE 2.1.** Let \( X = \{a, b\} \) with topology \( \mathcal{T}_1 = \{\emptyset, X, \{a\}, \{b\}\} \) and \( Y = \{a, b, c\} \) with topology \( \mathcal{T}_2 = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\} \). Then, clearly, the graph of the identity mapping \( i : x \rightarrow y \) is semi-closed and strongly semi-closed but is neither closed nor strongly-closed.

**EXAMPLE 2.2.** Let \( X \) be the space given in Example 2.1, and let \( Y = \{a, b, c\} \) with topology \( \mathcal{T}_2 = \{\emptyset, Y, \{c\}, \{a, c\}, \{b, c\}\} \). Then, obviously, the graph of the identity mapping \( i : x \rightarrow y \) is semi-closed and, also, closed but is not strongly semi-closed.
From Examples 2.1 and 2.2, it is also clear that a mapping having a strongly semi-closed graph need not have a closed graph and a mapping having a closed graph may fail to have a strongly semi-closed graph.

Again, every mapping with a strongly-closed graph also has a closed graph, but the converse may fail [12]. Also a mapping having a closed graph also has a semi-closed graph but the converse may not be true [7].

Hence, we have the following implications:

Diagram 1:

A mapping $f : X \rightarrow Y$ is set-a-connected [Chapter I and also [5] ] iff the inverse image under $f$ of every semi-clopen subset of $f(X)$ is semi-clopen in $X$. 


A mapping \( f : X \rightarrow Y \) is defined to be \( \mathbb{C} \)-monotone \([1]\) if \( f^{-1}(V) \) is a connected subset of \( X \) for every connected set \( V \) of \( Y \).

**Remark 2.1:** Several types of known mappings, viz., a continuous mapping, a \( \Theta \)-continuous mapping \([8]\), a weakly continuous mapping \([15]\), a mapping almost continuous in the sense of Frolik \([9]\), a semi-continuous mapping \([16]\), a mapping almost continuous in the sense of Husain \([11]\), a mapping almost continuous in the sense of M.K. Singal and A.R. Singal \([23]\), a \( c \)-continuous mapping \([10]\), a \( c^* \)-continuous mapping \([22]\), an \( \pi \)-continuous mapping \([18]\), an \( s \)-continuous mapping \([14]\), a set-connected mapping \([13]\), a set-\( s \)-connected mapping \([5]\), an irresolute mapping \([4]\), a connected mapping \([21]\), a \( \mathbb{C} \)-monotone \([1]\), a semi-connected mapping \([17]\), and a weak semi-connected mapping \([14]\), may fail to have a strongly semi-closed graph. It is shown by the following example.

**Example 2.3:** Let \( X \) be a space, containing more than one point, with the indiscrete topology, and let \( i : X \rightarrow X \) be the identity mapping. Here, \( G(1) \) is not strongly semi-closed.

**Remark 2.2:** A mapping having a strongly semi-closed graph need not be continuous, \( \Theta \)-continuous \([8]\), weakly continuous \([15]\),
almost continuous in the sense of Frolik [9], semi-continuous [16], almost continuous in the sense of Husain [11], almost continuous in the sense of M.K. Singal and A.R. Singal [23], c-continuous [10], c*-continuous [22], H-continuous [18], s-continuous [14], set-connected [13], set-s-connected [5], irresolute [4], connected [21], semi-connected [17], weak semi-connected [14]. It may be seen by the following example.

**Example 2.4.** Let \( X = \{a, b, c\} \) with topologies

\[ \mathcal{T}_1 = \emptyset, X, \{a\}, \{b\}, \{a, b\} \] and \[ \mathcal{T}_2 = \text{a discrete topology}. \]

Then, obviously, the identity mapping \( 1: (X, \mathcal{T}_1) \rightarrow (X, \mathcal{T}_2) \)

has a strongly semi-closed graph but is none of the mappings mentioned in Remark 2.2.

It has been observed earlier in the Remark 2.1 that various 'nice' mappings may fail to have strongly semi-closed graphs. Our next study, initiated with this view of point, deals with the conditions imposed on the range space so that a certain mapping may have strongly semi-closed graph. A range space being \( T_2 \) or semi-\( T_2 \) or extremally s-disconnected semi-\( T_2 \) space is shown to be a sufficient condition so that a semi-continuous or an irresolute or a set-s-connected surjection, respectively, has strongly semi-closed graph.
**Theorem 2.2.** Let \( f : X \to Y \) be a semi-continuous mapping with \( Y \) as \( T_2 \). Then \( G(f) \) is strongly semi-closed.

**Proof:** Let \( x \in X \) and \( y \in Y \) such that \( y \neq f(x) \). Then, \( Y \) being \( T_2 \), there is an open set \( V \) containing \( y \) such that \( f(x) \notin \text{cl } V \). And so, \( Y - \text{cl } V \) is open and contains \( f(x) \).

Hence, by semi-continuity of \( f \), there exists a semi-open set \( U \) containing \( x \) such that \( f(U) \subseteq Y - \text{cl } V \). Thus \( f(U) \cap \text{scl } V = \emptyset \). Consequently, \( G(f) \) is strongly semi-closed.

**Theorem 2.3.** If \( f : X \to Y \), where \( Y \) is semi-\( T_2 \), is an irresolute mapping, then \( G(f) \) is strongly semi-closed.

**Proof:** Similar to that of Theorem 2.2, using the definitions of a semi-\( T_2 \) space and an irresolute mapping.

**Definition 2.2.** A space \( X \) is extremally s-disconnected if the semi-closure of every semi-open set is semi-open [Chapter I and also [6]].

**Theorem 2.4.** Let \( f : X \to Y \) be a set-s-connected surjection and \( Y \) be an extremally s-disconnected, semi-\( T_2 \) space. Then \( G(f) \) is strongly semi-closed.
PROOF: Let $x \in X$ and $y \in Y$ such that $y \neq f(x)$. Then, $Y$ being semi-$T_2$, there is a semi-open set $N$ containing $y$ such that $f(x) \notin \text{scl } N = V$, say. Since $Y$ is extremally $s$-disconnected, $V$ is semi-clopen in $Y$ not containing $f(x)$. Therefore, $f$ being set-$s$-connected surjection, $f^{-1}(V)$ is semi-clopen in $X$ and $x \notin f^{-1}(V)$. Taking $U = X - f^{-1}(V)$, $U$ is semi-open in $X$ containing $x$ and then $f(U) \cap V = \emptyset$. Consequently, $G(f)$ is strongly semi-closed.

It is worth to mention here that one may have the results given earlier in the Theorems 2.2, 2.3, and 2.4 of the Chapter II as the immediate consequences of the above Theorems 2.4, 2.2 and 2.3, respectively.

DEFINITION 2.3. [20]: A space $X$ is locally $s$-connected if, for each $x \in X$ and each open set $O$ containing $x$, there exists an open $s$-connected set $G$ such that $x \in G \subseteq O$.

DEFINITION 2.4. [20]: Two subsets $A$ and $B$ of the space $X$ are termed semi-separated if $A \cap \text{scl } B = \emptyset = \text{scl } A \cap B$.

A set $A$ in the space $X$ is said to be $s$-connected if $A$ is $s$-connected as a subspace of $X$ [20].

LEMMA 2.1 [20]: A space $X$ is not $s$-connected iff it is the union of two non-empty, disjoint, semi-closed sets.
Lemma 2.2 [20]: Semi-closure of an open, s-connected set in a space X is s-connected.

Lemma 2.3 [20]: If \( A, B \) are open, s-connected and non-semi-separated sets in the space X, then \( A \cup B \) is s-connected.

A space \( X \) is Urysohn if, for each pair of distinct points \( x, y \) of \( X \), there exist open sets \( U \) and \( V \) containing \( x \) and \( y \), respectively, such that \( cl U \cap cl V = \emptyset \).

Definition 2.5: A mapping \( f : X \rightarrow Y \) is almost irresolute on \( X \) if for each \( x \in X \) and each semi-neighbourhood \( V \) of \( f(x) \), \( scl f^{-1}(V) \) is a semi-neighbourhood of \( x \) [Chapter II and [7]].

Evidently, every irresolute is almost irresolute but not conversely.

In view of Example 2.3, an almost irresolute mapping may fail to have a strongly semi-closed graph, and, in view of Example 2.4, a mapping having a strongly semi-closed graph need not be almost irresolute. However, we have the following result.

Theorem 2.5: Let \( f : X \rightarrow Y \) be an almost irresolute mapping, where \( Y \) is a locally s-connected Urysohn space.
If $f$ maps open $s$-connected sets onto $s$-connected sets and $f^{-1}$ maps $s$-connected sets onto open $s$-connected sets, then $G(f)$ is strongly semi-closed.

**Proof:** Let $x \in X$ and $y \in Y$ such that $y \neq f(x)$. Then, $Y$ being Urysohn locally $s$-connected, there exist open, $s$-connected sets $U$ and $V$ containing $y$ and $f(x)$, respectively, such that $\text{cl } U \cap \text{cl } V = \emptyset$. This gives $\text{cl } U \cap \text{cl } V = \emptyset$. Therefore $f^{-1}(\text{cl } U) \cap f^{-1}(\text{cl } V) = \emptyset$. Further, we claim that $f^{-1}(\text{cl } U) \cap \text{cl } f^{-1}(\text{cl } V) = \emptyset$. For, if not, then $f^{-1}(\text{cl } U)$ and $f^{-1}(\text{cl } V)$ are non-semi-separated (by Definition 2.4), and open, $s$-connected (by Lemma 2.2 and hypothesis) subsets of $X$, and so, by Lemma 2.3, $f^{-1}(\text{cl } U) \cup f^{-1}(\text{cl } V)$ is open, $s$-connected. But, again by hypothesis, $f[f^{-1}(\text{cl } U) \cup f^{-1}(\text{cl } V)] = \text{cl } U \cup \text{cl } V$ is $s$-connected which is a contradiction in view of Lemma 2.1. Therefore, it follows that $f^{-1}(\text{cl } U) \cap \text{cl } f^{-1}(\text{cl } V) = \emptyset$. Now, $f$ being almost irresolute, $\text{cl } f^{-1}(V)$ and hence $\text{cl } f^{-1}(\text{cl } V)$ is a semi-neighbourhood of $x$ and so there exists a semi-open set $T$ in $\text{cl } f^{-1}(\text{cl } V)$ containing $x$. Therefore $f^{-1}(\text{cl } U) \cap T = \emptyset$ which gives $f(T) \cap \text{cl } U = \emptyset$. Consequently, $G(f)$ is strongly semi-closed.

**Definition 2.6:** A mapping $f : X \rightarrow Y$ is weakly irresolute if, for each $x \in X$ and each semi-neighbourhood $V$ of $f(x)$, there exists a semi-neighbourhood $U$ of $x$ such that $f(U) \subseteq \text{cl } V$. 

Evidently, every irresolute is weakly irresolute but not conversely.

A weakly irresolute mapping may not have a strongly semi-closed graph. For, the identity mapping, in Example 2.3, is weakly irresolute but its graph is not strongly semi-closed. However, we have the following result.

**Theorem 2.6.** If \( f: X \rightarrow Y \) is weakly irresolute and \( Y \) is a Urysohn space, then \( G(f) \) is strongly semi-closed.

**Proof:** Let \( x \in X \) and \( y \in Y \) such that \( y \neq f(x) \). Then, since \( Y \) is Urysohn, there exists open sets \( V_1 \) and \( V_2 \) containing \( f(x) \) and \( y \), respectively, such that \( \text{cl } V_1 \cap \text{cl } V_2 = \emptyset \) which gives \( \text{scl } V_1 \cap \text{scl } V_2 = \emptyset \). Since \( f \) is weakly irresolute, there exists a semi-open set \( U \) containing \( x \) such that \( f(U) \subset \text{scl } V_1 \). Consequently, \( f(U) \cap \text{scl } V_2 = \emptyset \) and so \( G(f) \) is strongly semi-closed.

The converse to Theorem 2.6 does not hold, in general, as is clear from Example 2.4.

**Definition 2.7.** A mapping \( f: X \rightarrow Y \) is \( \Theta \)-irresolute if, for each \( x \in X \) and each semi-neighbourhood \( V \) of \( f(x) \), there is a semi-neighbourhood \( U \) of \( x \) such that \( f(\text{scl } U) \subset \text{scl } V \).
Obviously, every $\Theta$-irresolute is weakly irresolute but converse may fail. Also, every irresolute is $\Theta$-irresolute but not conversely.

Hence we have the following implications diagram.

\[
\text{Irresolute} \quad \quad \quad \quad \text{$\Theta$-irresolute} \quad \quad \quad \quad \text{Weakly irresolute}
\]

\[
\text{mapping} \quad \quad \quad \quad \text{mapping} \quad \quad \quad \quad \text{mapping}
\]

In view of Example 2.3, a $\Theta$-irresolute mapping need not have a strongly semi-closed graph. However, in view of Theorem 2.6 and the fact that every $\Theta$-irresolute is weakly irresolute, we have the result given below.

**COROLLARY 2.1.** If $f : X \to Y$ is $\Theta$-irresolute where $Y$ is Urysohn, then $G(f)$ is strongly semi-closed.

The converse to Corollary 2.1 need not be true.

It is clear by Example 2.4.

Recall that a closed graph may fail to be strongly semi-closed and a strongly semi-closed graph may fail to be closed. However, we have the following result.

**THEOREM 2.7.** Let $f : X \to Y$ be pre-semi-open and have a closed graph $G(f)$. Then $G(f)$ is strongly semi-closed.
PROOF: Let \( x \in X \) and \( y \in Y \) such that \( y \not\in f(x) \). Then, 
\( G(f) \) being closed, there exist open sets \( U \) and \( V \) containing \( x \) and \( y \), respectively, such that \( f(U) \cap V = \emptyset \). Now, since \( f \) is pre-semi-open, \( f(U) \) is semi-open. Therefore \( f(U) \cap \text{scl} \ V = \emptyset \). Consequently, \( G(f) \) is strongly semi-closed.

Now we shift our interest to the investigation alongside the problem 'what is the property implied on domain or range space by strongly semi-closed graph property itself of a certain mapping?'

**Theorem 2.9:** Let \( f : X \to Y \) be a surjection with strongly semi-closed \( G(f) \). Then \( Y \) is semi-\( T_2 \), and hence semi-\( T_1 \).

**Proof:** Let \( y_1, y_2 \) be distinct points of \( Y \). Then, \( f \) being surjective, there exists an \( x_1 \in X \) such that \( f(x_1) = y_1 \). Thus, \( (x_1, y_2) \not\in G(f) \). Therefore, \( G(f) \) being strongly semi-closed, there exist semi-open sets \( U \) and \( V \) containing \( x_1 \) and \( y_2 \), respectively, such that \( f(U) \cap \text{scl} \ V = \emptyset \). Consequently, \( y_1 \not\in \text{scl} \ V \). This shows \( Y \) is semi-\( T_2 \). Since every semi-\( T_2 \) space is semi-\( T_1 \), \( Y \) is semi-\( T_1 \).

**Theorem 2.9:** The space \( X \) is semi-\( T_2 \) iff the identity mapping \( i : X \to X \) has a strongly semi-closed graph.
**Proof**: Immediately follows from Theorem 2.3 and Theorem 2.8.

**Theorem 2.10**: Let \( f: X \rightarrow Y \) be injective with strongly semi-closed \( G(f) \). Then \( X \) is semi-\( T_1 \).

**Proof**: Let \( x_1, x_2 \) be distinct points of \( X \). Then, \( f \) being injective, \( f(x_1) \neq f(x_2) \). Thus \( (x_1, f(x_2)) \notin G(f) \). Therefore, there exist semi-open sets \( U \) and \( V \) containing \( x_1 \) and \( f(x_2) \), respectively, such that \( f(U) \cap \text{scl } V = \emptyset \). Thus \( x_2 \notin U \). Consequently, \( X \) is semi-\( T_1 \).

**Theorem 2.11**: Let \( f: X \rightarrow Y \) be bijective with strongly semi-closed graph. Then both \( X \) and \( Y \) are semi-\( T_1 \).

**Proof**: It is evident by Theorem 2.8 and Theorem 2.10.

**Theorem 2.12**: Let \( f: X \rightarrow Y \) be an injective weakly irresolute with strongly semi-closed graph \( G(f) \). Then \( X \) is semi-\( T_2 \).

**Proof**: Let \( x_1, x_2 \) be two distinct points of \( X \). Then \( f(x_1) \neq f(x_2) \). Thus \( (x_1, f(x_2)) \notin G(f) \). And so, there exist semi-open sets \( U \) and \( V \) containing \( x_1 \) and \( f(x_2) \),
respectively, such that $f(U) \cap \text{scl } V = \emptyset$. Thus $f^{-1}(\text{scl } V) \subseteq X - U$.

Now, since $f$ is weakly irresolute, there exists a semi-open set $N$ containing $x_2$ such that $N \subseteq f^{-1}(\text{scl } V) \subseteq X - U$. But then $x_1 \in U$ and $x_2 \in N \subseteq f^{-1}(\text{scl } V) \subseteq X - U$ so that $X$ is semi-$T_2$.

**Corollary 2.2.** If $f : X \to Y$ is an injective $\Theta$-irresolute with strongly semi-closed graph $G(f)$, then $X$ is semi-$T_2$.

**Proof:** Theorem 2.12 and the fact that every $\Theta$-irresolute is weakly irresolute.

**Corollary 2.3.** Let $f : X \to Y$ be an injective irresolute with strongly semi-closed $G(f)$. Then $X$ is semi-$T_2$.

**Proof:** It is evident by Corollary 2.2 and the fact that every irresolute is $\Theta$-irresolute.

**Theorem 2.13.** Let $f : X \to Y$ be an injective almost irresolute with strongly semi-closed graph $G(f)$. Then $X$ is semi-$T_2$.

**Proof:** Proceeding as in Theorem 2.12 and then using Definition 2.5, $\text{scl } f^{-1}(V)$ and hence $\text{scl } f^{-1}(\text{scl } V)$ is a
semi-neighbourhood of $x_2$. Since $X - U$ is semi-closed, $\text{scl } f^{-1}(\text{scl } V) \subseteq X - U$. But then $x_1 \in U$ and $x_2 \in N(\text{a semi-open set})$, $N \subseteq \text{scl } f^{-1}(\text{scl } V) \subseteq X - U$ (by definition of a semi-neighbourhood), so that $X$ is semi-$T_2$.

**Theorem 2.14**: Let $f : X \to Y$ be bijective irresolute (almost irresolute, $\mathcal{E}$-irresolute or weakly irresolute) with strongly semi-closed $G(f)$. Then $X$ and $Y$ both are semi-$T_2$.

**Proof**: This is clear by Theorem 2.8 and Corollary 2.3 (Theorem 2.13, Corollary 2.2 or Theorem 2.12).
REFERENCES


****
***
*