Iterative model based Image reconstruction method and its applicability to refine artifacts
## CHAPTER 3

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3.1 Introduction

The Medical Images Obtained for doing this work is from Medall & Rainbow Diagnostic Center (India). The figures used in this work illustrate how Image reconstruction takes place for CT Image head section, Bony tissues. The Proposed work describes the experimental findings involving Degrading effects in CT Image from literature survey gives data statistics Beam Hardening, Noise, Scatter, Incompleteness of projection data, High Scatter–to–Primary x-Ray Ratios, Superimposition and Conspicuity. The above artifacts in CT imaging are removed by using Iterative model based Reconstruction where this method is used to reconstruct the CT data with model based approach for removing beam hardening, streaking and helical cone beam artifacts for better reconstruction of CT data and also gives better performance when compared with other methods, Expectation maximization & Total Variance method for providing appropriate Image Contrast and resolution to the CT Image. The CT-Opt reconstruction method used to detect the smaller specimens in the Human Body.

3.1.1 CT Reconstruction

The cross sectional view of an object is generated by Image reconstruction. The body is divided in to smaller blocks. These blocks are assigned a number to the X-ray attenuated beams which are called as voxels. The attenuation is determined by thickness and composition.
To determine the $\mu$ pixels in image matrix many equations are calculated. The Imaging process reconstructs the data by recording the views of projection data in memory. An Object is reconstructed from multiple projections of the object. A narrow X-ray beam is passed through cross section of the body and is collected by detector. The detector system gives the energy of photons transmitted and data is passed to computer to generate an image.

![Diagram of CT Image Reconstruction](image)

**Fig. 3.1:** Shows CT Image Reconstruction

The Formation of CT image is in distinct three phases the reconstruction is carried in three different phases.

1. Scanning phase: It deals with projection data in the image.
2. Reconstruction Phase: Acquiring data and process a digital image.
3. DAC Phase: Visible analog Images are displayed with gray shades.
These phases define the Quality of Image is characterized from this phases.

Fig. 3.2: Shows Reconstruction of Image in different Phases

The below diagram describes the reconstruction with an example

Fig. 3.3: gives the Reconstruction Example

To produce image with numerical data of object. The values of horizontal and vertical directions are added to gray shades of an image. These values are given numbers to obtain an Image, The Horizontal and vertical directions are shown, this used as
mathematical method to reconstruct an Image.

The process of reconstruction which carries in sequence of steps which are as follows

The Transform of Radon  \( P(U, \theta) = F(u, 0) \) (3.1)

Where \( F(u,v) \) is 2D-FFT of \( f(x,y) \)

Data acquisition at angle: 0 – 180 degree

Obtain \( F(u,v) \) and then 2D IFFT -> reconstruction

This Transform works in similar to filtered back projection.

The equations describe the reconstruction of data with FBP the equations find relations with attenuated intensity, projection data, Filtered data and back Projection.

Attenuated pixel Intensity of the reconstruction data can be found with the process of reconstruction  \( I(X) = I_0(X) e^{-\int f(x,y)dy} \) (3.2)

The reconstructed projection data be given as

\[ P(X, \theta) = \int f(x, y) dy = \ln \frac{I_0(X)}{I(x)} \] (3.3)

The filtered data can be reconstructed from the projections by means of ramp filtering \( P_f(X, \theta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(U, \theta) . U . e^{iUX} dX \) (3.4)

Where e \( P(u, \theta) \) is 1D FFT of \( P(X, \theta) \) Filtered data

The projections use the formula for back projection or the reconstructed data is given by  \( f(x,y) = \int_0^\pi p_f (x \cos \theta + y \sin \theta, \theta) d\theta \) (3.5)
3.1.2. Mathematical Methods of Image reconstruction Algorithms

The calibration and data acquisition with detector system involves Iterative and analytical methods. Analytical method is further divided into Iterative filtered back Projection and FBP.

These methods provided improved Image quality and high Computational Efficiency. In Similar Understanding Iterative reconstruction has algebraic Model based which provides simpler modelling and simple statistics. The model based reconstruction has full statistics, Complex optimum value and limited physics. Many methods are used with exact formulas in Analytical Reconstruction.

1. 2D Fourier Analysis

2. Filtered Back Projection

The reconstruction iteratively uses an approximation then compares with measured values. The values of assumed and measured are brought together until they become equal. The problem in this method is more time taking for calculation and amplification of noise.

Iterative reconstruction uses three methods

1. Point to point correction
2. Simultaneous reconstruction
3. Ray by Ray correction
### 3.2 Concept of back projection

Early reconstruction method is back projection and it is known as Linear Interpolation method.

Fig. 3.4: shows Reconstruction from projections from 8 to 256 projections

Fig. 3.5: Shows the Back projection with one View

Fig. 3.6: Shows the Back projection with two Views

Fig. 3.7: Back projection modeling (a) Data Acquisition process (b) Reconstruction
3.2.1 Back Projection Example with n*n Matrix

\[
A_{ij} = \sum_t p_{ai}(t)
\]

\[
A_{11} = p_0(0) + p_{90}(0) = 7
\]

\[
A_{12} = p_0(0) + p_{90}(1) = 9
\]

\[
A_{21} = p_0(1) + p_{90}(0) = 11
\]

\[
A_{22} = p_0(1) + p_{90}(1) = 13
\]

\[\begin{array}{ccc}
7/4 & 9/4 \\
11/4 & 13/4 \\
\end{array}\]

Normalize by 10/40

The original image can be reconstructed from various projections and this provides a problem of blurring in some parts of the image reconstructed. By passing the data to a filter the solution can be obtained; this removal of patterns is ramp filter.

Fig. 3.8: A projection, p(s, \(\phi\)), is formed from integration along all parallel LOR’S at an angle \(\phi\).

FBP algorithm uses sinogram with the inverse of radon transform and this algorithm uses finite number of projection are less than the reconstructed image has poor quality and uses filters to improve resolution and remove noise.
FBP is very fast it uses very less memory depends on FFT computations. The drawbacks in FBP as it requires more projection for accurate reconstruction and it is difficult to incorporate the prior information about solution.

### 3.2.2 Filtered back projection and its drawbacks

The back projection data is filtered or modified to obtain FBP. It also adjusts the effects of reconstruction data.

![Fig. 3.9: Back projection modelling](image)

For reconstruction using the FBP the following steps are used:

1. The projections are mapped with FFT with appropriate angles.
2. Perform Inverse of FFT from F(u,v) to f(x,y).
3. This projections work at low frequencies.
4. Perform Filtering to the data.
5. Hence Filtered back projection (FBP) can be obtained.
3.2.3 The Filtered Back Projection (FBP) Method

The view of Back Projection $i$th is given

\[(3.6)\]

The object in 2D is smeared with projection angle. The temporary Images form the projection by sum of data. To better understand $b_i(x, y)$, note that when $I = 1$

\[(3.7)\]

The sinogram is generated and convoluted to form cone filter

$f(x, y) \rightarrow \text{Projection} \rightarrow p_\phi(r) \rightarrow \text{Backprojection} \rightarrow f_k(x, y) \rightarrow \text{Cone filter} \rightarrow f(x, y).$

The cone filter is moved to obtain projections with linear shape

$f(x, y) \rightarrow \text{Cone filter} \rightarrow \hat{f}(x, y) \rightarrow \text{Projection} \rightarrow \hat{p}_\phi(r) \rightarrow \text{Backprojection} \rightarrow f(x, y),$

By applying the Fourier slice transform we get 1D FFT

\[(3.8)\]

where, assuming $w(\Phi) = 1$ hereafter, the filtered object $\hat{f}(x, y)$ has the following the spectrum:

\[(3.9)\]

These values of projections are applied to ramp filter due to its shape

$f(x, y) \rightarrow \text{Projection} \rightarrow p_\phi(r) \rightarrow \text{Ramp filters} \rightarrow \hat{p}_\phi(r) \rightarrow \text{Backprojection} \rightarrow f(x, y).$

FBP is used in tomography to reconstruct data accurately and it uses fourier slice theorem A formal derivation of the FBP method uses the Fourier-slice theorem as follows:

\[(3.10)\]
\[(3.11)\]
\[(3.12)\]
where we define the filtered projection $\tilde{p}_\phi(r)$ as follows:

$$\tilde{p}_\phi(r) = \int_\infty^{\infty} P_\phi(v)|v|e^{i2\pi vr} \, dv$$ \hspace{1cm} (3.14)

The following steps are used in FBP

1. The projections angle for 1D FFT is computed
2. Multiple Projection angle with ramp Filter
3. Then perform periodic convolution the filtering cause aliasing and wrap around artifacts
4. The DC component nulls the value of Projection

The filtered sinogram gives the back projection

This practical sense used in pixel driven back projection to the domain

Backprojection the filtered sinogram $\{\tilde{p}_\phi(r)\}$ to get $f(x, y)$, i.e.

$$\tilde{f}(x, y) = \int_0^\pi \tilde{p}_\phi(x \cos \varphi + y \sin \varphi) \, d\varphi$$ \hspace{1cm} (3.15)

Hann or Hanning: $A(v) = \left[ \frac{1}{2} + \frac{1}{2} \cos \left( \frac{\pi v}{v_0} \right) \right] \text{rect} \left( \frac{v}{2v_0} \right)$ \hspace{1cm} (3.16)

1. Hamming: $A(v) = [0.54 + 0.46 \cos (\frac{\pi v}{v_0})] \text{rect} \left( \frac{v}{2v_0} \right)$ \hspace{1cm} (3.17)

2. Generalized Hamming:

$$A(v) = [\alpha + (1 - \alpha) \cos (\frac{\pi v}{v_0})] \text{rect} \left( \frac{v}{2v_0} \right), \text{for } \alpha \in [0,1]$$ \hspace{1cm} (3.18)

3. Butterworth: $A(v) = \frac{1}{\sqrt{1 + (\frac{v}{v_0})^{2n}}}, \text{for } n \geq 0$ \hspace{1cm} (3.19)

4. Parzen: $A(v) = \begin{cases} 1 - 6 \left( \frac{v}{v_0} \right)^2 \left( 1 - \frac{|v|}{v_0} \right) & |v| \leq \frac{v_0}{2} \\ 2(1 - \frac{|v|}{v_0})^3 v_0 \leq |v| \leq v_0 \\ 0, & \text{otherwise} \end{cases}$ \hspace{1cm} (3.20)

5. Sheep Logman[57]: $A(v) = \left| \sin \left( \frac{v}{2v_0} \right) \right|$ or $\left| \sin \left( \frac{v}{2v_0} \right) \right|^3$ \hspace{1cm} (3.21)
6 Modified Sheep Logan: 
\[ A(v) = \text{sine} \left( \frac{v}{2v_o} \right) [0.4 - 0.6 \cos \left( \frac{\pi v}{v_o} \right) ] \]  \hspace{1cm} (3.22)

### 3.2.4 Problems with Classical Methods on Real Problems

The Samples will only a finite number to projection views and the integral of projection angles is obtained.

Usually the projection angles are Uniformly spaced over the interval [0, \( \pi \)], i.e., \( \varphi_i = \left( \frac{i-1}{n_\varphi} \right) \pi, \quad i = 1, \ldots, n_\varphi \)  \hspace{1cm} (3.23)

For Identical samples various projections based approaches are used in discrete. In such cases, the usual approach is to use the following Riemann sum approximation to

\[ f_b(x,y) \approx \frac{\pi}{n_\varphi} \sum_{i=1}^{n_\varphi} p_\varphi(x\cos\varphi_i + y\sin\varphi_i) \]  \hspace{1cm} (3.24)

Ignoring noise and blur, we are given the discrete sonogram.

\[ y_i[n] = p_\varphi(r) [\varphi = \varphi_i, r = r[n], \quad i = 1, \ldots, n_\varphi, \quad n = 0, \ldots, n_R - 1. \]  \hspace{1cm} (3.25)

where the radial sample locations are given by

\[ r_c[n] = (n - n_0) \Delta_R \]  \hspace{1cm} (3.26)

The values can be assumed appropriately to the band limited

\[ p_{\varphi i}(r) = \sum_{n=-\infty}^{\infty} y_i[n] \text{sinc} \left( \frac{r - r_c[n]}{\Delta_R} \right) \]  \hspace{1cm} (3.27)

The Interpolation technique is used for inappropriate objects in real space. This method requires infinite number of samples the simple methods used is linear and spline methods with oversampling of FFT with ramp Filter.
3.3 Tissue characterization in Computed Tomographic (CT) Images

The beam of X-ray passes through the body and it works with Lambert-Beer Law.

\[ I_t = I_0 \exp (-\mu l) \]  \hspace{1cm} (3.28)

\( I_t \): Transmitted intensity of the X-ray

\( I_0 \): Incident Intensity

\( l \): Length of the path of the beam and

\( \mu \): Linear attenuation coefficient

3.3.1 Hounsfield Units (HU)

The value of \( \mu \) with X-ray energy is represented by CT number

\[ \text{CT number} = K \frac{\mu - \mu_w}{\mu_w} \]  \hspace{1cm} (3.29)

\( \mu_w \): Linear attenuation coefficient of water

\( K \): Scaling constant

If \( k=1000 \): Hounsfield units (HU)

3.3.2 CT NUMBER

The computer represents a relationship between pixel and water. The values of CT number are represented from -1000 to +1000 in 256 gray levels. The CT number at the base line defined window width and the center pixel value as window level. The collected data is computed pixels linear attenuation coefficient. The value of CT number calculates of each pixel converted as CT number. The calculation values are represented by a picture in gray scale.

\[ \text{CT number} = \frac{k(m_p - m_w)}{M_w} \]  \hspace{1cm} (3.30)

The projection data statistics gives a view of difficulty to access
region and imaging conditions and ambiguities related to positioning and projection defines artifacts and describes the low contrast tissues and acceptable radiation exposure to various organs and structures.

Table 3.1 Describes the CT Values of Abdominal Tissues

<table>
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<tr>
<th>Tissue</th>
<th>CT Value(HU)</th>
<th>SD</th>
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<tr>
<td>Air</td>
<td>-1006</td>
<td>2</td>
</tr>
<tr>
<td>Fat</td>
<td>-90</td>
<td>18</td>
</tr>
<tr>
<td>Skin</td>
<td>+16</td>
<td>11</td>
</tr>
<tr>
<td>Spinal canal</td>
<td>+23</td>
<td>15</td>
</tr>
<tr>
<td>Kidney</td>
<td>+32</td>
<td>10</td>
</tr>
<tr>
<td>Blood(Aorta)</td>
<td>+42</td>
<td>18</td>
</tr>
<tr>
<td>Muscle</td>
<td>+44</td>
<td>14</td>
</tr>
<tr>
<td>Spleen</td>
<td>+46</td>
<td>12</td>
</tr>
<tr>
<td>Necrosis</td>
<td>+45</td>
<td>15</td>
</tr>
<tr>
<td>Liver</td>
<td>+60</td>
<td>14</td>
</tr>
<tr>
<td>Viable Tumor</td>
<td>+91</td>
<td>25</td>
</tr>
<tr>
<td>Calcification</td>
<td>+345</td>
<td>155</td>
</tr>
<tr>
<td>Bone</td>
<td>+1005</td>
<td>103</td>
</tr>
</tbody>
</table>

3.3.3 Artifacts in CT Imaging from Literature survey give data statistics

The projection data statistics gives a view of difficulty to access region and imaging conditions and ambiguities related to positioning and projection defines artifacts and describes the low contrast tissues and acceptable radiation exposure to various organs and structures.

The difficulties with CT numbers and attenuation coefficients values provide artifacts in CT imaging. The number of detector measurements also provides artifacts to CT image.
1. Streaking: This occurs with the problems in single measurements.

2. Shading: The channels which are deviating from true measurements.

3. Rings: The errors provide from detector calibrations individually.

4. Distortion: This is because of Helical Reconstruction process.

The Origins of Artifacts in CT image reconstruction

1. Physics based Artifacts: It is due to process involved in acquisition of CT data.

2. Patient based Artifacts: This occurs due to presence of metallic materials and movement of patient.

3. Scanner based Artifacts: This artifacts result from scanner imperfections.

4. Helical and multisection artifacts: It deals with the process in Image reconstruction.

In general these artifacts are motion based, streaking and beam hardening and ring artifacts

![Fig. 3.10 (a) Beam hardening + scatter (b) Noise (c) Truncation (d) detectors](image)

1. **Energy**: The accelerating voltage in the electron beam that penetrates the target of the X-ray generator. This capability of energy penetrating in to the object is measured by KVP. Kilo Volt peak is used
to measure maximum photon energy with the given voltage. The skin surface absorbs the low energy X-ray Photons whereas the higher energy results in poor visibility of tissues and contrast and result in low differential attenuation.

Many factors affect the photon energy which is distance from the patient to X-ray source and thickness of patient and grids used. The energy levels in projection radiography are

- Abdomen: 60-100 KVP
- Chest: 80-120KVP
- Skull: 70-90 KVP

The limitations in photon energy mainly concentrate on soft tissues and maximized contrast of image.

The X-ray photons of Low energy are absorbed faster than photons of high energy of tissues related to breast and skin Therefore it increases the patient dose of radiation.

Fig. 3.11: Shows the concept of X-Ray Attenuation
2. **Exposure:** The tube voltages have X-ray photons projected from the source to the current in the tube (mA) with respect to the time of exposure. This element gives the product of mA's. The number of photons near to the film decides the mA’s quantity. If the photons are less in the film the image results very poor light in the image which is known as faint image. Similarly more quantity of mA’s results in darker image or overexposed image. The exposure values typically are 2-120mA. Presently most of the systems automatically determine the exposure radiation for the required object being imaged.

3. **Beam Hardening:** This Issue is mostly seen in various systems where the system gets very less X rays as they get very low energy levels as the distance will be very high from the point of entry. This leads to beam hardening as this results in improper estimation of the attenuation coefficients. The photons of low energy absorbed very early when compared with high energy photons and the beam becomes harder when the value of mean energy increases.

   The tissues such as bones have different attenuation values which result in streaks this causes due to incorrect and improper in set of projections. This problem can be overcome by software preprocessing Built in features for minimizing beam hardening is Calibration Correction which is given as follows.

   By using phantoms with various sizes the beam hardening can be reduce to a level but best method is software pre-processing. Over correction results in high residual cupping artifacts with central CT value.
Beam Hardening Correction software is an iterative algorithm used for the bony regions is minimized and also reduces the blurring with brain soft tissues in reducing the dark bands in cross sections.

4. Scatter and the use of grids: The absorption and scattering occurs when more photons are lost in x-ray beam propagation. Scattering results more photon noise and contrast and discrimination of energy. These effects will reduce the intensity of background, contrast and introduces noise when it hits the detector.

5. Photon detection noise: The scattering results when some photons will pass and become unaffected. The photons at the detector and x-ray beam interaction results in photon noise from scattering and absorption.

These results in low noise and contrast where the detectability of structures will become difficult this somehow can be reduced by current modulation techniques and amount of attenuation required and the patients image quality to projection data.
Fig. 3.13: (a,b,c) shows the blurring artifacts which also known as volume averaging.

6. Ray stopping by Heavy Implants

The presence of materials such as screws, pins and surgical clips will completely attenuate or block x-ray beam. The body parts such as shoulders attenuate the photons and result in noise angulations the dose is increased by passing through less attenuation parts still there is noise due to heavy implants the remedy is to increase the current in the tube for the scan duration. The attenuation provides insufficient photons to the detectors and result in more noisy projections.

Fig. 3.14: Shows Metallic Artifacts
7 **Physiological artifacts**: This occurs due to patient movement and blood circulation and heavy breathing and any cardiac activity will contribute to such artifacts.

![Conventional Scanning vs Spiral Scanning](image)

Fig. 3.15: Shows the Through-plane motion artifacts are suppressed by single-breath-hold spiral scanning.

8. **Cupping artifacts**: The cylindrical phantom is hardened when it passes through middle portion rather than edges in a material. This profile gives CT numbers in the phantom and the beam becomes more intense and results in cupped shape with dense objects.

9. **Incomplete projections**: These deals with when the object is outside the region concentrated it results in incomplete information such as shading and streaking artifacts. The projection views will have whole object than partial result in truncation artifacts.

10. **Helical Artifacts in the axial plane**: The interpolation process results in consistent projections and additional artifacts with resulting process. Sometimes the image is contributed with narrow and wider part of the plane resulting in to projection angles.
Fig. 3.16: shows the abdomen section of body with metals for Dental Phantom with Linear Interpolation.

3.4 Exploration to the technologies used

Image reconstruction method defines the artifacts in CT Imaging such as streaking, Beam Hardening and physiological artifacts which are studied in the literature.

A optimized solution is obtained by defining a novel reconstruction method to overcome those artifacts and perform modelling by deriving a criterion for filtering and multi-phase formulation.

3.4.1 Criterion for filtering edge information

The overlapping with the edges to define boundaries with interpolation that deals with shape and describes to baye’s probability to combine the slices which are greater than or equal to user defined surfaces.

To define connectivity of edge components the boundaries are detected which gives the probability of the contours with user initialized surfaces. The classification defines the approximation to the given boundaries for edge detection. The linked boundaries are achieved with correspondence to function derived.
3.5 Proposed algorithms

3.5.1 Iterative model based reconstruction

To obtain real optics and image quality the statistics are to be obtained. In major quantum and electronic noise provides major difficulty which works the CT system slow and computationally inefficient with low quality image.

The accuracy and physics is a major thing that gives the complexity to given work and implementation. To model the system optics and X-ray physics the data which is acquired is essential for the process. The problem in Image reconstruction is done directly by using constrained optimization function and other is with sinogram data the process involves filtering and interpolation to reduce the noise and blur to make the system stable.

The method uses a function to define boundaries and a model is specified with local intensity property to present surfaces, the formulation to energy and the various level set methods are multiplied together.

3.5.2 Model Based Specification

The non-invasive technique used to provide better surgeries in clinical practice. It is a diagnosis system which gives information about disease pathology with low radiation. The elements such as software and hardware components give the accuracy to diagnostic tool by appropriate projection data of CT. The quality of image can be enhance by iterative model based reconstruction for low mA scenarios.

The pixels in an image is given by $S$ as
\[ I = (I_1, \ldots, I_N) \text{ and } I^* = (I_1^*, \ldots, I_N^*) \]  
(3.31)

The intensity values varies with ideal and observed in image the approximation is done to avoid distortion by pixel in a summing form

\[ I_i \times I_i^* \times d_i \]  
(3.32)

The \( d_i \) given is the intensity of gain in homogeneous pixel

\[ y = y^* + d \]  
(3.33)

where \( y \) and \( y^* \) is the value transformed and observed with respect to intensities parameter \( \theta(x_i) = (\mu_{x_i}, \sigma_{x_i}), \mu_{x_i}, \sigma_{x_i} \) being the mean and the variance of class \( x_i \), respectively

\[ p(y_i^*|x_i) = g(y_i^*; \theta(x_i)) \]  
(3.34)

\[ \text{where } g(y; \theta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right). \]  
(3.35)

The transformed intensity defines the segmentation field with distortion and classification the values assigned are given pixel by pixel in labels. The values of S are denoted by X and the pixel values each are given by increasing the assumption value of Gaussian distribution the variance value and mean can be achieved with class as

\[ p(y_i|x_i,d_i) = g(y_i - d_i; \theta(x_i)) \]  
(3.36)

The Gaussian distribution is given with intensity value of change intensities is given The distribution function independently.

\[ p(y_i|d_i) = \Sigma_{j \in L} \{g(y_i - d_i; \theta(x_i))p(x_i = j)\} \]  
(3.37)

The field of Gaussian is distributed which varies with intensity of pixel with the given Image.
The probability of Baye’s given prior information about the field of distortion to given values of Intensity
\[ \hat{d} = \arg \max_d p(d|y). \] (3.40)

The probability of information is given with intensity and Maximum a posterior (Map) to define the distortion of the given field.

The optimum solution \( \hat{d} \) satisfies the following condition
\[ \left[ \frac{\partial}{\partial d_i} \ln p(d|y) \right] = 0 \quad \forall_i. \] (3.41)

The equation with prior information is given
\[ w_{ij} = \frac{p(y_i|x_i,d_i)p(x_i=j)}{p(y_i|d_i)} \] (10)

\[ d_i = \frac{|FR|_i}{|F_{\psi}|_i}, \text{ with } I= (1,1,\ldots,1)^T. \] (3.42)

The mean and variance of the fields with pixel intensities given covariance
\[ R_i = \sum_j L_{ij} \frac{w_{ij}(y_i-y_j)}{\sigma_j^2} \] (3.43)

\[ \psi^{-1}_{ik} = \left( \sum_{\ell} W_{ij} \sigma_j^{-2} \right), \text{ if } i = k \text{ otherwise} \] (3.44)

### 3.5.3 Boundary edge correspondence

The edges and boundaries matches with the shape of Interpolation and its deviations. The points in the boundary are mapped together to find the edge points in the given distance which is minimal and detect false edges the iterations given will have a
proximity for slices in Images The criterion Bayesian will have weighted distance transform to define matrix.

\[ M_{pq} = \frac{1}{\left\| b_p - b_q \right\| + 1} \quad (3.45) \]

The algorithm is adaptive as it is working to various tissues in human body relating to this attenuation coefficient values. The motivation is to reduce noise and blur and detect the edges by differentiating frequency regions as low and high. The statistics gives the acquisition process requires a better reconstruction process for evaluating the tissues of human body from CT.
Fig. 3.17: CT reconstruction model based approach

3.5.4 Local Intensity clustering property

The methods gives a description in a region specific for segmenting the image intensities the overlapping between pixel intensities will give segmented image.

By estimating the bias field with variance to find the local intensity property of surfaces to define neighbourhood pixel values
The region with constant values has

\[ b(X) \approx b(y) \text{ for } X \in \varphi_y. \]  

(3.46)

The intensities \( b(X) \) in each sub region \( \varphi_y \cap \Omega_i \) are close to the constant \( b(y) C_i \)

The image models have less in view

\[ I(X) \approx b(y) C_i + n(X) \text{ for } X \in \varphi_y \cap \Omega_i \]  

(3.48)

Where \( n(X) \) is additive zero-mean Gaussian noise. Therefore, the intensities in the set \( I_i = \{ I(x) : x \in \varphi_y \cap \Omega_i \} \)

form a cluster with cluster centre \( m_i \approx b(y) C_i \).

(3.50)

Then, the clusters \( I_i, \ldots, I_N \), are partitioned centre cluster \( m_i \approx b(y) C_i \), \( i = 1, \ldots, N \), the constants \( C_1, \ldots, C_N \) are distinct and the variance of the Gaussian noise is relatively small.

With Gaussian distribution of the pixel intensities. The segmentation of image with bias field to estimate the proper value.

### 3.5.5 The Formulation of Energy

The property of Intensity is classified from the surface centers which yields the property of K-means to classify the pixel values from the neighbourhood values. The process involves the clustering for the function

\[ F_y = \sum_{i=1}^{N} \int_{\varphi_y} I(X) - m_i^2 u_i(X) dx \]  

(3.51)

The function values of membership give the center of cluster

\[ F_y = \sum_{i=1}^{N} \int_{\Omega_i \cap \varphi_y} I(X) - m_i^2 dx \]  

(3.52)

\( m_i \) is the cluster center of the \( i-th \) cluster, \( u_i \) is the \( X \in \Omega_i \) and \( u_i(X) = 0 \) for \( X \notin \Omega_i \). Since \( u_i \) is the membership function of the region \( \Omega_i \), we
can rewrite $F_y$ as $\epsilon = \sum_{i=1}^{N} \int_{\Omega_i \cap \phi_y} K(y - x) I(x) - b(y) C_i^2 dx \quad (3.53)$

The function defines the intensity classification and clustering criterion function $\epsilon$ can be rewritten as

$$\epsilon = \sum_{i=1}^{N} \int_{\Omega_i} K(y - x) I(x) - b(y) C_i^2 dx \quad (3.54)$$

The formulation takes place with the given neighbourhood method.

The functions with two level set values and membership functions have $\epsilon = \sum_{i=1}^{N} \int_{\Omega_i} K(y - x) I(x) - b(y) C_i^2 dx \quad (3.55)$

### 3.5.6 Experimental Results

Iterative Back projection Reconstruction Describes Reconstruction of CT image with given number of projections CT image before image reconstruction CT image after image reconstruction.

CT Image 1, (a) Sinogram1.(b)First projection1.(c)First Projection

Back projection 1.(d) Reconstructed image 1.(e)

Fig. 3.18: (a, b, c, d, e) gives illustration of how Image reconstruction takes place for CT Image (head section).
Table 3.2 & 3.3 gives details of CT image Before and After Image Reconstruction respectively

<table>
<thead>
<tr>
<th>S No</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Name</td>
<td>CT image of the head</td>
</tr>
<tr>
<td>2</td>
<td>Size</td>
<td>250 x 250</td>
</tr>
<tr>
<td>3</td>
<td>Pixels</td>
<td>62500</td>
</tr>
<tr>
<td>4</td>
<td>Minimum</td>
<td>0 @ 1, 0</td>
</tr>
<tr>
<td>5</td>
<td>Maximum</td>
<td>255 @ 2, 0</td>
</tr>
<tr>
<td>6</td>
<td>Background</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>Average</td>
<td>28.377184</td>
</tr>
<tr>
<td>8</td>
<td>Sum</td>
<td>1773574.0199</td>
</tr>
<tr>
<td>9</td>
<td>Standard deviation</td>
<td>50.905673</td>
</tr>
</tbody>
</table>

Table 3.2 gives details of CT image Before Image Reconstruction

<table>
<thead>
<tr>
<th>S No</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Name</td>
<td>cone beam CT image of the head section</td>
</tr>
<tr>
<td>2</td>
<td>Size</td>
<td>135 x 135</td>
</tr>
<tr>
<td>3</td>
<td>Pixels</td>
<td>17955.</td>
</tr>
<tr>
<td>4</td>
<td>Minimum</td>
<td>0 @ 19 , 0</td>
</tr>
<tr>
<td>5</td>
<td>Maximum</td>
<td>55 @ 65 , 30</td>
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<tr>
<td>6</td>
<td>Background</td>
<td>16</td>
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<tr>
<td>7</td>
<td>Average</td>
<td>54.801158</td>
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<tr>
<td>8</td>
<td>Sum</td>
<td>983954.80001</td>
</tr>
<tr>
<td>9</td>
<td>Standard deviation</td>
<td>73.342285</td>
</tr>
</tbody>
</table>

Table 3.3 Details of CT image After Image Reconstruction respectively
3.6 3D Reconstruction using IMBR Synthetic Data Analysis

To analyze the efficiency of our algorithm (Projection reconstruction) in a controlled environment, extensive computer simulations were performed using the popular Shepp-Logan phantom, as shown in Figures. For a fair evaluation of the performance of Projection reconstruction, it is also implemented with three standard reconstruction methods: 5.4 (a) filtered back-projection with linear interpolation and a Ram-Lak filter (LIN-FBP), (b) filtered back-projection with spline interpolation with a Ram-Lak filter (SPLINE-FBP), and (c) filtered back-projection with cubic interpolation with a Shepp-Logan filter (SPLINE-FBP).

Here, LIN-FBP uses linear interpolation, whereas SPLINE-FBP uses spline-based interpolation. The reason for including SPLINE-FBP is to check whether higher order interpolation might improve the reconstruction quality.

FBP is used in tomography to reconstruct data accurately and it uses Fourier slice theorem. A formal derivation of the FBP method uses the Fourier-slice theorem as follows:

\[
f(x,y) = \int \int F(u,v) e^{-t2\pi(xu+yv)} \, du \, dv
\]

\[
= \int_0^\pi \int_{-\infty}^{\infty} F(v \cos \varphi, v \sin \varphi) e^{-t2\pi v(x \cos \varphi + y \sin \varphi)} \, |v| \, dv \, d\varphi
\]

\[
= \int_0^\pi \int_{-\infty}^{\infty} P_\varphi(v) e^{-t2\pi v(x \cos \varphi + y \sin \varphi)} \, |v| \, dv \, d\varphi
\]

\[
= \int_0^\pi \tilde{p}_\varphi(x \cos \varphi + y \sin \varphi) \, d\varphi
\]

where we define the filtered projection \( \tilde{p}(\varphi) \) as follows:
\[
\bar{p}_\varphi(r) = \int_{-\infty}^{\infty} p_\varphi(v) |v| e^{it2\pi vr} dv \tag{3.60}
\]

The following steps are used in FBP

5. The projections angle for 1D FFT is computed

6. Multiple Projection angle with ramp Filter

7. Then perform periodic convolution the filtering cause aliasing and wrap around artifacts.

8. The DC component nulls the value of Projection.

The filtered sinogram gives the Iterative model based back projection

This practical sense used in pixel driven back projection to the domain. Back projection the filtered sinogram \{p\varphi(r)\} to get \(f(x, y)\), i.e.

\[
\hat{f}(x,y) = \int_0^\pi \bar{p}_\varphi (x \cos \varphi + y \sin \varphi) d\varphi \tag{3.61}
\]

### 3.6.1 Different filters

1. Hann or Hanning: \(A(v) = \left[ \frac{1}{2} + \frac{1}{2} \cos \left( \frac{\pi v}{v_0} \right) \right] \text{rect}(\frac{v}{2v_0}) \) \tag{3.62}

2. Hamming: \(A(v) = [0.54 + 0.46 \cos(\frac{\pi v}{v_0})] \text{rect}(\frac{v}{2v_0}) \) \tag{3.63}

3. Generalized Hamming:

\[
A(v) = [\alpha + (1 - \alpha) \cos(\frac{\pi v}{v_0})] \text{rect}(\frac{v}{2v_0}); \text{for} \ \alpha \in [0,1] \tag{3.64}
\]

4. Butterworth: \(A(v) = \frac{1}{\sqrt{1+(\frac{v}{v_0})^{2n}}} \), \text{for} \ n \geq 0 \tag{3.65}

5. Parzen: \(A(v) = \begin{cases} 
1 - 6 \left( \frac{v}{v_0} \right)^2 (1 - |v|/v_0) & |v| \leq \frac{v_0}{2} \\
2(1 - |v|/v_0)^3 & \frac{v_0}{2} \leq |v| \leq v_0 \\
0 & \text{otherwise}
\end{cases} \tag{3.66}
\]

6. Sheep Logman[57]: \(A(v) = \left| \text{sine} \left( \frac{v}{2v_0} \right) \right| \text{or} \left| \text{sine} \left( \frac{v}{2v_0} \right)^3 \right| \) \tag{3.67}
7. Modified Sheep Logan: $A(v) = \sin\left(\frac{v}{2v_0}\right) \left[ 0.4 - 0.6\cos\left(\frac{\pi v}{v_0}\right) \right]$ (3.68)

3.6.2 3D reconstruction Using FBP with different filters and interpolation methods for projection geometry

Fig. 3.19: Sinogram data from Shepp-Logan phantom. Projection data from (a) 180 views, and (b) 90 views, respectively.

Fig. 3.20: 3D Reconstruction using FBP results from 45° & 120° views with phantom & head section of human body using (a) the filtered back projection algorithm (linear interpolation, Shepp-Logan filter), (b) the filtered back projection algorithm (nearest interpolation, Shepp-Logan filter), (c) the filtered back projection algorithm (spline interpolation, Shepp-Logan filter), (d) the filtered back projection algorithm (cubic interpolation, Shepp-Logan filter).
3.6.3 3D reconstruction Using Iterative model based reconstruction with different filters

The below figure shows a Shep Logan Phantom (128x128) for Phantom of head section with sinogram.

(a) Phantom of head section (b) Sinogram (c) Using Ram-Lak Filter

(d) Cosine Filter (e) Hamming filter (f) Shepp-Logan Filter (g) Hanning Filter

Fig. 3.21: 3D Reconstruction using FBP results from 180° views with phantom of head section of human body using the filtered back projection algorithm (linear interpolation, Shepp-Logan filter), nearest interpolation, Shepp-Logan filter (spline interpolation, Shepp-Logan filter), (cubic interpolation, Shepp-Logan filter).
Table 3.4 Computational Efficiency Measurement

<table>
<thead>
<tr>
<th>S.No</th>
<th>Measurements</th>
<th>CT of head</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mean square error</td>
<td>1.1057e+003</td>
</tr>
<tr>
<td>2</td>
<td>peak signal to noise ratio</td>
<td>17.6944</td>
</tr>
<tr>
<td>3</td>
<td>Normalized cross co-relation</td>
<td>0.6899</td>
</tr>
<tr>
<td>4</td>
<td>Average difference</td>
<td>0.6255</td>
</tr>
<tr>
<td>5</td>
<td>Structural content</td>
<td>1.4177</td>
</tr>
<tr>
<td>6</td>
<td>Maximum difference</td>
<td>221</td>
</tr>
<tr>
<td>7</td>
<td>Normalized absolute error</td>
<td>0.4883</td>
</tr>
</tbody>
</table>

3.6.4. The Image quality metrics

This provides some measure of closeness between two digital images by exploiting the differences in the statistical distribution of pixel values. The most commonly used error metrics used for comparing compression are Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR). Image quality metrics are Figurers of merit used for the evaluation of imaging systems or processes. The image quality metrics can be broadly classified into two categories, subjective and objective. Subjective image quality is a method of evaluation of images by the viewers and it emphatically examines fidelity and at the same time considers image intelligibility. In objective measures of image quality metrics, some statistical indices are calculated to indicate the reconstructed image quality.
Table 3.5: The above table shows the noise and measured resolution for the artifacts identified in region of interest using the image quality metrics.

<table>
<thead>
<tr>
<th>Implementation method</th>
<th>RMSE</th>
<th>PNSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MATLAB’S radon- iradon</td>
<td>0.0439</td>
<td>147.153</td>
</tr>
<tr>
<td>Ram –Lak + linear interpolation</td>
<td>0.1528</td>
<td>72.006</td>
</tr>
<tr>
<td>Ram–Lak+ Nearest Neighbor interpolation</td>
<td>0.1594</td>
<td>70.2642</td>
</tr>
</tbody>
</table>

The factors affecting the assessment of the image resolution are defined.

3.6.5 Computational efficiency measurement

Some statistical indices are calculated to indicate the reconstructed image quality.

Table 3.6: Measurement of the image quality metrics.

<table>
<thead>
<tr>
<th>Measurements</th>
<th>CT of phantom</th>
<th>MRI of hand</th>
<th>CT of head</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean square error</td>
<td>803.2403</td>
<td>3.3288e+003</td>
<td>1.1057e+003</td>
</tr>
<tr>
<td>Peak signal to noise ratio</td>
<td>19.0823</td>
<td>12.9079</td>
<td>17.6944</td>
</tr>
<tr>
<td>Normalized cross correlation</td>
<td>0.7090</td>
<td>0.3285</td>
<td>0.6899</td>
</tr>
<tr>
<td>Average difference</td>
<td>1.0272</td>
<td>8.8198</td>
<td>0.6255</td>
</tr>
<tr>
<td>Structural content</td>
<td>1.5533</td>
<td>3.0208</td>
<td>1.4177</td>
</tr>
<tr>
<td>Maximum difference</td>
<td>179</td>
<td>255</td>
<td>221</td>
</tr>
<tr>
<td>Normalized absolute error</td>
<td>0.3322</td>
<td>0.6877</td>
<td>0.4883</td>
</tr>
</tbody>
</table>
3.7 Multiphase Level Set Formulation

The set formulation with these functions is given as

\[ M_i(\emptyset_1(y), \ldots, \emptyset_k(y)) = \begin{cases} 1, & y \in \Omega_i \\ 0, & \text{else} \end{cases} \]  

(3.69)

The functions \( \emptyset_1, \ldots, \emptyset_k \) to define \( N \) membership functions \( m_i \) of the regions \( \Omega_i, i = 1, \ldots, N \).

Membership functions with energy formulation is given with level set method

\[ M_1(\emptyset_1, \emptyset_2) = H(\emptyset_1)H(\emptyset_2), M_2(\emptyset_1, \emptyset_2) = H(\emptyset_1)(1 - H(\emptyset_2)) \]  

(3.70)

Where in case of \( N = 3 \), we use two level set functions \( \emptyset_1 \) and \( \emptyset_2 \) to defined method. For the four-phase case \( N = 4 \), the definition of \( M_i \) can be defined as

\[ M_1(\emptyset_1, \emptyset_2) = H(\emptyset_1)H(\emptyset_2), M_2(\emptyset_1, \emptyset_2) = H(\emptyset_1)(1 - H(\emptyset_2)), M_3(\emptyset_1, \emptyset_2) \]

\[ = (1 - H(\emptyset_1))H(\emptyset_2), \]

and \( M_4(\emptyset_1, \emptyset_2) = (1 - H(\emptyset_1))(1 - H(\emptyset_2)) \).  

(3.71)

The formulation of energy is given by

\[ M_i(\emptyset_1(y), \ldots, \emptyset_k(\emptyset)) \text{can be written as} \quad M_i(\Phi) \]. The energy \( \epsilon \) in can be converted to a multiphase level set formulation

\[ \epsilon(\Phi, c, b) = \int \sum_{i=1}^{N} e_i(X)M_i(\Phi(X))dx \]  

(3.72)

The energy functional \( F \) in our multiphase level set formulation is defined by

\[ F(\Phi, b, c) \triangleq \epsilon(\Phi, b, c) + R_p(\Phi). \]  

(3.73)

The minimization of the energy \( F(\Phi, c, b) \) in (25) with respect to the variable \( \Phi = (\emptyset_1, \ldots, \emptyset_k) \) can be performed by solving the
The gradient pixel value is selected by filtering equation

$$\frac{\partial \phi_1}{\partial t} = -\sum_{i=1}^{N} \frac{\partial M_i(\phi)}{\partial \phi_1} e_i + v \partial(\phi_1) div\left(\frac{\nabla \phi_1}{\nabla \phi_1}\right)$$

$$+ \mu div\left((d_p(|\nabla \phi_1|)\nabla \phi_1)\right)$$

$$\frac{\partial \phi_k}{\partial t} = -\sum_{i=1}^{N} \frac{\partial M_i(\phi)}{\partial \phi_k} e_i + v \partial(\phi_k) div\left(\frac{\nabla \phi_k}{\nabla \phi_k}\right)$$

$$+ \mu div\left(d_p(|\nabla \phi_k|)\nabla \phi_k\right)$$

(3.74)

(3.75)

3.7.1 CT bias variance measures with Level set Evolution: This shows the reduction of Beam hardening artifact by Iterative based reconstruction the presence of materials such as bones, which have attenuation properties significantly different from that of other tissues results in additional streaks in the reconstructions.
(d) Blurring Of Input Image for Checking   (e) Bias Corrected Image

Fig. 3.22: (a-e) shows bias variance with inner and outer set iterations for ROI detection.

The above results shows the reduction of Beam hardening Artifact by applying iterative Reconstruction and below table gives the Perceptual image quality measures by identifying the Beam hardening artifact.

### 3.7.2 Image Quality and Information Content

Table 3.7 Perceptual Quality measures

<table>
<thead>
<tr>
<th>Measurements</th>
<th>CT of knee bone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean square error</td>
<td>1.0430e+003</td>
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<tr>
<td>peak signal to noise ratio</td>
<td>17.9481</td>
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<tr>
<td>Normalized cross co-relation</td>
<td>0.9613</td>
</tr>
<tr>
<td>Average difference</td>
<td>5.6401</td>
</tr>
<tr>
<td>Structural content</td>
<td>1.0432</td>
</tr>
<tr>
<td>Maximum difference</td>
<td>243</td>
</tr>
<tr>
<td>Normalized absolute error</td>
<td>0.1394</td>
</tr>
</tbody>
</table>
Several factors affect image quality and information content. Good understanding of such factors and appropriate characterization of the concomitant loss in image quality essential in order to design image processing techniques to remove the degradation and/or improve image quality. Inherent problem in characterizing image quality: judged by human observers in a subjective manner.

To quantify the notion of image quality is a difficult proposition. Multifaceted characteristics of information in terms of: statistical, structural, perceptual, semantic, and diagnostic connotations.

3.7.3 Preprocessing Steps

1. Grow air region
2. Erode skin
3. Grow peripheral muscle
4. Original CT volume
5. Grow peripheral fat
6. Preprocessed CT volume

Fig. 3.23 shows surface removal of Peripheral artifacts, skin layer, the peripheral fat, the peripheral muscle
3.7.4 Removal of Peripheral Artifacts and the External Air Region

By definition, air has CT number of $-1000$ HU. Each CT volume is thresholded with the range $-1200$ HU to $+1200$ HU. 2D binary opening-by-reconstruction is applied on a slice-by-slice basis using the thresholded volume as mask and the four corners of each slice as markers. The air region is morphologically dilated in 2D to include the skin. The peripheral fat is the next layer after the skin from the outside of the body; varies in thickness from 3 to 8 mm in children.

- Fat has a mean CT value of $\mu = -90$ HU with $\sigma = 18$ HU.
- Voxels within a distance of 8 mm from the inner skin boundary are examined; if they fall within the range $-90 \pm 2 \times 18$ HU, they are classified as peripheral fat.

Removal of the Peripheral Muscle

Peripheral muscle has a mean CT value of $\mu = +44$ HU with $\sigma = 14$ HU; thickness varies from 6 to 10 mm.
3.7.5 Removal of peripheral artifacts and tissues

Fig. 3.24 shows the artifacts removal from peripherals and tissues

3.8 Expectation and Maximization (EM+TV) Method overview

To estimate the prior information to likelihood a computational algorithm is used. The data with the information lost gives idea to deal with missing data which is computed with expectation.

The likelihood is with the data in complete for maximizing. Perform the convergence by iterating

Proposed EM+TV reconstruction method:

The variation in probability with given sinogram can be found with optimal point of scalarization with given objective function

\[
\begin{align*}
\text{minimize} & \int_{\Omega} |\nabla x| + \alpha \sum_{i=1}^{M} (Ax)_i - b\log(Ax)_i, \\
\text{subject to} & \quad x_j \geq 0, \quad j = 1, \ldots, N, 
\end{align*}
\]

(3.76)

With \( \alpha > 0 \) a tuning parameter
The problem optimized is given with the solution of Karush-Kuhn-Tucker (KKT) conditions:

\[-\text{div} \left( \frac{\nabla x}{|\nabla x|} \right)_j + \alpha \sum_{i=1}^{M} a_{ij} \left( 1 - \frac{b_i}{(Ax)_i} \right) - y_j = 0, \quad j = 1, \ldots, N, \tag{3.77}\]

\[y_j \geq 0, \quad x_j \geq 0, \quad j = 1, \ldots, N,\]

\[y^T x = 0.\]

The movement which is slack has PDE

\[-x_j \text{div} \left( \frac{\nabla x}{|\nabla x|} \right)_j + \alpha \sum_{i=1}^{M} a_{ij} \left( 1 - \frac{b_i}{(Ax)_i} \right) x_j = 0, \quad j = 1, \ldots, N, \tag{3.78}\]

of \(\{x_j\}, \{y_j\}\) and the complementary slackness condition \(y^T x = 0\), we have \(x_j y_j = 0\) for every \(j = 1, \ldots, N\).

The step with expectation and maximization is given

\[-\frac{x_j}{\sum_{i=1}^{M} a_{ij}} \text{div} \left( \frac{\nabla x}{|\nabla x|} \right)_j + \alpha x_j - \frac{\sum_{i=1}^{M} a_{ij} b_i}{x_j a_{ij} a_x} x_j = 0, \quad j = 1, \ldots, N. \tag{3.79}\]

The optimum solution with EM is given as

\[x_j^{EM} = \frac{\sum_{i=1}^{M} a_{ij} b_i}{\sum_{i=1}^{M} a_{ij}} x_j \tag{3.80}\]

The iterations are performed in discrete manner to work with discrete equation

\[-\frac{x_j}{\sum_{i=1}^{M} a_{ij}} \text{div} \left( \frac{\nabla x}{|\nabla x|} \right)_j + \alpha x_j - \alpha x_j^{EM} = 0, \quad j = 1, \ldots, N. \tag{3.81}\]

\[
\text{minimize}_x \int_\Omega |\nabla x| + \alpha \sum_{j=1}^{N} \sum_{i=1}^{M} a_{ij} (x_j - x_j^{EM} \log x_j). \tag{3.82}\]

The computed \(x^{n+1}\) with \(n \geq 0\) from the following linearized discrete equation,

The steps used in iterations have given values to update and provide an algorithm to satisfy the optimum solution.
\[-\frac{1}{\alpha} x_{ij} \sqrt{e + (x_{i+1,j}^{n} - x_{i,j}^{n})^2 + (x_{i,j+1}^{n} - x_{i,j}^{n})^2} \]
\[+ \frac{1}{\alpha} x_{ij} \sqrt{e + (x_{i,j}^{n+1} - x_{i-1,j}^{n})^2 + (x_{i,j}^{n+1} - x_{i-1,j}^{n})^2} \]
\[-\frac{1}{\alpha} x_{ij} \sqrt{e + (x_{i,j}^{n+1} - x_{i,j}^{n+1})^2 + (x_{i,j+1}^{n+1} - x_{i,j}^{n+1})^2} \]
\[+ \frac{1}{\alpha} x_{ij} \sqrt{e + (x_{i,j}^{n+1} - x_{i,j}^{n+1})^2 + (x_{i,j+1}^{n+1} - x_{i,j}^{n+1})^2} + x_{i,j}^{n+1} - x_{i,j}^{EM} \]
\[= 0. \quad (3.83) \]

Where
\[v_{ij} = \sum_{l=1}^{M} a_{ij}. \quad (3.84) \]

### 3.8.1 Model Based approach with maximum likelihood Parameter Estimation subsets

For the reconstructed data which can reconstruct a better image using fewer views in the computed tomography setting and thus reducing the overall dose of the radiation. The above figure shows the appropriate resolution rate for given dose of radiation by using Model based iterative based approach.
Fig. 3.25: The above Figure shows better image Reconstruction using fewer views in the computed tomography thus setting and reducing the overall dose of the radiation.

Fig. 3.26: shows a CT-Opt Reconstruction Filtered back projection
The above results show the appropriate maximum intensity radiation with reduced views with appropriate resolution.

3.9 CT-OPT Reconstruction: This is a noninvasive imaging technique that enables imaging of small specimens.

This reveals unknown background or illumination intensity distributions over the field of view.

Fig. 3.27: shows Human abdomen section for optical reconstruction

Fig. 3.28: shows a sinogram with Gray scale with intensity variations on y-axis and Rotation angles on X-axis
Fig. 3.29: shows Segmentation of OPT of Human abdomen section for better reconstruction

Fig. 3.30: shows binary gradient mask of OPT Human abdomen section

Fig. 3.31: shows Images of human abdomen extracting high frequency components

3.10 Iterative Model based reconstruction

To obtain real optics and image quality the statistics are to be obtained. In major quantum and electronic noise provides major difficulty which works the CT system slow and computationally inefficient with low quality image. The accuracy and physics is a major thing that gives the complexity to given work and implementation. To
model the system optics and X-ray physics the data which is acquired is essential for the process.

The algorithm is adaptive as it is working to various tissues in human body relating to this attenuation coefficient values. The motivation is to reduce noise and blur and detect the edges by differentiating frequency regions as low and high. The statistics gives the acquisition process requires a better reconstruction process for evaluating the tissues of human body from CT.

The problem in Image reconstruction is done directly by using constrained optimization function and other is with sinogram data the process involves filtering and interpolation to reduce the noise and blur to make the system stable. The formulation is used to produce the results in iteration process. The CAD systems developed need a accurate method for disease pathology to refine artifacts the spatial resolution and high noise reduce the image quality.

To improve resolution and reduce noise and increase contrast a significant reconstruction process is needed. The FBP suffers from high noise and modelling physics with presence of materials in resulting streaking and shading artifacts related to various tissues and organs of Human body. To improve accuracy and image quality the noise is to be reduced by improving the contrast with less suppression and appropriate dose reduction to have high quality images. The reconstruction is done iteratively by depending on various factors. It requires appropriate algorithm to define the appropriate dose. The X-rays in tube and patient reconstruction process gives real
data to acquire the desired image. The noise statistics and physics based model and radiation high deals with few approximations for presenting a better image. The data is computed or matched to get the appropriate projection data of CT imaging.

The optimized solution for the reconstruction of CT Image

$$u^* \leftarrow \frac{-\sum_{k \in N_j} W_{jk} b_{jk} + \theta z_{x_j} - \theta 1}{2 \sum_{k \in N_j} W_{jk} a_{jk} + \theta 2}$$

$$[U_{min}, U_{max}]$$

(3.85)

(3.86)

This approach uses an inverse process for projection with the data synthesized to combine real data with the assumed data to identify the noise and represent a desirable image.

The complexity and the solution are available to estimate the answer of image the device algorithm is best solution to find the formulation of the function of CT to provide low noise and better accuracy the image is reconstructed with improved quality.

The quality related to image is increased with iterative reconstruction algorithm getting the reality of image either continuous or discontinuous. The elements of detector provide less projection to acquire the perfect data. The models used in distribution of pixel values have acquired data. The effects of absorption and scattering have more effect in introducing noise in to image the statistics have revealed that the distribution model have the approach to reach the degree with given content of noise.

The algorithms have produced more noise and define the properties which deal with complexity and detail of noise is improved. The convergence values of FBP have less number of weights. To
minimize the noise and represent better image the detector systems should get the projected source radiation. The data is divided in blocks which is known as voxel and beam will result with high noise in the final image.

It also considers the physics based modelling with prior knowledge by using homogeneous regions related to the boundaries of disease pathology. The projection data combined estimates the occurrence of noise in its distribution.

3.10.1 Finding the optimal approach

The approach used to improve the Image quality and reduce noise and dose of radiation related to projection space and converge the termination with proper desired estimates. The converged images will obtain the results with less cost and optimal image quality. The model uses the specified work integrated to have optimized solutions. For accurate segmentation the physics modelling and geometric optics with clinical protocol.

The solution for the reconstruction of CT Image

\[
\frac{-\sum_{k \in N_j} W_{jk} b_{jk} + \theta_2 x_j - \theta_1}{2 \sum_{k \in N_j} W_{jk} a_{jk} + \theta_2}
\]  

(3.87)

\[
[U_{min}, U_{max}]
\]  

(3.88)

3.10.2 To reduce the complexity and improve the image quality reduce computation intensity

The sharpness and spatial resolution and beam hardening and streak artifacts can be reduced with the proposed method. It eliminates blurs and noise and develops better image resolution with projection data. The phantom takes from multiple dose levels resulting
indifferent thickness and composition of image. The image quality scan can be improves Using low dose radiation for abnormal CT images and image quality. The quality can be enhanced by improving the resolution and maintaining the noise and blur measures in providing the acquisition protocol and range measurements. The soft and bony tissue attenuates more for the given projection data which results in streaking and shading artifacts.

3.10.3. CT performance Criteria

The image quality and Radiation dose decides the performance criteria of CT imaging

1. Dose Efficiency: Some of the rays are sensitive to noise because of finite number of projections and the radiation dose is more when high fraction of X-rays comes out of the object. The two factors involving the dose efficiency are efficiency related to the geometry optics with transmitting x-rays interaction and the scintillation occurs is the how many x-rays captured by detectors the contrast deals with absorption efficiency by detectors.

2. Scanner- The scatter is to completely remove with conventional reconstruction methods and removal grids.

3. Spatial Resolution- The spacing of measurements and higher spatial resolution and contrast, object size, speed and temporal resolution will give the measurement of the CT slice plane.
3.10.4 Image Quality with respect to dose radiation

Fig. 3.32: Illustration of Image quality in low dose abdominal CT Scanning at different dose of 200mAs

Fig. 3.33: Illustration of Image quality in low dose abdominal CT Scanning at different dose of 50 mA from a multidose prospective

Fig. 3.34: Head CT for a stroke patient comparing FBP and IMBR images caused result from the physical processes involved in the acquisition of CT data
Fig. 3.35: (a) FBP Reconstruction       (b) IBMR Reconstruction

The above figure shows Abdominal CT examination with FBP AND MBIR reconstruction in the presence of a metal hip implant images presence of metallic materials blocks parts of projection data which results in Stair Step Artifacts Image Resolution

Fig. 3.36 shows scanner based artifacts with Paediatric Trauma imaged for both FBP and IMBR images

Fig. 3.37: Phantom Scanned and imaged at multiple dose levels from 25 to 360mAs 120KVDose results for two representative scan techniques between FBP standard and IMBR
The Terms are FBP – Filtered Back Projection
ART- Algebraic Reconstruction Technique
IBMR- Iterative Model based reconstruction technique

3.11 Performance Measures

Image Noise Comparison with other two methods which are extensively used in the market for image reconstruction process which are Algebraic Reconstruction Technique and Filtered back projection.

Fig. 3.38: Graph gives the detail of noise comparison with dose radiation (0-120mA)
The Profile describes the edge response with given dose radiation (0-230mAs) representing the Tissue characterization in Hounsfield units

Fig. 3.39: Describes the Profile of edge response with given dose radiation (0-230mAs) representing the Tissue characterization in Hounsfield units
The terms referred as FBP – Filtered Back Projection

ART- Algebraic Reconstruction Technique

IMBR- Iterative Model based reconstruction technique

**3.12 Comparison Results**

A simple Schematic describes the total advantages of the proposed work carried in combination with Analytical and iterative Reconstruction methods are Full statistics, Limited physics, Complex optimum value, Removal Artifacts and computational efficiency.

Fig. 3.40: Gives comparison of related algorithms
3.13 **Image quality characterization**

The attributes related to the quality of image requires comparison with many method present and parameter settings should be dynamically change according to the requirement.

The enhancement algorithms discussed will generate better image quality with very low data loss.

3.14 **Summary**

The artifacts from various sources will reduce the image quality and the elements used in scanning object should be appropriate because the grids and detectors will give more noise. In order to reduce the blur and noise better reconstruction softwares with interpolation techniques are used which can minimize and remove the unwanted data. The accurate results promise one in common is spatial resolution, low dose radiation, noise and blur reduction and increased contrast and suppressing the artifacts to the bright future of CT imaging.