Chapter 5

$CP$-Violating Asymmetries in
\[ \gamma\gamma \rightarrow tt \]

So far we have been discussing $CP$ violation in the top-antitop quark pair production in $e^+e^-$ colliders, where we constructed $CP$-odd asymmetries involving final state momenta. These asymmetries, if observed, will tell what the values of the electric and the weak dipole form factors of the top quark are and, if not observed, will help in putting limits on them. The next generation of $e^+e^-$ colliders must be linear colliders for synchrotron radiation makes it difficult for a circular collider to increase its beam energy beyond a certain value. High-energy photon beams could be produced using such linear colliders together with the already available laser technology [4]. This could be done by back-scattering of an intense laser beam off a high-energy electron beam. These photon beams can then be made to collide with either an electron beam or another photon beam. Such photon linear colliders are being discussed extensively in the literature [44 - 49] and are expected to be built in the future. Advantages of such a collider, from the physics point of view, include clean signals for Higgs production, SUSY particle detection, study of SM triple-gauge-boson couplings, etc.
The topic of $CP$ violation in $\gamma \gamma \rightarrow t\bar{t}$ has also elicited some interest recently. Anlauf et al. [47, 48] have discussed $CP$ violation in a Higgs mediated $\gamma \gamma \rightarrow t\bar{t}$ process where they study triple product correlations as well as asymmetries. Choi and Hagiwara [49], and Baek et al. [46] have studied top quark EDFF in $\gamma \gamma \rightarrow t\bar{t}$ with linearly polarized photon beams.

In this chapter we shall discuss some of the possible $CP$-violating effects in the top-antitop pair production by constructing asymmetries out of the momenta of the top (antitop) quark decay products in a $\gamma \gamma$ collider.

The next section will briefly describe the main features of a $\gamma \gamma$ collider and the following section will discuss $CP$ violation in $\gamma \gamma \rightarrow t\bar{t}$ in the presence of top diole moment. In the last section we shall discuss the numerical results and conclusions.

### 5.1 Features of a $\gamma \gamma$ Collider

In a $\gamma \gamma$ collider, high-energy photons would be produced by Compton backscattering of intense low-energy laser beams off high energy electrons [4]. The energy spectrum of a compton scattered photon is given by

$$\frac{1}{\sigma_c} \frac{d\sigma_c}{dy} = f(x, y) = \frac{2\pi \alpha^2}{\sigma_c x m_e^2} \left[ \frac{1}{1 - y} + 1 - y - 4r(1 - x) - 2\lambda r x(2r - 1)(2 - y) \right].$$

(5.1)

Here

$$x = \frac{4E_{\gamma_0}}{m_e^2} = 15.3 \left( \frac{E_\gamma}{\text{TeV}} \right) \left( \frac{\omega_0}{\text{eV}} \right),$$

(5.2)
where $E_b$ is the electron beam energy and $\omega_0$ is the energy of the laser beam. $y$ is given in terms of the energy of the scattered photon, $\omega (\leq E_b \frac{x}{1+y})$, as

$$ y = \frac{\omega}{E_b} $$

and

$$ r = \frac{y}{x(1-y)} \leq 1. $$

$\lambda_e$ and $\lambda_l$ are the initial electron and laser photon helicities respectively. Energy distribution in terms of the variable $y$ is related to that in terms of $\omega$ by

$$ f(\omega) = \frac{1}{\sigma_e} \frac{d\sigma_e}{d\omega} = \frac{1}{E_b} f(x, y). \quad (5.3) $$

Total cross section, $\sigma_e$ is given by

$$ \sigma_e = \sigma_{\text{np}} + 2\lambda_e\lambda_l \sigma_1, $$

with

$$ \sigma_{\text{np}} = \frac{2\pi \alpha^2}{xm_e^2} \left[ \left( 1 - \frac{4}{x} - \frac{8}{x^2} \right) \log(x+1) + \frac{1}{2} + \frac{8}{x} - \frac{1}{2(x+1)^2} \right] $$

and

$$ \sigma_1 = \frac{2\pi \alpha^2}{xm_e^2} \left[ \left( 1 + \frac{2}{x} \right) \log(x+1) - \frac{5}{2} + \frac{1}{1+x} - \frac{1}{2(x+1)^2} \right]. $$
Figure 5.1: Figure on the left shows the energy distribution of Compton scattering for different helicity combinations of the initial electron beam and the laser beam. In the figure on the right side the scattered photon helicity is plotted against the energy of the scattered photon for different helicity combinations of the laser beam and the initial electron beam. Solid and doted lines correspond to \( 2 \lambda_e \lambda_l = 1 \) with \( \lambda_e = 1/2 \) and \( \lambda_e = -1/2 \) respectively while dash line and dash-dot line correspond to \( 2 \lambda_e \lambda_l = -1 \) with \( \lambda_e = 1/2 \) and \( \lambda_e = -1/2 \) respectively.

Here \( \sigma^{np} \) is the unpolarized cross section. The energy spectrum \( f(x, y) \) is plotted against \( y \) in Figure 5.1. \[4\].

It is clear from the figure that when \( \lambda_e \lambda_l < 0 \), there are more number of hard photons than soft photons, while for \( \lambda_e \lambda_l > 0 \) the number of hard photons are less than the number of soft photons. Also in the case of \( \lambda_e \lambda_l < 0 \) the spectrum peaks at higher energies resulting in nearly monochromatic beams.

Polarized photon beams are more appropriate for \( CP \) violation studies. Dependence of the helicity of the Compton scattered photon on the energy of the photon is discussed in \[4\] and is given by

\[
\lambda_s(\omega) = \frac{\lambda_l (1 - 2r) (1 - y + \frac{1}{1-y}) + 2 \lambda_e r x [1 + (1 - y) (1 - 2r)^2]}{1 - y + \frac{1}{1-y} - 4 r (1 - r) - 2 \lambda_e \lambda_l r x (2 r - 1) (2 - y)},
\]

(5.4)
At $2\lambda c\lambda t = -1$, hard photons will have $\lambda = \lambda_c$ (See Figure 5.1). As already mentioned the number of hard photon is much higher than the soft ones at $2\lambda c\lambda t = -1$.

Another important aspect of a collider is its luminosity. The luminosity distribution of a $\gamma\gamma$ collider depends on different factors like the conversion distance, i.e., the distance from the scattering point to the interaction point, energy distribution of the beams, etc. Assuming a Gaussian profile for the electron beam with azimuthal symmetry, the luminosity distribution of a $\gamma\gamma$ collider is given in terms of the photon energy distribution by [4]

$$
\frac{1}{L_{ee}} \frac{dL_{\gamma\gamma}}{d\omega_1 d\omega_2} = f_1(\omega_1) f_2(\omega_2) I_0 \left( \frac{d_1 d_2}{\sigma_1^2 + \sigma_2^2} \right) e^{-\frac{d_1 d_2}{2(\sigma_1^2 + \sigma_2^2)}}. \tag{5.5}
$$

Here $f_1(\omega_1)$ and $f_2(\omega_2)$ are the energy distributions of the two photon beams (see Equation 5.3). $I_0$ is the zeroth order modified Bessel function with $d_i = \omega_i/i$, where $z_i$ is the conversion distance and $\theta_i$, is the scattering angle of the photon beam and $\sigma_i$ is the half width of the Gaussian profile. $L_{ee}$ is the geometrical luminosity of the original electron-electron collider. Making a variable change from $\omega_1$ and $\omega_2$ to $\eta$ and $W$, where $\eta = tan^{-1}(-\frac{\omega_2 - \omega_1}{\omega_1 + \omega_2})$ is the $\gamma\gamma$ rapidity and $W = 2\sqrt{\omega_1\omega_2}$ is the $\gamma\gamma$ invariant mass, we get the luminosity distribution as

$$
\frac{1}{L_{ee}} \frac{dL_{\gamma\gamma}}{dW d\eta} = \frac{W}{2} f_1 \left( \frac{We^{\eta}}{2} \right) f_2 \left( \frac{W e^{-\eta}}{2} \right) I_0 \left( \frac{d_1 d_2}{\sigma_1^2 + \sigma_2^2} \right) e^{-\frac{d_1 d_2}{2(\sigma_1^2 + \sigma_2^2)}}. \tag{5.6}
$$

Taking the conversion distance to be zero for simplicity, the expression for the luminosity distribution becomes

$$
\frac{1}{L_{ee}} \frac{dL_{\gamma\gamma}}{dW d\eta} = \frac{W}{2} f_1 \left( \frac{We^{\eta}}{2} \right) f_2 \left( \frac{W e^{-\eta}}{2} \right). \tag{5.7}
$$
Figure 5.2: Luminosity distributions is plotted against the $\gamma\gamma$ invariant mass, $W = 2\sqrt{\omega_1\omega_2}$. Initial electron beam energy, $E_b$ is taken to be 250 GeV and a laser beam of energy 1.24 eV is assumed. Solid line is for $2\lambda_e\lambda_l = -1$ while doted curve is for $2\lambda_e\lambda_l = 1$. The curve is for zero conversion distance.

Figure 5.2 gives luminosity distribution after rapidity is integrated out. Luminosity peaks at higher values of invariant mass in case of $2\lambda_e\lambda_l = -1$ and the peak value could be as high as 90% of $L_{ee}$, while for $2\lambda_e\lambda_l = 1$ the spectrum is almost a Gaussian peaking at low energies.

For large values of $L_{ee}$, which are possible to achieve, we expect large $t\bar{t}$ production in a $\gamma\gamma$ collider. In the next section we shall discuss some of the $CP$-violating effects which could be tested in these colliders.

5.2 Charge Asymmetries in $\gamma\gamma \to t\bar{t}$

In this section we shall discuss some of the asymmetries defined in the earlier chapters, in the context of a $\gamma\gamma$ collider with features described in the previous section. Rest of this section will discuss the charge asymmetry arising due to the top quark dipole moment in $\gamma\gamma \to t\bar{t}$ process with subsequent decay of $t$ and $\bar{t}$. 

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Charge asymmetry as defined in Chapter 3 is the total leptonic charge asymmetry in the process with $t\bar{t}$ decaying semileptonically.

As in the earlier cases, here again, considering on-shell production of $t\bar{t}$ pairs, the production and the decay parts of the amplitude can be separated (see Section 2.2). The production helicity amplitudes are calculated using a method developed by Vega and Wudka [44] as discussed below.

This method makes use of the identity introduced by Michel and Bouchiat [45], viz.,

$$u(p, \lambda')\bar{u}(p, \lambda) = \frac{1}{2}(\gamma + m)(\delta_{\lambda\lambda'} + \sum \gamma_5 \eta' \sigma'_{\lambda\lambda'}).$$

(5.8)

Here the $\sigma^i$ are the three Pauli matrices and $\eta^i$ are the spin vectors corresponding to the four-momentum $p$, defined such that

$$\eta_i \cdot \eta_j = -\delta_{ij}$$

$$\eta_i \cdot p = 0.$$  

(5.9)

In the case where $\lambda' = \lambda$ expression (5.8) reduces to the usual projection operator for a state of momentum $p$ and helicity $\lambda$. For

$$p = |p| \left( \frac{E}{|p|}; \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta \right),$$

a standard representation of the $\eta_i$ is given by

$$\eta_i^\mu = (0; \cos \theta \cos \phi, \cos \theta \sin \phi, -\sin \theta).$$
\[
\eta^\mu_L = (0; -\sin \phi, \cos \phi, 0)
\]
\[
\eta^\mu_S = \left( \frac{m^2}{|p|}, \frac{E - \vec{p}}{|p|} \right),
\]

(5.10)

where \( \eta^\mu_L = (0; \delta^\mu_L) \) for the case when \(|p| = 0\).

For spinors with different momenta, this gives

\[
u(p', \lambda) \otimes \bar{u}(p, \lambda) = -\lambda \frac{i}{2} \gamma_0 (\not{\lambda} + m) \gamma_5 \eta^\lambda \ e^{i\phi}
\]
\[
u(p', \lambda) \otimes \bar{u}(p, -\lambda) = -\lambda \frac{i}{2} \gamma_0 (\not{\lambda} + m) (1 - \lambda \gamma_5 \eta^\lambda) \ e^{i\phi},
\]

(5.11)

where \( \eta^\lambda = \eta_1 - i \lambda \eta_2 \). Here we have assumed that the masses of the spinors are the same and also \( \vec{p}' = -\vec{p} \).

This method is made use of in calculating the amplitude of the process \( \gamma \gamma \rightarrow t\bar{t} \) with the effective \( t\bar{t} \gamma \) coupling,

\[
\Gamma^j = c^j \gamma_\mu + s^j \gamma_\mu \gamma_5 + \frac{c^j}{2m_t} i \gamma_\nu \gamma_5 (p_t + p_{\bar{t}}), \quad j = \gamma, Z
\]

arising from the Lagrangian given by Equation 1.7.

There are two diagrams, one corresponding to the \( t \)-channel process and the other for the cross channel process, which are shown in Figure 5.3.

The production helicity amplitude \( M(\lambda_{\gamma_1}, \lambda_{\gamma_2}, \lambda_t, \lambda_{\bar{t}}) \) is given in terms of the amplitudes corresponding to the two diagrams in Figure 5.3 by

\[
M(\lambda_{\gamma_1}, \lambda_{\gamma_2}, \lambda_t, \lambda_{\bar{t}}) = M_1(\lambda_{\gamma_1}, \lambda_{\gamma_2}, \lambda_t, \lambda_{\bar{t}}) + M_2(\lambda_{\gamma_1}, \lambda_{\gamma_2}, \lambda_t, \lambda_{\bar{t}})
\]
\[ T_{\mu\nu}(\lambda_1, \lambda_2, \lambda_l, \lambda_f) = \overline{u}(p_1, \lambda_1) \Gamma^\nu \left( \frac{\not{p}_1 - \not{k}_1 + m}{(p_1 - k_1)^2 - m^2} \right) \Gamma^\mu v(p_f, \lambda_f) \]

and

\[ T_{2\mu\nu}(\lambda_1, \lambda_2, \lambda_l, \lambda_f) = \overline{u}(p_1, \lambda_1) \Gamma^\nu \left( \frac{\not{p}_1 - \not{k}_2 + m}{(p_1 - k_2)^2 - m^2} \right) \Gamma^\mu v(p_f, \lambda_f). \]

\( \theta_s \) is the scattering angle in the c.m. frame of \( t\bar{t} \). Using the relation

\[ v(\not{p}, \lambda) = (-)^{\frac{1}{2} - \frac{1}{2}} \gamma^5 u(\not{p}, \lambda) \]

we obtain

\[ T_{1\mu\nu} = \text{Tr} \left\{ (-)^{\lambda_2 - \frac{1}{2}} \gamma^5 u(\not{p}, \lambda_1) \bar{u}(p, \lambda_f) [\Gamma^\nu (\not{p}_1 - \not{k}_1 + m) \Gamma^\mu] \right\} \]
and

\[ T^{\mu\nu}_2 = \text{Tr} \left\{ (-)^{\frac{N}{2}} \gamma^5 u(p, \lambda_t) \bar{u}(p, \lambda_t) \left[ \Gamma^{\mu}(\not{k}_2 - \not{k}_1 + m) \Gamma^{\nu} \right] \right\}. \]

Making use of Equation 5.11 and choosing the polarization vectors

\[ \epsilon^{\mu}(\lambda_{\gamma_1}) = \frac{1}{\sqrt{2}} (0; 1, i \lambda_{\gamma_1}, 0) \]

and

\[ \epsilon^{\mu}(\lambda_{\gamma_2}) = \frac{1}{\sqrt{2}} (0; 1, -i \lambda_{\gamma_2}, 0), \]

we get the helicity amplitudes as

\[
M(\lambda_{\gamma_1}, \lambda_{\gamma_2}, \lambda_t, -\lambda_t) = \frac{4m_t e^2 Q_t^2}{\sqrt{s}(1 - \beta_t^2 \cos^2 \theta_t)} \left\{ (\lambda_{\gamma_1} + \lambda_t \beta_t) \right.
\]

\[
- i d_t 2m_t \left[ 2 + \frac{s}{4m_t^2} \beta_t (\beta_t - \lambda_t \lambda_{\gamma_1}) \sin^2 \theta_t \right]
\]

\[
+ d_t \frac{s \lambda_{\gamma_1}}{2} \left[ \frac{4m_t^2}{s} + \beta_t (\beta_t - \lambda_{\gamma_1} \lambda_t) \sin^2 \theta_t \right] \right\}
\]

\[
M(\lambda_{\gamma_1}, -\lambda_{\gamma_2}, \lambda_t, -\lambda_t) = \frac{4m_t e^2 Q_t^2}{\sqrt{s}(1 - \beta_t^2 \cos^2 \theta_t)} \times \beta_t \sin \theta_t \cos \theta_t \left[ \lambda_{\gamma_1} \gamma_t - i d_t - m_t d_t^2 \right]
\]

\[
M(\lambda_{\gamma_1}, -\lambda_{\gamma_2}, \lambda_t, -\lambda_t) = \frac{4m_t e^2 Q_t^2}{\sqrt{s}(1 - \beta_t^2 \cos^2 \theta_t)} \times \left[ \lambda_t \beta_t + i d_t - \frac{s}{2m_t} \beta_t^2 - d_t^2 \frac{s}{2} \lambda_t \beta_t \right] \sin^2 \theta_t
\]

\[
M(\lambda_{\gamma_1}, -\lambda_{\gamma_2}, \lambda_t, -\lambda_t) = \frac{2\beta_t e^2 Q_t^2}{(1 - \beta_t^2 \cos^2 \theta_t)} \sin \theta_t \left\{ (\lambda_{\gamma_1} \lambda_t + \cos \theta_t) \right.
\]

\[
- d_t^2 \frac{s}{2} \left[ \frac{4m_t^2}{s} \cos \theta_t + \lambda_{\gamma_1} \lambda_t (1 - \beta_t^2 \cos^2 \theta_t) \right] \right\}. \quad (5.13)
\]

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These expressions agree with those in [46].

The number of events for the process $\gamma \gamma \rightarrow X$, in terms of the luminosity distribution (Equation 5.7) and the Stokes parameters of the two photon beams, $\xi_i$ and $\bar{\xi}_i$, is given by Ginzburg et al. [4] as

$$dN_{\gamma \gamma \rightarrow X} = dL_{\gamma \gamma} \left( d\sigma_{00} + \Lambda d\tau + \xi_2 \bar{\xi}_2 d\tau_{22} + \xi_4 d\sigma_{20} + \bar{\xi}_2 d\sigma_{02} \right). \quad (5.14)$$

If the photon beam is circularly polarized, and if there is no transverse electron polarization and linear laser beam polarization, then $\xi_2$ corresponds to the helicity of the photon beam. At zero conversion distance $dN$ becomes

$$dN_{\gamma \gamma \rightarrow X} = dL_{\gamma \gamma} \left( d\sigma_{00} + \xi_2 \bar{\xi}_2 d\tau_{22} + \xi_4 d\sigma_{20} + \bar{\xi}_2 d\sigma_{02} \right). \quad (5.15)$$

Expressions for $d\sigma_{ij}$ and $\xi_2$ are given in Section B.4. The density matrix elements of the production process corresponding to different $d\sigma_{ij}$, in case of $t$ decaying leptonically with $\bar{t}$ decaying hadronically ($\rho^+$) and vice versa ($\rho^-$), are given in Appendix B, Section B.5. Only terms upto order $d_4$ are kept assuming higher order terms are negligible.

Expressions for the decay amplitudes are given by Equations 2.13 and 2.14. Using all these we get the following differential cross sections (see Equation 2.17).

$$\frac{d\sigma_{ij}^\pm}{d\cos\theta_i dE_i d\cos\theta_u d\phi_i} = \frac{3\alpha^2}{16x_w^2 \sqrt{s} \Gamma_i \Gamma_W m_W} \left( \frac{1}{1 - \beta \cos\theta_u} - \frac{4E_i}{\sqrt{s} (1 - \beta^2)} \right)$$

$$\times \left\{ \left[ \rho_{ij}^\pm(++) + \rho_{ij}^\pm(--) \right] (1 - \beta \cos\theta_u) + \left[ \rho_{ij}^\pm(++) - \rho_{ij}^\pm(--) \right] (\cos\theta_u - \beta) + 2 \text{Re} \left( \rho_{ij}^\pm(+-) \right) (1 - \beta^2) \right\}$$
\[
\times \sin \theta_t \sin \theta_i \left( \cos \theta_t \cos \phi_t - \sin \theta_t \cot \theta_t \right) \\
+ 2 \text{Im} \left( \rho_{ij}^\pm \left( \pm \right) \right) \left( 1 - \beta^2 \right) \sin \theta_t \sin \theta_i \sin \phi_t \right). \quad (5.16)
\]

This cross section is integrated over the luminosity spectrum (Equation 5.7) to get the number of events. i.e.,

\[
dN = \int \frac{dL_{\gamma \gamma}}{dW} d\sigma(W) dW,
\]

where \(W = \sqrt{s}\) is the invariant mass of the two photon state.

Charge asymmetry, as defined in Equation 3.3, is redefined in the present case to take care of the luminosity distribution to be

\[
A_{ch} = \frac{1}{2N} \left\{ \int \frac{dL_{\gamma \gamma}}{d\omega_1 d\omega_2} d\omega_1 d\omega_2 \int_{-1}^{1} d\cos \theta_t \\
\times \int_{\theta_0}^{\pi - \theta_0} d\theta_t \left[ \frac{d\sigma^-}{d\cos \theta_t d\theta_t} (\theta_t) - \frac{d\sigma^+}{d\cos \theta_t d\theta_t} (\pi - \theta_t) \right] \right\} \quad (5.17)
\]

Here luminosity distribution in terms of \(\omega_1\) and \(\omega_2\) is taken, rather than that in terms of \(W\) and \(\eta\), anticipating the ease in doing the computation.

Expression for the angular distribution given by Equation 5.16 is in the c.m. frame whereas the expression for \(A_{ch}\) above (Equation 5.17) is in the lab frame. Changing the variable of integration to \(\cos \theta_t\) and realising that the lab frame is obtained by boosting the c.m. frame by \(\beta_\gamma = \frac{\omega_i - \omega_f}{\omega_i + \omega_f}\), we get the lower and the upper limits of integration as

\[
f(\theta_0) = \frac{\cos \theta_0^{cm} + \beta_\gamma}{1 + \beta_\gamma \cos \theta_0^{cm}}
\]
and
\[
 g(\theta_0) = \frac{\cos(\pi - \theta_0^\text{cm}) + \beta_p}{1 + \beta_p \cos(P_\gamma - \theta_0^\text{cm})} = \frac{-\cos \theta_0^\text{cm} + \beta_p}{1 - \beta_p \cos \theta_0^\text{cm}}.
\]

Making use of the fact that
\[
 f(\pi - \theta_0) = \frac{-\cos \theta_0^\text{cm} + \beta_p}{1 - \beta_p \cos \theta_0^\text{cm}} = g(\theta_0)
\]

and
\[
 g(\pi - \theta_0) = \frac{\cos \theta_0^\text{cm} + \beta_p}{1 + \beta_p \cos \theta_0^\text{cm}} = f(\theta_0)
\]

we get the final expression for \( A_{ch} \) as (refer Equation 4.9)
\[
 A_{ch} = \frac{1}{2N} \left\{ \int \frac{dL_{\gamma\gamma}}{d\omega_1 d\omega_2} d\omega_1 d\omega_2 \int_{-1}^{1} d\cos \theta_t \right. \\
 \left. \times \int_{f(\theta_0)}^{g(\theta_0)} d\cos \theta_1 \left[ \frac{d\sigma^-}{d\cos \theta_t d\cos \theta_1} (\theta_t) - \frac{d\sigma^+}{d\cos \theta_t d\cos \theta_1} (\theta_t) \right] \right\} (5.18)
\]

where the differential cross section is in the c.m. frame. A similar expression holds for \( A_{fb} \).

\[
 A_{fb} = \frac{1}{2N} \left\{ \int \frac{dL_{\gamma\gamma}}{d\omega_1 d\omega_2} d\omega_1 d\omega_2 \int_{-1}^{1} d\cos \theta_t \right. \\
 \left. \times \left\{ \int_{f(\theta_0)}^{g(\theta_0)} d\cos \theta_1 \left[ \frac{d\sigma^-}{d\cos \theta_t d\cos \theta_1} (\theta_t) - \frac{d\sigma^+}{d\cos \theta_t d\cos \theta_1} (\theta_t) \right] \right. \right. \\
 + \left. \left. \int_{\beta_t}^{g(\theta_0)} d\cos \theta_1 \left[ \frac{d\sigma^-}{d\cos \theta_t d\cos \theta_1} (\theta_t) - \frac{d\sigma^+}{d\cos \theta_t d\cos \theta_1} (\theta_t) \right] \right\} \right. (5.19)
\]
These asymmetries are used to fix the DFF’s using

\[ A_{ch}(c_d^7, c_d^7) = \frac{1.65}{2\sqrt{N}} \]

as described in Section 1.6. The following section discusses the results we obtained.

### 5.3 Results

Charge asymmetry and forward-backward asymmetry combined with charge asymmetry are studied for different initial beam helicities. Also, fixing a particular helicity combination, asymmetries are studied at different electron beam energies and for different laser beam energies. In doing so, the value of \( x \) is kept constant. Variation of asymmetries with \( x \), fixing variables like the helicities and cut-off angle is also considered. Asymmetries are also studied at different cut-off angles with beam energy and other parameters kept constant. All the calculations are done assuming a geometrical integrated luminosity of 20 \( fb^{-1} \) for the electron-electron collider. We shall discuss the results in the following.

Table 5.1 displays asymmetries obtained for different helicity combinations. There is no combined asymmetry when both \( \lambda_e^1 = \lambda_e^2 \) and \( \lambda_e^3 = \lambda_e^4 \). This is expected as the forward and backward directions cannot be distinguished in this case because the two colliding photons are identical. In SM electromagnetic interactions respect parity and hence the cross section is symmetric under \( \lambda_e^i \leftrightarrow -\lambda_e^i \). Thus the total number of events, which gets contribution only from SM, remains the same under this transformation.

The best limit obtained is \( 2.3 \times 10^{-17} \ e \ cm \) coming from \( A_{fb} \) with initial beam
Table 5.1: Asymmetries and corresponding 90% C.L. limits obtained on the DFF’s for various combinations of initial beam helicities. Top quark mass is kept at $m_t = 174$ GeV and an initial electron beam of energy $E_b = 250$ GeV and a laser beam of energy $w_0 = 1.24$ eV are considered. The cut-off angle take is $\theta_0 = 30^\circ$. $N$ is the total number of events. Asymmetries are for $\Im d_4 = \frac{e}{2m_t}$.

| $\lambda_e^1$ | $\lambda_e^2$ | $\lambda_l^1$ | $\lambda_l^2$ | $N$  | $A_{ch}$ | $A_{fb}$ | $|A_{ch}|$ | $|A_{fb}|$ |
|--------------|--------------|--------------|--------------|-----|---------|---------|---------|---------|
| -0.5         | -0.5         | -1           | -1           | 76  | -0.019  | 0       | 2.76    |         |
| -0.5         | -0.5         | 1            | -1           | 252 | -0.025  | -0.129  | 1.19    | 0.23    |
| -0.5         | -0.5         | -1           | 1            | 252 | -0.025  | 0.129   | 1.19    | 0.23    |
| -0.5         | -0.5         | 1            | 1            | 631 | -0.035  | 0       | 0.54    |         |
| 0.5          | -0.5         | -1           | -1           | 73  | -0.024  | 0.013   | 2.31    | 4.25    |
| 0.5          | -0.5         | 1            | -1           | 32  | -0.021  | -0.080  | 3.89    | 1.03    |
| 0.5          | -0.5         | -1           | 1            | 163 | -0.021  | 0.033   | 1.73    | 1.12    |
| 0.5          | -0.5         | 1            | 1            | 73  | -0.024  | 0.013   | 2.31    | 4.25    |
| -0.5         | 0.5          | -1           | -1           | 73  | -0.024  | -0.033  | 1.73    | 1.12    |
| -0.5         | 0.5          | 1            | -1           | 32  | -0.021  | 0.080   | 3.89    | 1.03    |
| -0.5         | 0.5          | -1           | 1            | 73  | -0.024  | -0.033  | 1.73    | 1.12    |
| 0.5          | 0.5          | -1           | -1           | 631 | -0.035  | 0       | 0.54    |         |
| 0.5          | 0.5          | 1            | -1           | 252 | -0.025  | -0.129  | 1.19    | 0.23    |
| 0.5          | 0.5          | -1           | 1            | 252 | -0.025  | 0.129   | 1.19    | 0.23    |
| 0.5          | 0.5          | 1            | 1            | 76  | -0.019  | 0       | 2.76    |         |

Unpolarized    194  -0.028  0       1.19
Table 5.2: Variation of DFF limits obtained at different beam energies keeping $x$ value at 4.75 (by choosing suitable laser beam energy in each case). Top quark mass of 174 GeV is considered and the cut-off angle is taken to be 30°. Asymmetries are for $\Im d_t = \frac{e}{2m_t}$. Helicities of the initial electron and laser beams are $\lambda_e^1 = -0.5$, $\lambda_e^2 = -0.5$, $\lambda_l^1 = -1$, and $\lambda_l^2 = 1$.

$$\text{Cut-off angle is varied to study the variation of limits on DFF's. The result is}$$
Table 5.3: Limits on EDFF of the top quark from the charge asymmetry and the combined asymmetry for different cut-off angles. Helicities of the initial electron and laser beams are $\lambda_e = -0.5, \lambda_L = -0.5$ and $\delta_1 = 1$. A top quark mass of 174 GeV and an electron beam energy of 250 GeV are used. Laser beam energy is taken to be 1.24 eV, which corresponds to $x = 4.75$. Asymmetries are at 1 mtr.

Asymmetries are tabulated in Table 5.3. Charge asymmetry, which is the total leptonic charge in the semi-leptonic decay of $t\bar{t}$ is zero when there is no cut-off. Charge asymmetry is found to give best limits on the dipole form factors around a cut-off of 60° whereas the combined asymmetry is better at lower cut-offs.

For a fixed beam energy and at a fixed cut-off angle, variation of DFF limits with $x$ is studied in Table 5.4. From the table it is clear that the limits are better at higher $x$ values.\(^1\)

To conclude, we find that for an electron beam energy of 250 GeV, and for a suitable

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\(^1\)For $x > 4.83$ $e^+e^-$ production due to the collision of high energy photon beam with laser beam is considerable [4]. This introduces additional $e^+e^-$ beam backgrounds as well as degrading the photon spectrum.
Asymmetries from $x_{N2}$.

| $x$  | $N$  | $A_{ch}$ | $A_{fb}$ | $|A_{ch}|$ | $|A_{fb}|$ |
|------|------|---------|---------|----------|----------|
| 2.60 | 1.1  | -0.039  | 0.0343  | 112.81   | 12.83    |
| 3.20 | 28.8 | -0.0111 | 0.0717  | 7.85     | 1.22     |
| 4.74 | 250.8| -0.2468 | 0.128   | 1.20     | 0.23     |

Table 5.4: DFF's calculated at different $x$ values for a fixed beam energy, $E_b = 250$ GeV at a cut-off angle of 30°. Top quark mass is taken to be 174 GeV and the helicities of the initial beams are $\lambda_e^1 = -0.5, \lambda_e^2 = -0.5, \lambda_t^1 = -1$ and $\lambda_t^2 = 1$.

The choice of longitudinal polarizations of the laser photons and electron beams, and assuming a geometrical luminosity of 20 fb$^{-1}$ for the electron beam, it is possible to obtain limits on the imaginary part of the top EDFF of the order of $10^{-17}$ e cm. An order of magnitude improvement is possible if the beam energy is increased to 500 GeV. In that case the sensitivity would be comparable to that obtained in $e^+e^- \rightarrow t\bar{t}$ with $\sqrt{s} = 500$ GeV.

For comparison, we mention the result of Choi and Hagiwara [49]. They have obtained limits on the real part of the dipole moments of the top quark from a number asymmetry and with linearly polarized photons, to be of the order of $10^{-18}$ e cm for a beam energy of 250 GeV.