Chapter 2

$CP$-violating Asymmetries in $e^+e^- \rightarrow t\bar{t}$

This chapter discusses some $CP$-violating asymmetries arising due to electric and weak dipole moments of the top quark in top-antitop pair production in an $e^+e^-$ collider. We make use of an effective Lagrangian (Equation 1.7), which includes dipole interaction terms in addition to the SM terms. As mentioned in Section 1.5, the top quark system is chosen because heaviness of the top quark implies that it decays before hadronization takes place [21] and the decay products preserve top quark spin information which can be used to study $CP$ properties of the interaction. In the following we have defined $CP$-odd asymmetries which signal $CP$ violation when the initial state is a $CP$ eigenstate. These asymmetries are used to determine the dipole moments, which are the parameters of the theory. We have four parameters altogether, viz., the real and the imaginary parts of the electric and the weak dipole form factors and hence we require at least four asymmetries to fix them.

Rest of the chapter discusses some $CP$-odd asymmetries which may be constructed
out of final state momenta. Section 2.1 will introduce the asymmetries. Expressions for the asymmetries and details of the calculations are given in Section 2.2. In the last section we shall discuss the results and possible conclusions drawn from it.

2.1 CP-Odd Asymmetries

In the process $e^+e^- \rightarrow t\bar{t}$ spins and momenta of the top quark and the antitop quark and the momentum of the electron are the variables we would be able to use to construct the asymmetries with. It is essential to consider spins as it is not possible to construct $CP$-odd asymmetries (or correlations) only using the momenta [25].

$CP$ violation effects in $e^+e^-$ colliders ¹ using $CP$-odd correlations and $CP$-odd asymmetries have been studied by various groups [27, 28]. One of the asymmetries which directly depends on the top quark polarization is the following.

Among the different helicity combinations of $t\bar{t}$ pair produced, $t_L\bar{t}_L$ and $t_R\bar{t}_R$ are $CP$ conjugates of each other, while $t_L\bar{t}_R$ and $t_R\bar{t}_L$ are $CP$ self-conjugates. Thus any asymmetry in the production rates of $t_L\bar{t}_L$ and $t_R\bar{t}_R$ will signal $CP$ violation. This possibility is discussed by Schmidt and Peskin [29] in the context of hadron colliders and in a supersymmetric model, and in the case of an $e^+e^-$ collider by Chang et al. [30]. This number asymmetry can be converted into a lepton energy asymmetry in the limit of bottom quark mass going to zero as explained below [29].

Top quark being heavy, decays mostly into longitudinally polarized $W^+$ and $b$ and the $W^+$ in turn decays into leptons or hadronic jets. Antitop quark also follows a

¹For studies on hadron colliders see, e.g., Ref. [26].
similar decay process.

\[
\begin{align*}
    t & \to W^+ b \\
    \bar{t} & \to W^- \bar{b} \\
    W^+ & \to l^+ \nu X \\
    W^- & \to l^- \bar{\nu} \bar{X}
\end{align*}
\]

In the limit \( m_b \to 0 \), the \( b \) quark is left-handed and hence will move opposite to the top quark spin direction. Thus \( t_R \) decay will have a \( W \) moving in the top quark momentum direction while \( t_L \) will produce a \( W \) which moves against the top momentum direction. Hence the antilepton produced in the decay of \( W \) from \( t_R \) will be more energetic than the one produced in the decay of \( W \) from \( t_L \). On the other hand leptons produced in the decay of \( \bar{t}_L \) will be more energetic than that from \( \bar{t}_R \). Therefore \( t_R \bar{t}_R \) will have more energetic antileptons than leptons and \( t_L \bar{t}_L \) will have more energetic leptons than antileptons. Thus the number asymmetry in the \( t_R \bar{t}_R \) and \( t_L \bar{t}_L \) becomes an asymmetry in the energy of leptons and antileptons produced. Energy asymmetry between distributions of \( l^+ \) and \( l^- \) at the same value of \( x = x(l^+) = x(l^-) = 4 E(l^+)/\sqrt{s} \) is given by

\[
A_E(x) = \frac{1}{\sigma} \left[ \frac{d\sigma}{dx(l^+)} - \frac{d\sigma}{dx(l^-)} \right].
\]

Extending the work of [30], we have discussed this energy asymmetry in the presence of longitudinal beam polarization.

The other asymmetry discussed in [30] is the so called up-down asymmetry. Asymmetry in the number of leptons and antileptons taken together between the two hemispheres separated by the \( t\bar{t} \) production plane is a \( CP \)-odd quantity. Here up/down refers to \((p_{l^\pm})_y \geq 0\), \((p_{l^\pm})_y \) being the \( y \) component of \( p_{l^\pm} \), with respect to a coordinate system chosen in the \( e^+e^- \) center-of-mass (c.m.) frame so that the \( z \)-axis is along \( p_t \), and the \( y \)-axis is along \( p_x \times p_t \). The \( t\bar{t} \) production plane is thus
the \( xz \) plane.

Schematically the up-down asymmetry [30] is

\[
A_{ud} = \int_{-1}^{+1} A_{ud}(\theta) \, d \cos \theta,
\]

(2.2)

where

\[
A_{ud}(\theta) = \frac{1}{2\sigma} \left[ \frac{d\sigma(l^+, \text{up})}{d \cos \theta} - \frac{d\sigma(l^+, \text{down})}{d \cos \theta} + \frac{d\sigma(l^-, \text{up})}{d \cos \theta} - \frac{d\sigma(l^-, \text{down})}{d \cos \theta} \right].
\]

(2.3)

Here \( \theta \) refers to the scattering angle, i.e., the angle between \( \vec{p}_l \) and \( \vec{p}_e \) in the c.m. frame.

As already mentioned we have four parameters here: real and imaginary parts of the electric and the weak dipole form factors. Depending on the \( CPT \) property (here \( T \) is the naive time reversal with only the momenta and spins reversed) of the asymmetry it will be proportional to either the real or the imaginary part of DFF's. That is because a \( CPT \)-odd observable must be proportional to the absorptive part. (For details see, e.g., Rindani [31].) The up-down asymmetry defined above is \( CPT \)-even and hence proportional to the real parts of DFF's. To disentangle the electric and weak dipole form factors we need at least one more \( CP \)-odd asymmetry which is \( CPT \)-even, and to fix the imaginary parts we need another two \( CP \) and \( CPT \)-odd asymmetries. We therefore propose the following new asymmetries [32].

A combination of the up-down and forward-backward asymmetry \( A_{ud}^{FB} \) is again \( CPT \) even which could be used together with \( A_{ud} \) to disentangle the real parts of
DFF's. Given the definition of $A_{ud}(\theta)$ (Eqn. 2.3), $A_{ud}^{lh}$ is defined as

$$A_{ud}^{lh} = \int_0^1 A_{ud}(\theta) \, d\cos\theta - \int_{-1}^0 A_{ud}(\theta) \, d\cos\theta.$$  \hspace{1cm} (2.4)$$

Next we define a left-right asymmetry along the lines of the up-down asymmetry, but now with the hemispheres separated by the $yz$ plane in the same coordinate system described earlier. i.e., left/right refers to $(p_{ls})_r \geq 0$. This asymmetry along with the forward-backward combined left-right asymmetry helps to fix the imaginary parts of DFF's, as they are CPT odd. The following equations define these asymmetries:

The left-right asymmetry is

$$A_{lr} = \int_{-1}^{+1} A_{lr}(\theta) \, d\cos\theta,$$  \hspace{1cm} (2.5)$$

where

$$A_{lr}(\theta) = \frac{1}{2\sigma} \left[ \frac{d\sigma(l^+, \text{left})}{d\cos\theta} - \frac{d\sigma(l^+, \text{right})}{d\cos\theta} + \frac{d\sigma(l^-, \text{left})}{d\cos\theta} - \frac{d\sigma(l^-, \text{right})}{d\cos\theta} \right].$$  \hspace{1cm} (2.6)$$

and the combined left-right and forward-backward asymmetry is

$$A_{lr}^{fb} = \int_0^1 A_{lr}(\theta) \, d\cos\theta - \int_{-1}^0 A_{lr}(\theta) \, d\cos\theta.$$  \hspace{1cm} (2.7)$$

In case of $e^+e^-$ colliders with longitudinal beam polarization we need to have only one asymmetry in each category (CPT-odd or CPT-even) to disentangle the four parameters. These asymmetries measured at different beam polarizations – either differing in magnitudes or simply differing in sign – will help to disentangle the parameters. As we will see later, using polarized beams also improves
the sensitivity of the measurement of asymmetries. (A similar thing happens in
the case of $CP$-violating asymmetries in $\tau\tau$ production [33].) Presently more than
80% electron beam polarization is available at SLAC [2], where they use strained
GaAs as photocathode. KEK/Nayoga/NEC collaboration has achieved 71% po-
larization [3] using a GaAs-AlGaAs superlattice photocathodes. NLC is expected
to have similar degree of polarization. However, we use the conservative value of
50% for the polarization expected at NLC.

Next section discusses these asymmetries in detail.

2.2 Calculation of Asymmetries

The cross section for the process $e^+e^- \rightarrow t\bar{t}$ and subsequent decay of the quarks is
calculated treating top quark and antitop quark as produced on shell in the narrow
width approximation. In that case it is possible to split the production and decay
parts at the amplitude level [34, 35]. Spinor techniques developed by Gastman and
Wu [36] can be used to compute helicity amplitudes of processes involving mass-
less fermions. This technique was later on extended to include massive fermions
by Kleiss and Stirling [37]. We use this technique to calculate the helicity ampli-
tudes. Furthermore we assume on-shell production of $W$'s in $t$ and $\bar{t}$ decay, again
in the narrow width approximation.

To the first order in $c_\alpha^2$ and $c_\beta^2$ the production helicity amplitudes $e^{2 \hat{M}_p(\lambda_e, \lambda_\tau, \lambda_t, \lambda_\bar{t})}$,
where $\lambda_e$, $\lambda_\tau$, $\lambda_t$ and $\lambda_\bar{t}$ are twice the electron, positron, top quark and antitop
quark helicities respectively, are given by

\[ \hat{M}_p(-+++) = \left[ (c_\gamma^2 + r L c_\beta^2 - \beta r L c_\gamma^2) \right] (1 + \cos \theta), \]  

(2.8)
Here $c^z_v$, $c^z_a$, $c^z_{\omega}$ are the vector and axial vector couplings of the top quark to photon and Z-boson whose values are

\[
\begin{align*}
c^z_v &= \frac{2}{3}, \quad c^z_a = 0, \\
c^z_{\omega} &= \frac{\left(1 - \frac{2}{3} x_w\right)}{\sqrt{x_w(1 - x_w)}}, \\
c^z_{\omega} &= -\frac{1}{4\sqrt{x_w(1 - x_w)}},
\end{align*}
\]

with $x_w = \sin^2\theta_w$, $\theta_w$ being the weak mixing angle. $t = \frac{m_t^2}{\sqrt{s}}$, where $m_t$ is the top quark mass and $\sqrt{s}$ is the c.m. energy. $\beta = \sqrt{1 - 4t^2}$ is the top quark velocity. $c^j_t$ ($j = \gamma, Z$) are the electric and the weak dipole form factors and $\theta$ is the scattering angle. $r_L$, the product of the Z propagator and the electron coupling with the Z boson is given by

\[
r_L = \frac{\left(1 - \frac{1}{2} x_w\right)}{\left(1 - \frac{m_t^2}{s}\right) \sqrt{x_w(1 - x_w)}}.
\]

The above amplitudes are of the helicity combination $e_L e_R$. For the other combination, $e_R \bar{e}_L$ we have helicity amplitudes

\[
M_p(- - -) = - \left[ (c^z_v + r_L c^z_{\omega} - \beta r_L c^z_{\omega}) \right] (1 - \cos \theta),
\]

\[
M_p(- + -) = i \left[ 2t \left( c^z_v + r_L c^z_{\omega} \right) - \frac{i \beta}{2t} \left( c^z_a + r_L c^z_{\omega} \right) \right] \sin \theta,
\]

\[
M_p(- + +) = i \left[ 2t \left( c^z_v + r_L c^z_{\omega} \right) + \frac{i \beta}{2t} \left( c^z_a + r_L c^z_{\omega} \right) \right] \sin \theta.
\]
\[ M_p(+-++) = \left[ (c_v^2 + r_L c_d^2 + \beta r_L c_d^2) \right] (1 + \cos \theta), \]
\[ M_p(---) = i \left[ 2t \left( c_v^2 - r_L c_v^2 \right) - \frac{i \beta}{2} \left( c_d^2 + r_L c_d^2 \right) \right] \sin \theta, \]
\[ M_p(+-+) = i \left[ 2t \left( c_v^2 + r_L c_v^2 \right) + \frac{\beta}{t} \left( c_d^2 + r_L c_d^2 \right) \right] \sin \theta, \]

where

\[ \tau_R = \frac{-x_w}{\left( 1 - \frac{m_t^2}{x_w} \right)^{1/2} x_w (1 - x_w)} \] (2.12)

These amplitudes agree with those given in [30] upto a phase. The main decay channel of heavy top quark is \( t \to bW^+ \) with on shell \( W^+ \). We consider the case where these \( W^+ \)s further decay into leptons. The decay density matrix elements are given by

\[ \Gamma_d(++) = \left[ \frac{m_t^2}{2} - m_t E_{l^+} \right] 8m_t E_{l^+} (1 + \cos \theta_l) \] (2.13)
\[ \Gamma_d(--) = \left[ \frac{m_t^2}{2} - m_t E_{l^+} \right] 8m_t E_{l^+} (1 - \cos \theta_l), \]
\[ \Gamma_d(+-) = \left[ \frac{m_t^2}{2} - m_t E_{l^+} \right] (-8i) m_t E_{l^+} \sin \theta_l e^{i \phi_l}. \]

Here \( \pm \) in the l.h.s. denote the helicity of the top quark and \( \theta_l \) and \( \phi_l \) are the polar and the azimuthal angles of the top quark. A common constant of \( \left( \frac{2 \pi n}{x_w} \right)^2 \pi \delta \left( \frac{p_{W^+}^2 - m_{W^+}^2}{l_{W^+} M_{W}} \right) \) has been factored out.

Corresponding density matrix elements for \( \bar{t} \) decay are given by

\[ \bar{\Gamma}_d(++) = \left[ \frac{m_t^2}{2} - m_t E_{l^-} \right] 8m_t E_{l^-} (1 - \cos \theta_l), \] (2.14)
\[
\Gamma_d(--) = \left[ \frac{m_t^2}{2} - m_t E_{t^-} \right] 8 m_t E_{t^-} (1 + \cos \theta_t),
\]

\[
\Gamma_d(++) = \left[ \frac{m_t^2}{2} - m_t E_{t^-} \right] (-8i) m_t E_{t^-} \sin \theta_t e^{-i\phi_t}.
\]

With these the differential cross section for \(tt\) production followed by \(t\) decay will be

\[
d\sigma^+ = \frac{(4\pi \alpha)^4}{4 \pi^2} \frac{\pi^2}{\Gamma_1 m_t \Gamma_W m_W} \delta(p_{W^+}^2 - m_W^2) \delta(p_t^2 - m_t^2) \times \left\{ \rho(++) \Gamma_d(++) + \bar{\rho}(---) \bar{\Gamma}_d(---) \right\} + 2 \text{Re} \left[ \rho(+-) \Gamma_d(+-) \right] d\mathcal{L}ip,
\]

where \(\rho(\lambda_t, \lambda_t')\) are production density matrices with top quark spin state shown explicitly;

\[
\rho(\lambda_t, \lambda_t') = \frac{1}{4} \sum_{\lambda_e, \lambda_e'} \left[ M_p(\lambda_e, \lambda_e', \lambda_t, \lambda_t') M_p^*(\lambda_e, \lambda_e', \lambda_t', \lambda_t) \right] (1 + \lambda_e P_e) (1 + \lambda_e P_e)
\]

and \(d\mathcal{L}ip\) is the Lorentz invariant phase space element. The differential cross section for \(tt\) production followed by \(t\) decaying into \(\bar{b}l^-\bar{\nu}\) through \(W^-\) is given by

\[
d\sigma^- = \frac{(4\pi \alpha)^4}{4 \pi^2} \frac{\pi^2}{\Gamma_1 m_t \Gamma_W m_W} \delta(p_{W^-}^2 - m_W^2) \delta(p_t^2 - m_t^2) \times \left\{ \bar{\rho}(++) \bar{\Gamma}_d(++) + \bar{\rho}(---) \bar{\Gamma}_d(---) \right\} + 2 \text{Re} \left[ \bar{\rho}(+-) \bar{\Gamma}_d(+-) \right] d\mathcal{L}ip,
\]

with

\[
\bar{\rho}(\lambda_t, \lambda_t') = \frac{1}{4} \sum_{\lambda_e, \lambda_e'} \left[ \bar{M}_p(\lambda_e, \lambda_e', \lambda_t, \lambda_t') \bar{M}_p^*(\lambda_e, \lambda_e', \lambda_t', \lambda_t) \right] (1 + \lambda_e P_e) (1 + \lambda_e P_e).
\]
Substituting expressions for the helicity amplitudes given in Equations 2.8–2.14, we get expression for the cross section as

\[
\frac{d\sigma^\pm}{d\cos\theta_id\theta_i d\phi_i} = \frac{3\alpha^4\beta}{16\alpha^2\sqrt{\beta}} \frac{E_i}{\Gamma_f\Gamma_W m_W} \left( \frac{1}{1 - \beta \cos\theta_{H}} - \frac{4E_i}{\sqrt{\beta}(1 - \beta^2)} \right) \\
\times \left\{ \left( A_0 + A_1 \cos\theta_i + A_2 \cos^2\theta_i \right) (1 - \beta \cos\theta_{H}) \right. \\
+ \left( B_0^\pm + B_1 \cos\theta_i + B_2^\pm \cos^2\theta_i \right) (\cos\theta_{H} - \beta) \\
+ \left( C_0^\pm + C_1^\pm \cos\theta_i \right) (1 - \beta^2) \sin\theta_i \sin\theta_i (\cos\theta_i \cos\phi_i - \sin\theta_i \cos\theta_i) \\
+ \left( D_0^\pm + D_1^\pm \cos\theta_i \right) (1 - \beta^2) \sin\theta_i \sin\theta_i \sin\phi_i \right\}. \tag{2.17}
\]

Here \( \pm \) refers to the \( l^+ \) and \( l^- \) distributions. Expressions for \( A, B, C \) and \( D \) are given in Section B.1. Asymmetries defined in the previous section (Equations 2.2-2.7) are given by the following expressions [30, 32]:

\[
A_E(x) = \frac{2\beta}{C} \left\{ f_L(x, \beta) - f_R(x, \beta) \right\} \\
\times \left\{ \text{Im} c_d^r \left[ (1 - P_e P_e) \left( 2 c_v^2 + (r_L + r_R) c_v^2 \right) \right] \\
+ (P_e - P_e) (r_L - r_R) c_v^2 \right\} \\
+ \text{Im} c_d^l \left[ (1 - P_e P_e) \left( (r_L + r_R) c_v^2 + (r_L^2 + r_R^2) c_v^2 \right) \right] \\
+ (P_e - P_e) \left( (r_L - r_R) c_v^2 + (r_L^2 - r_R^2) c_v^2 \right), \tag{2.18}
\]

where

\[
C = (1 - P_e P_e) \left\{ (3 - \beta^2) \left[ (c_v^2 + r_L c_v^2)^2 + (c_v^2 + r_R c_v^2)^2 \right] \\
+ 2\beta^2(c_a^l)^2 \left( r_L + r_R \right) \right\} \\
+ (P_e - P_e) \left\{ (3 - \beta^2) \left[ (c_v^2 + r_L c_v^2)^2 - (c_v^2 + r_R c_v^2)^2 \right] \right\}
\]

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\[ + 2\beta^2 (c^2_{\alpha})^2 \left( r_L^2 - r_R^2 \right) \] 

and the lepton energy distribution in t decay is given for left and right top helicities by [38]

\[
f_{L,R}(x, \beta) = \int_{\frac{1-\beta}{1+\beta}}^{\frac{1+\beta}{1-\beta}} f(x_0) \frac{\beta x_0 \pm (x - x_0)}{2 x_0^2 \beta^2} \, dx_0,
\]

\[ f(x_0) \] being the distribution in the t rest frame,

\[
f(x_0) = \frac{x_0(1 - x_0)}{\frac{1}{6} - \frac{1}{2} \left( \frac{\beta}{m} \right)^4 + \frac{1}{3} \left( \frac{\beta}{m} \right)^6} \theta(1 - x_0) \theta(x_0 - \frac{m^2}{m^2_t}).
\]

\[ \beta = (1 - 4m^2_t/s)^{1/2} \] is the top velocity in the c.m. frame.

\[
A_{ud} = -\frac{3\pi\beta\sqrt{s}}{16m_tC} c^2_{\alpha} \left\{ \text{Re} \, c^2_d \left[ (1 - P_e P_e)(r_L - r_R)c^Z_{\nu} \right] + (P_e - P_e)(2c^Z_{\nu} + (r_L + r_R)c^Z_{\nu}) \right\} + \text{Re} \, c^Z_{\nu} \left[ (1 - P_e P_e) \left( (r_L - r_R)c^Z_{\nu} + (r_L^2 - r_R^2)c^Z_{\nu} \right) \right] + (P_e - P_e) \left( (r_L + r_R)c^Z_{\nu} + (r_L^2 + r_R^2)c^Z_{\nu} \right),
\]

\[
A_{ub} = \frac{\beta^2\sqrt{s}}{4m_tC} c^2_{\alpha} \left\{ \text{Re} \, c^2_d \left[ (1 - P_e P_e)(r_L + r_R) \right] + (P_e - P_e)(r_L - r_R) \right\} + \text{Re} \, c^Z_{\nu} \left[ (1 - P_e P_e)(r_L^2 + r_R^2) + (P_e - P_e)(r_L^2 - r_R^2) \right],
\]

\[
A_{tr} = -\frac{3\pi\beta^2\sqrt{s}}{16m_tC} c^2_{\alpha} \left\{ \text{Im} \, c^2_d \left[ (1 - P_e P_e)(r_L - r_R) \right] + (P_e - P_e)(r_L + r_R) \right\} + \text{Im} \, c^Z_{\nu} \left[ (1 - P_e P_e)(r_L^2 - r_R^2) + (P_e - P_e)(r_L^2 + r_R^2) \right],
\]

(2.23)
As mentioned earlier these asymmetries depend linearly on either the real or the imaginary part of DFF's.

These asymmetries can be measured at a future $e^+e^-$ collider. The next linear collider (NLC) with an integrated luminosity of 10 fb$^{-1}$ is assumed in our calculations. A higher luminosity, if available, will improve the limits which can be obtained.

The next section discusses the numerical results we obtained.

### 2.3 Numerical Results

In our calculations we have assumed the NLC with c.m. energy, $\sqrt{s} = 500$ GeV and an integrated luminosity $\int \mathcal{L} = 10$ fb$^{-1}$. We look at only semi-leptonic events, viz., either of $t$ or $\bar{t}$ decays leptonically, while the other decays hadronically. Furthermore, we do not consider the top quark decaying into tau leptons as the experimental detection is difficult in that case.

The cross section for a top mass of 174 GeV is plotted as a function of $\sqrt{s}$ in Figure 2.1.

Figure 2.2 shows the asymmetries plotted against the centre of mass energy of the
Figure 2.1: Cross section for $t\bar{t}$ production is plotted against c.m. energy. The top mass is taken to be 174 GeV. Curves are plotted for different beam polarizations.

collider for typical values of the real and the imaginary parts of DFF's.

Procedure to obtain limits on the values of DFF's from the experimental measurements of asymmetries and the sensitivities of the experiments is already described in section 1.6. Equations 1.10 and 1.11 in case of the asymmetries described in this chapter become

$$\delta c_d^j = \frac{1.64 c_d^j}{2\sqrt{N_A}}$$

and

$$A(c_d^j, c_d^j) = \frac{2.15}{2\sqrt{N}}, \quad (2.26)$$

where $A$ stands for any one of $A_{udt}$, $A_{udb}$, $A_{lr}$ or $A_{b}$. We plot contours obtained using Equation 2.26 showing the allowed region at 90% C.L., if no signal for $CP$ violation is seen experimentally.

Figure 2.3 shows bands in the $|\text{Re } c_d^j| - |\text{Re } c_d^j|$ plane which correspond to 90% C.L.
Figure 2.2: $A_{ud}$ (left) and $A_{ub}^f$ (right) are plotted against c.m. energy at a top mass of 174 GeV for the case of unpolarized electron beams. Solid lines correspond to $c_d^2 = 0.005$ and dashed lines correspond to $c_d^2 = 0.005$ with the other dipole form factor taken to be zero in each case.

limits obtained from $A_{ud}$ and $A_{ub}^f$ with and without longitudinal beam polarization. In case of unpolarized beams, while $A_{ud}$ or $A_{ub}^f$ taken singly can limit one of $|\text{Re} c_d^2|$ or $|\text{Re} c_f^2|$ when the other is known, both the asymmetries put together can provide independent limits on $|\text{Re} c_d^2|$ and $|\text{Re} c_f^2|$ of the order of 5 and 1.5 respectively. Figure 2.3 also shows bands from $A_{ud}$ for $e^-$ polarization $P_e = \pm 0.5$ (with $P_\perp = 0$). The limits obtainable are improved by an order of magnitude. The top quark mass is taken to be 174 GeV in these cases. A similar analysis is done for top quark masses of 180 GeV and 200 GeV the plots of which are shown in Figures 2.4-2.5.

Figures 2.6-2.8 show contours obtained from $A_{tr}$ and $A_{tr}^f$ for different top quark masses. This puts 90% C.L. limits on $|\text{Im} c_d^2|$ and $|\text{Im} c_f^2|$. Again, for $P_e = 0$, only a simultaneous search for both these asymmetries can put independent limits on $|\text{Im} c_d^2|$ and $|\text{Im} c_f^2|$, of the order of 0.7 and 6, respectively. Limits on $A_{tr}$ with $P_e = \pm 0.5$, also shown in these figures, can improve these numbers by a factor of about 4 - 7.
Figure 2.3: Contours showing the allowed region of $\text{Re}c^u_D - \text{Re}c^d_D$ plane. Top mass is taken to be 174 GeV. Unpolarized beam is considered in case of figure on the left where contours are obtained from the asymmetries $A_{ud}$ and $A^d_{ud}$, while only $A_{ud}$ is considered with different beam polarizations in the other figure. A c.m. energy of 500 GeV and a luminosity of $10 \text{ fb}^{-1}$ are assumed.

Figure 2.4: Contours similar to those of Figure 2.3 but with top quark mass of 180 GeV.
Figure 2.5: Contours similar to those of Figures 2.3 and 2.4 but with top quark mass of 200 GeV.

Figure 2.6: Contours showing the allowed region of $|\text{Im} c_3^p| - |\text{Im} c_3^f|$ plane. Top mass is taken to be 174 GeV. Unpolarized beam is considered in case of figure on the left where contours are obtained from the asymmetries $A_t$, and $A_t^f$, while only $A_t$ is considered at different beam polarizations in the other figure. A c.m. energy of 500 GeV and a luminosity of $10 \, fb^{-1}$ are assumed.
Figure 2.7: Contours similar to Figure 2.6. Top quark mass here is 180 GeV.

Figure 2.8: Contours similar to Figures 2.6 and 2.7. Top quark mass here is 200 GeV.
Table 2.1: Limits on the dipole couplings obtained from different asymmetries. In the unpolarized case the asymmetries $A_{ud}$ and $A_{ub}^{fb}$ are together used to get the limits on the real parts of $c_{d}^{z}$ and $c_{d}^{x}$, and $A_{t}$ and $A_{t}^{fb}$ to obtain the limits on the imaginary parts. In the case of polarization, the limits obtained from $A_{ud}$ and $A_{t}$ are denoted by (a) and the ones from $A_{ub}^{fb}$ and $A_{t}^{fb}$ are denoted by (b).

The effect of polarization in the case of combined up-down (left-right) and forward-backward asymmetries for non-zero polarization is similar.

Thus, by using polarization, one can obtain independent limits of the order of 0.2-0.25 on three of the four dipole coupling parameters. The remaining parameter, $\text{Im } c_{d}^{z}$ can be constrained to about 0.8.

The various independent limits that can be obtained with and without beam polarization are collected in Table 2.1.

Having considered independent limits, we now consider limits obtained from the energy asymmetry on either the electric or the weak dipole moment, assuming the other dipole moment to be zero.

For the energy asymmetry, we have estimated limits in the following fashion. The
asymmetries corresponding to $x$ values in a range of 0.1 to 1.5 at definite intervals are obtained. Limit on the DFF is obtained in each bin using the formula

$$
\delta(c_d')_x = \frac{1.65}{\sqrt{N_x}} \frac{c_d'}{A_E(x)},
$$

where $N_x$ is the number of events in that particular bin and $A_E(x)$ is the asymmetry at $x$ with DFF value, $c_d'$ the other $c_d'$ being kept zero. An average obtained using the formula

$$
\delta c_d' = \frac{1}{N_b} \frac{1}{\sqrt{\sum_x (1/\delta(c_d')_x)^2}},
$$

where $N_b$ is the number of bins, gives the actual limit on the DFF. We have considered a bin size of .1 with $x$ value in a range of .1 to 1.5. The improvement in sensitivity is about a factor of 3 for $P_e = -0.5$ as compared to $P_e = 0$ for $|\text{Im } c_d'|$, whereas measurement of $|\text{Im } c_d'|$ is insensitive to polarization. We also find that in some cases the sensitivity is greatly enhanced by isolating the polarization-dependent part of the distribution. Thus, if we take a polarization asymmetrized sample, corresponding to $|d\sigma(P_e, P_e) - d\sigma(-P_e, -P_e)|$, and evaluate all asymmetries with respect to this new sample, we get a different set of asymmetries with different sensitivities. The sensitivity from this sample for $|\text{Im } c_d'|$ is improved by a factor of about 12 as compared to the unpolarized case, giving the best attainable 90% C.L. limit as 0.06. The limits are given in Table 2.2.

The polarization asymmetrized distributions for $P_e = 0.5$ leads to an improvement in the sensitivities from the measurement of $A_{ud}$ and $A_{ub}$, whereas the sensitivity is worse in the case of $A_{lr}$ and $A_{tb}$. For example, $A_{ud}$ can give a limit on $\text{Re } c_d'$ of 0.04 as compared to 0.10 obtained without the asymmetrization procedure.
Table 2.2: Limits on the dipole couplings obtained from energy asymmetry at 90% C.L. at a c.m. energy of 500 GeV.

One can also consider combinations of the different procedures mentioned above to maximize the sensitivity available.

We end the chapter with the following conclusions. We have calculated several CP-violating asymmetries which can arise in the process $e^+e^- \rightarrow t\bar{t}$, with subsequent $t,\bar{t}$ decay, in the presence of electric and weak dipole couplings of the top quark. In order to disentangle the CP-violating dipole couplings from each other, at least two T-odd asymmetries are needed for the real parts and two T-even asymmetries are needed for the imaginary parts, and we calculated possible asymmetries which could be used for the purpose. It was shown that longitudinal polarization of the electron can help in separating the various parameters, and in addition, leads to higher sensitivity. At the NLC with $\sqrt{s} = 500$ GeV and polarized electron beams with $\pm 50\%$ polarization, 90% C.L. sensitivities of the order of 0.25 are obtainable on independent determinations $|\text{Re } c^e_d|, |\text{Re } c^\gamma_d|$, and $|\text{Im } c^e_d|$, respectively, and a sensitivity of about 0.8 for $|\text{Im } c^\gamma_d|$.

Of these, the measurements of the real parts of $c^e_d, c^\gamma_d$ are free from CP-invariant back-
ground contributions. As for the T-even asymmetries depending on the imaginary parts of $c_{d}^{\mu-z}$, the backgrounds from order-$\alpha$ collinear initial-state photon emission have to be calculated and subtracted. This will be treated in Chapter 4.