Chapter 1

Introduction

The Standard Model (SM) of High Energy Physics is a highly successful theoretical model describing the dynamics of fundamental particles under the basic forces. It has been tested to a very high accuracy in various experiments for the past two and a half decades. Even then there are certain issues like understanding $CP$ violation in nature which need greater attention. Different models considering effects outside the regime of SM have been proposed to study the behaviour of nature at the level of fundamental particles. Predictions of such models have to be tested in experiments which can be carried out at proposed colliders which will be running at higher energies.

Electron-positron colliders have been used in the past to obtain a number of important results. They have an advantage over hadron colliders because leptons are more nearly pointlike than hadrons. Consequently, their interactions are better understood than those of hadrons. Moreover, hadronic backgrounds are larger in hadronic colliders. Thus $e^+e^-$ colliders provide a much cleaner environment.

Circular $e^+e^-$ colliders cannot go beyond a certain centre-of-mass energy because
of the synchrotron radiation losses and hence future colliders must be linear colliders. The proposed electron-positron linear colliders, for example SLAC's Next Linear Collider (NLC), will be running at centre of mass energies of several hundred GeV's. A large number of top-antitop quark pairs are expected to be produced in such colliders. The top quark being very massive [1] is considered to be a good place to test many of the new physics effects. Studying $CP$ violation in such systems is very promising. In this thesis we have studied $CP$-violating asymmetries arising due to electric and weak dipole form factors (DFF) of the top quark, which might be observable at NLC.

A high degree of longitudinal beam polarization can be achieved for an electron in linear colliders as confirmed in, e.g., experiments at SLC [2] and KEK [3]. The effect of electron beam polarization on $CP$-violating signals is therefore also included in this work.

Apart from electron-positron colliders, the possibility of a $\gamma\gamma$ collider has also been considered in the literature [4]. The idea is that high-energy photon beams could be obtained by the Compton backscattering of an intense laser beam off a high-energy electron (or positron) beam. The $\gamma\gamma \rightarrow t\bar{t}$ system is studied in this thesis for $CP$-violating signals in the presence of the top quark DFF's.

In an attempt to investigate whether any new models can lead to large DFF's, we consider a scenario where third-generation leptoquarks are present. These leptoquarks couple top quarks with $\tau$ leptons. One-loop correction to the $\tau\tau\gamma(Z)$ vertex arising from the above coupling is studied for $CP$ violation. From the existing lim-

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1Various proposals for building a high-energy linear $e^+e^-$ collider are being examined. Notable of these are for SLC (Stanford), TESLA (DESY), JLC (KEK) and CLIC (CERN). While the term NLC was first used for the proposed collider at Stanford, we will use this term to denote any of the future linear colliders.
Leptons | Quarks
--- | ---
\( l^L_L \equiv \left( \begin{array}{c} \nu_i \\ e_i \end{array} \right) \) & \( q^L_L \equiv \left( \begin{array}{c} u_i \\ d_i \end{array} \right) \)
\( (1,2,1) \) & \( (1,1,2) \)
\( e^R_e \) & \( u^R_u \)
\( (3,2,-\frac{1}{3}) \) & \( (3,1,-\frac{4}{3}) \)
\( d^R_d \) & \( (3,1,\frac{2}{3}) \)

Table 1.1: Fermion content of the Standard Model

In the following four sections we shall review the relevant aspects of SM and then discuss the phenomenology of \( CP \) violation. In sections 1.5 and 1.6 we shall introduce the method used in our studies.

Throughout the thesis, Bjorken and Drell conventions are used for the metric and Dirac gamma matrices.

1.1 The Standard Model

Particle interactions in high energy physics are studied using field theory techniques and are currently believed to arise from gauge theories. In a gauge theory, a certain local gauge symmetry is assigned to the Lagrangian of the system studied. The standard model of Particle Physics assumes a gauge group \( SU(3)_C \times SU(2)_L \times U(1)_Y \) to study particle dynamics. Fundamental fields are classified in SM in the following way.
Table 1.1 summarizes the fermion content of SM. In the table the subscripts $L$ and $R$ denote the chiralities of the particles. In the case of a particle represented by the Dirac spinor $\psi$, chiralities are defined by the following projections:

$$\psi_{R,L} = \frac{1}{2} \left( 1 \pm \gamma^5 \right) \psi. \quad (1.1)$$

In SM left-chirality particles are doublets under $SU(2)_L$ whereas right-chirality particles are singlets. Notice that SM does not have right-helicity neutrinos in it. In Table 1.1, colour $SU(3)_c$ indices are suppressed. All the quarks are colour triplets while leptons are colour neutral. Last row in Table 1.1 indicates to which multiplet of $SU(3)_c$ and $SU(2)_L$ the particles belong and what their $U(1)_Y$ hypercharges are. $i = 1, 2, 3$ correspond to three families. Charges of the particles are related to the hypercharge ($Y$) and the third component of the $SU(2)_L$ quantum number, ($T_3$) by the Gell-Mann–Nishijima relation, $Q = T_3 - \frac{Y}{2}$. Apart from these particles SM has two charged gauge bosons in it, the $W^\pm$ and two neutral gauge bosons, $Z$ and photon, which are the mediators of the electroweak interactions, and eight gluons, which mediate the strong interactions. To generate masses for the particles SM uses a technique known as the Higgs mechanism. For this purpose, scalar fields, the Higgs fields ($\phi$), are introduced into the theory. They are colour neutral and are $SU(2)_L$ doublets with hypercharge $-1$ in the minimal version of SM.

The Lagrangian describing the fundamental particles and their behaviour under the basic forces (except gravity) is invariant under the above group transformations, apart from being Lorentz invariant. This thesis discusses the phenomenon of $CP$ violation in the electroweak sector of high energy physics. Rest of this section will therefore discuss only the electroweak part of SM.

The general form of an electroweak Lagrangian (see Appendix A, Equation A.1 for
the QCD part of the Lagrangian) is

\[ \mathcal{L} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_Y + \mathcal{L}_H, \]  

(1.2)

where

\[ \mathcal{L}_G = -\frac{1}{4} B_{\mu \nu} B^{\mu \nu} - \frac{1}{4} \bar{W}_{\mu \nu} \cdot \bar{W}^{\mu \nu}, \]

\[ \mathcal{L}_F = \sum \bar{\psi}_L i \slashed{D}_L \psi_L + \sum \bar{\psi}_R i \slashed{D}_R \psi_R, \]

\[ \mathcal{L}_Y = h_{i j}^L q_i^L \bar{\psi}_i R u_j R + \bar{h}_{i j}^L q_i^L \phi d_j^R + h_{i j}^T \bar{t}_i^L \phi c_{R i} + h.c., \]

\[ \mathcal{L}_H = (D^\mu_L \phi)^\dagger (D_{\mu L} \phi) - V(\phi). \]

In \( \mathcal{L}_F \) the summation is over all left-handed doublets \( \psi_L \) in the first term and over all right-handed singlets \( \psi_R \) in the second term.

Here \( \tilde{\phi} = i \gamma_5 \phi^* \), and the covariant derivatives are given by

\[ D_L^\mu = \partial^\mu + ig \frac{\tau^a}{2} \bar{W}^\mu + i g' \frac{Y}{2} B^\mu, \]

\[ D_R^\mu = \partial^\mu + ig' \frac{Y}{2} B^\mu, \]

where \( g \) and \( g' \) are coupling constants corresponding to \( SU(2)_L \) and \( U(1)_Y \) gauge groups, and \( \bar{W}^\mu, B^\mu \) are vector gauge fields corresponding to these groups. \( \bar{W}^\mu \) is an \( SU(2)_L \) triplet with \( Y = 0 \) and \( B^\mu \) is neutral under \( SU(2)_L \times U(1)_Y \).

\[ V(\phi) = \mu^2 \phi^4 + \lambda (\phi^4 \phi)^2 \]  

(1.3)
is the scalar potential, where $\mu$ and $\lambda$ are the two coupling constants. The gauge field strengths are

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

(1.4)

and

$$\tilde{W}_{\mu\nu} = \partial_\mu \tilde{W}_\nu - \partial_\nu \tilde{W}_\mu + g \tilde{W}_\mu \times \tilde{W}_\nu.$$

The Lagrangian $\mathcal{L}$ is invariant under local $SU(2)$ gauge transformations.

Under $SU(2)_L$, $\psi_L$ transforms like a doublet and $\psi_R$ transforms like a singlet, and hence a typical fermionic mass term, $m \overline{\psi}_L \psi_R$ is not gauge invariant. The gauge boson mass terms $(m^2 A_\mu^a A^a_\mu)$ are also not gauge invariant. Since we demand the gauge invariance of the SM Lagrangian it cannot accommodate these mass terms. At the same time we know that the real particles are massive. To save the situation, SM generates masses for particles by breaking the gauge symmetry using the Higgs mechanism in which the Higgs field is introduced in such a way that the field has a non-zero expectation value ($v$) in the true vacuum state. This is done by choosing $\mu^2 < 0$ and $\lambda$ positive. In that case for smaller values of $\phi$, where the quadratic term dominates the scalar potential will be negative while at larger values of $\phi$ the quartic term dominates giving positive values to the potential. This results in having a minimum for the potential at a non-zero value of the electrically neutral component of $\phi$ ($< \phi_0 > = v = -\frac{\mu^2}{\lambda}$). This phenomenon where the vacuum does not respect the symmetry, while the Lagrangian is still gauge invariant, is known as spontaneous symmetry breaking.

Out of the four components (the real and the imaginary parts of the upper and the
lower components of the doublet) of the Higgs field \( \phi \), three can be rotated away by using the gauge transformation. Since we want to retain the electromagnetic gauge invariance, we give vacuum expectation value to the neutral component of the Higgs field. A convenient choice of Higgs field is \( \phi = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v + \eta(x) \end{array} \right) \) and \( \bar{\phi} = i\tau_3 \phi^* \). Here \( \eta \) corresponds to the only Higgs particle remaining out of the four. When this Higgs field is used in the Lagrangian 1.2 we get the following mass terms for the fermions:

\[
\mathcal{L}_{\text{mass}} = \bar{u}_L^i M_{ij}^u u_R^j + \bar{d}_L^i M_{ij}^d d_R^j + \bar{e}_L^i M_{ij}^e e_R^j,
\]

where \( M_{ij}^f = \frac{\sqrt{2}}{f} \ h_{ij}^f \), with \( f = u, d, e \). There are no mass terms for neutrinos because SM does not have a right-handed neutrino.

Here the mass matrices \( M^u \), \( M^d \) and \( M^e \) are not necessarily diagonal. Physical particles should correspond to definite mass states and the corresponding fields will be the eigenvectors of these mass matrices. Diagonalization of mass matrices is done by a biunitary transformation, with a separate unitary transformation on the left-handed and the right-handed fermions. In this process "flavour" fields introduced in the original Lagrangian are transformed to physical fields. The transformations are given below:

\[
\begin{align*}
&u_L \rightarrow u_L' \equiv U_L u_L, & d_L \rightarrow d_L' \equiv D_L d_L, & e_L \rightarrow e_L' \equiv E_L e_L, \\
&u_R \rightarrow u_R' \equiv U_R u_R, & d_R \rightarrow d_R' \equiv D_R d_R, & e_R \rightarrow e_R' \equiv E_R e_R, \\
&M^u \rightarrow M^{u'} \equiv U_L M^u U_R^\dagger, & M^d \rightarrow M^{d'} \equiv D_L M^d D_R^\dagger, & M^e \rightarrow M^{e'} \equiv E_L M^e E_R^\dagger.
\end{align*}
\]

Going back to the Lagrangian given in Equation 1.2, now written in terms of the physical fields (the primed ones), the fermionic part includes a kinetic energy term,
a charged current interaction term and a neutral current interaction term:

\[ \mathcal{L}_F = \mathcal{L}_{KE} + \mathcal{L}_{CC} + \mathcal{L}_{NC}. \]

Here

\[ \mathcal{L}_{KE} = \sum \bar{\psi} i D \psi', \]

\[ \mathcal{L}_{CC} = -\frac{g}{2} \bar{\psi}_L \gamma^\mu \mathcal{V} d_L^* (W_\mu^1 + i W_\mu^2) - \frac{g}{2} \bar{\psi}_L \gamma^\mu \epsilon' L (W_\mu^1 + i W_\mu^2) + h.c. \]

and

\[ \mathcal{L}_{NC} = -g \sum \bar{\psi}_L \gamma^\mu \frac{T_3}{2} \psi' L W_\mu^3 - g' \sum \frac{Y}{2} \bar{\psi} \gamma^\mu \psi' B_\mu. \]

\[ W^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 + i W_\mu^2) \] are the two charged gauge bosons. The neutral current is written in terms of the photon and the Z-boson fields as

\[ \mathcal{L}_{NC} = e \sum \bar{\psi} \gamma^\mu \psi A_\mu + \sum g_R \bar{\psi}_R \gamma^\mu \psi'_L Z_\mu + \sum g_L \bar{\psi}_L \gamma^\mu \psi'_L Z_\mu, \]

where

\[ A_\mu = g' W_\mu^3 + g B_\mu \quad (1.5) \]

and

\[ Z_\mu = g W_\mu^3 + g' B_\mu \quad (1.6) \]
are the gauge boson corresponding to the electromagnetic force and the weak neutral gauge boson respectively, obtained by diagonalization of the $2 \times 2$ mass matrix of $W^3$ and $B$. This diagonalization gives a mass $m_Z = \frac{g}{2} \sqrt{g^2 + g'^2}$ for $Z$, while the photon is massless. The Higgs mechanism also leads to a mass $M_W = \frac{g_W}{2}$ for the $W^\pm$.

The neutral current couplings of various fermions are written in terms of their respective $T_3$ and $Q$ values by

$$g_L = \frac{e}{\sin \theta_W \cos \theta_W} \left( \frac{T_3}{2} - Q \sin^2 \theta_W \right)$$

and

$$g_R = \frac{e}{\sin \theta_W \cos \theta_W} \left(-Q \sin^2 \theta_W \right)$$

are the left- and right-handed couplings.

Family indices are suppressed in the above expression. $e$ is the electric charge of positron and is related to the $SU(2)_L$ and $U(1)_Y$ couplings by $e = \sqrt{g^2 + g'^2}$. $T_3$ is the third component of the weak isospin and $Y$, the hypercharge. Note that the neutral current terms continue to be diagonal.

$V = U_L D_L^1$ is a $3 \times 3$ unitary matrix (called CKM matrix, named after Cabibbo, Kobayashi and Maskawa.) Out of the 9 independent parameters of $V$, 3 are angles (an orthogonal matrix of dimension three has 3 independent parameters) and the rest are phases. 5 of these phases can be rotated away by redefining the quark fields leaving one phase which makes the matrix $V$ complex. No such matrix appears in the charge current coupling of the leptons because with massless neutrinos, such a
unitary matrix can be absorbed in the redefinition of the neutrino fields.

Thus, couplings of charged bosons to the quarks are complex and cause $CP$ violation. We will first discuss in brief the $P$ and the $C$ symmetries before going on to $CP$ violation.

1.2 $P$ and $C$ Symmetries

Dynamical equations of a physical system are invariant under Lorentz transformations. Transformation properties under other discrete symmetries are also important to understand a physical system. In this section we shall look at the space inversion and the charge conjugation transformations.

1.2.1 Space Inversion or Parity ($P$)

Under this transformation all space coordinates change sign, i.e., $(\vec{x}, t) \rightarrow (-\vec{x}, t)$. Invariance of the Dirac equation under a spatial transformation $x'^\mu = \Lambda_\mu^\nu x^\nu$ requires a matrix $S$ satisfying

$$ S^{-1} \gamma^\mu S = \Lambda_\mu^\nu \gamma^\nu. $$

Fermion fields transform under this transformation as

$$ \psi(x) \rightarrow \psi'(x') = S \psi(x). $$

For space inversion $\Lambda = \text{diag}(1, -1, -1, -1)$, implying $S^{-1} \gamma^\mu S = \gamma^\mu$, $S = \gamma^0$ satisfies this condition.
See A.2 for transformation properties of Dirac field bilinears under \( P \).

### 1.2.2 Charge Conjugation (\( C \))

Under charge conjugation all the additive quantum numbers of a particle change sign. For a free particle this amounts to changing the particle creation (annihilation) operator to an antiparticle creation (annihilation) operator and vice versa. Demanding that the particle and its charge conjugated counterpart under an external electromagnetic field obey the same Dirac equation, we get the transformation property of a spinor under charge conjugation as

\[
\psi \to C \overline{\psi}^T,
\]

where \( C \) is a \( 4 \times 4 \) matrix satisfying the condition (see Section A.3 in Appendix A.)

\[
C^{-1} \gamma^\mu C = - (\gamma^\mu)^T.
\]

Section A.4 gives the transformation properties of bilinears under \( C \).

From the experimental data available till then, especially, looking at the two different decay modes of \( K^+ \), viz., \( K^+ \to 2\pi \) and \( K^+ \to 3\pi \), Lee and Yang in 1956 [5] observed that parity was violated in weak interactions. A year later Wu et al. [6] confirmed \( P \) violation in the \( ^{60}Co \) to \( ^{60}Ni \) transition. Also, neutrinos in \( \beta \) decay are found to be left-handed and never right-handed while antineutrinos from the conjugate process is always right-handed. This clearly violates charge conjugation invariance.
In SM maximal violation of $P$ and $C$ are explicitly brought in by choosing a (V A) form for the weak interactions and by assuming the non-existence of right-handed neutrino. It was believed till 1964 that the combined symmetry $CP$ is not violated even though $C$ and $P$ are broken maximally when taken individually. Christenson et al. [7] provided experimental evidence for $CP$ violation in the $K$-meson system. Later on this small violation of $CP$ was explained successfully by SM using the $CKM$ phase.

### 1.3 $CP$ Violation Phenomenology

Christenson et al.’s [7] discovery of the $2\pi$ decay of the long lived neutral kaon, $K_L$, implying that it is not a $CP$ eigenstate, was the first experimental evidence for $CP$ violation. They measured a branching ratio, $R = \frac{K_L \rightarrow \pi^+\pi^-}{K_L \rightarrow \pi^0\pi^0} = (2.0 \pm 0.4) \times 10^{-3}$. This gives $|\epsilon| \sim 2.3 \times 10^{-3}$, where, $\epsilon$ is the mixing parameter in expressing the long lived, physical $K$-meson, $K_L$ in terms of the weak eigenstates, $K_0$ and $\bar{K}_0$:

$$K_L = \frac{1}{\sqrt{2}} \left[ (K_0 - \bar{K}_0) + \epsilon (K_0 + \bar{K}_0) \right].$$

$\epsilon$ is given in terms of the experimentally measurable quantities

$$\eta_{+-} = \frac{K_0^0 \rightarrow \pi^+\pi^-}{K_0^0 \rightarrow \pi^0\pi^0} \quad \text{and} \quad \eta_{00} = \frac{K_0^0 \rightarrow \pi^0\pi^0}{K_0^0 \rightarrow \pi^0\pi^0}$$

as

$$\eta_{+-} = \epsilon + \epsilon' \quad \text{and} \quad \eta_{00} = \epsilon - 2\epsilon'.$$
\( \epsilon \) is the ratio of transition amplitude of \( K^0 \) decay to isospin two state to the amplitude of \( K^0 \) decay to an isospin zero state.

Present experimental values of the parameters are [8]:

\[
|\eta_{+-}| = (2.285 \pm 0.019) \times 10^{-3}, \\
|\eta_{00}| = (2.275 \pm 0.019) \times 10^{-3}
\]

and

\[
\frac{\epsilon'}{\epsilon} = (1.5 \pm 0.8) \times 10^{-3}.
\]

\( CP \) violation needs complex coupling, which is supplied in SM by the \( CKM \) matrix, which has complex elements arising from a single phase.

Experimental results are compatible with the SM predictions. But SM does not say anything about the value of \( CKM \) phase and it has to be fixed from experiments. An independent test of the \( CKM \) picture of \( CP \) violation could be done in the proposed B-factories. Until then, alternative \( CP \) violation scenarios cannot be ruled out. To understand the phenomenon better, it is required to look for other \( CP \)-violating effects in nature. It can be seen that presence of electric dipole moment (EDM) indicates \( CP \) violation. The electric dipole interaction with electromagnetic field is given by the term \( d_{s}^{\psi} \bar{\psi} \sigma_{\mu\nu} \gamma^5 \psi F^{\mu\nu} \) which is odd under \( CP \) transformation. An analogous term can be written for the weak neutral current interaction, \( d_{s}^{\psi} \bar{\psi} \sigma_{\mu\nu} \gamma^5 \psi (\partial^\mu Z^\nu - \partial^\nu Z^\mu) \), which is also \( CP \)-violating. \( d_{s}^{\psi} \) is referred to as the weak dipole moment. Interference of this term with \( CP \)-even terms in the Lagrangian causes observable \( CP \) violation. It is a dimension 5 term and thus
\[ d_e^p = (-0.3 \pm 0.8) \times 10^{-26} \]
\[ d_n^p = (3.7 \pm 3.4) \times 10^{-19} \]
\[ d_e^e < 5 \times 10^{-17} \]
\[ d_n^n < 1.1 \times 10^{-17} \]
\[ \text{Re} d_e^\nu < 5.6 \times 10^{-18} \]
\[ \text{Im} d_e^\nu < 1.5 \times 10^{-17} \]

Table 1.2: Experimental limits on particle dipole moments in units of \( \mu \text{cm} \).

cannot be included as a fundamental interaction in a renormalizable theory. It has to come from higher order terms in the perturbation expansion. In SM there is no dipole term up to two-loop level and so particle dipole moments are predicted to be very small. Experimental bounds on particle dipole moments (\( d_f^Z \) correspond respectively to electromagnetic and weak dipole couplings) are given in the Table 1.2 [8, 9]

In SM dipole moments come as higher order corrections in the perturbation theory. There are no dipole moment at one-loop level as the diagrams in these cases are self-conjugate. The contribution from two-loop diagrams is also zero [10]. Since the only \( CP \) violating phase that is available in SM is the \( CKM \) phase which comes in the quark sector, leptons acquire dipole moments through the dipole couplings of quarks and/or gauge-boson fields. SM predicts electric dipole moment of electron to be \( \sim 10^{-38} \) while for neutron it is \( \sim 10^{-31} \).

From the table it is clear that SM predictions are far too small compared to the experimental bounds. There are many extensions of SM which give dipole moments of particles close to the experimental values. In the next section we shall have a look at some of the models trying to explain \( CP \) violation.
1.4 $CP$ Violation and Extensions of SM

There were many models prior to the Glashow-Weinberg-Salam model of particle physics (the Standard Model) trying to explain $CP$ violation, among which are the superweak hypothesis of Wolfenstein [11] (strength $\sim 10^{-7} G_F$), semi-strong interaction theory of Prentki and Veltman [12] and the milliweak theory of Wu and Yang [13]. Kobayashi and Maskawa [14] explained how $CP$ violation can be brought into SM through quark mixing by extending the fermions to three generations. Later on, Weinberg's [15] two Higgs doublet model treating $CP$ violation as a spontaneously broken symmetry proposed $CP$ violation in the quark sector through Higgs exchange. The strength of $CP$ violation in this case is milliweak. Mohapatra and Pati [16] have discussed the possibility of $CP$ violation in a left-right symmetric model. Here $CP$ violation comes through the inequality of the masses of the two gauge bosons, $W_L$ and $W_R$, which interact with the left and right chiral currents. The strength of $CP$ violating interactions, in this case, is milliweak or weaker than milliweak.

Coming to the recent models investigating $CP$ violation, multi-Higgs models [17, 18] have been discussed extensively. These models have been used in constructing $CP$-odd correlations and asymmetries in hadronic and $e^+e^-$ reactions. $CP$ violation effects generated in supersymmetric models [19] are also studied in the context of both hadronic and $e^+e^-$ colliders. There are also investigations of $CP$ violation effects in left-right symmetric models [20].

The effective Lagrangian method, which is made use of in the studies we have carried out in this thesis is described in the next section.
1.5 Effective Lagrangian Method

As seen in the previous section, $CP$ violation has been studied in a large number of models. A model independent way of looking at the whole scenario is the effective Lagrangian method. In this approach, an effective Lagrangian, having electric and weak dipole terms in addition to the SM terms, is considered. Dipole moments are parameters in this approach and are fixed from experimental measurements of $CP$-violating observables. We have chosen this approach in our studies.

We consider the effective Lagrangian;

$$L_{\text{eff}} = L_{\text{SM}} + L_{CP},$$  \hspace{1cm} (1.7)

where

$$L_{CP} = \frac{i e j}{2 m_t} \bar{\psi}_t \sigma^{\mu \nu} \gamma_5 \psi_t F_{\mu \nu} + \frac{i e Z}{2 m_t} \bar{\psi}_t \sigma^{\mu \nu} \gamma_8 \psi_t Z_{\mu \nu},$$

with

$$F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$  \hspace{1cm} (1.8)

and

$$Z_{\mu \nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu.$$

$c_d (j = \gamma, Z)$ are the electric and the weak dipole form factors and are, in general, complex and momentum dependent. This modifies the Standard Model $i\bar{\psi}\gamma(Z)$
coupling to $ie\Gamma_{\mu}$, where

$$\Gamma_{\mu} = c_d^j \gamma_\mu + c_a^j \gamma_{\mu} \gamma_5 + \frac{c_d^j}{2m_t} \sigma_{\mu\nu} \gamma_5 (p_\mu + p_\nu)\nu, \quad j = \gamma, Z.$$  (1.9)

The dipole moments are related to $c_d^j$ by $d^j = c_d^j / (2m_t)$. We have considered the top quark system because it is confirmed now that top quark is heavy with mass, $m_t = 180 \pm 12 \text{ GeV}$ [1] and therefore decays very fast. Its life time, $\tau \sim 10^{-24} \text{ sec}$, is less than the time it requires to hadronize $(R_{\text{comb}} = \Lambda_{QCD}^{-1} \sim 10^{-23} \text{ sec})$ [21]. Thus the top quark produced in a collider decays before hadronization, and the decay products preserve the spin information, which then can be used to study $CP$ properties of the interaction.

We study $CP$ violation using $CP$-odd asymmetries constructed with Lagrangian in Equation 1.7. How the asymmetries are used to determine allowed regions in the parameter space and the sentitivity of the experiment are discussed in the next section. In our work we consider the top quark decay to be the standard one. For discussions which include $CP$ violation effects in the decay, see [22, 23].

### 1.6 Sensitivity of the measurements

$CP$-odd asymmetries are studied in the following chapters of this thesis as signatures of $CP$ violation. Experimental measurements of these asymmetries can determine the dipole form factors, or if the asymmetries are found consistent with zero, they can put bounds on the dipole form factors of the top quark. Sensitivity of an experiment depends on the statistics. The number of asymmetric events must be greater than the statistical fluctuation by a certain factor for it to be observed. This factor determines the confidence level (C.L.) of the measurement. For a sys-
tem with one degree of freedom the number of asymmetric events, $N_A$ ($= N_A$, where $A$ is the asymmetry and $N$ is the total number of events.) should then be greater than $1.64\sqrt{N}$ for observing it at 90% C.L., where $\sqrt{N}$ corresponds to the standard deviation. Using this we can get limits on the dipole form factors. They are given by

$$\delta c^i_A = \frac{1.64\sqrt{N} c^i_A}{N_A},$$

where $c^i_A$ is the value of either electric or weak dipole form factor (DFF) at which $A$ is calculated. The other $c^j_A$ is fixed in this case.

To get simultaneous limits on two DFF's one should do the following. For two degrees of freedom 90% C.L. corresponds to $2.15\sigma$ and therefore to observe an asymmetry we must have an asymmetric number of events satisfying the condition

$$N_A > 2.15\sqrt{N}.$$

$A$ is a function of $c^i_A$ and $c^j_A$ (either the real or the imaginary parts). The minimum number of events for the asymmetry to be observable is given by

$$N_A(c^i_A, c^j_A) = 2.15\sqrt{N}.$$  \hspace{1cm} (1.11)

This gives a linear equation in $c^i_A$ and $c^j_A$. Since the number of events is the absolute value of $N A$, and $A$ could be either positive or negative, we get a band of allowed values in the $c^i_A - c^j_A$ plane. This does not really fix the DFF's as we can choose any value of $c^i_A$ and get the corresponding value of $c^j_A$ from the graph. Use of another asymmetry will give a different band restricting the values of DFF's.
intersecting region of the two bands helps getting the range of allowed values of DFF's independently.

The other possibility in the study of $CP$ violation is the investigation using correlations of $CP$-odd kinematic variables. Of particular importance are variables known as optimal variables [23, 24], whose correlations are minimized compared to the error in their measurement. Realizing that the statistical significance of the non-zero value of the correlation increases with the resolving power, $R$, we can say that the optimal variable is the correlation with maximum $R$. Correlations of optimal variables have an advantage over asymmetries mainly in case of distributions with several kinematic variables. We restrict ourselves to asymmetries which are conceptually simpler than correlations of optimal variables, and also allow us to obtain analytic expressions.

1.7 Plan of the Thesis

The plan of the thesis is as follows. In Chapter 2, $CP$ violation studies in $e^+e^- \rightarrow t\bar{t}$ and the subsequent decay of $t$ and $\bar{t}$ are made by constructing $CP$-violating asymmetries. Some other asymmetries, which may be simpler from the experimental point of view are discussed in Chapter 3. In the case when only electron beam is polarized a collinear helicity-flip photon emission from the initial state can in principle give rise to a background to the asymmetries considered. Here $CP$ non-invariant helicity combination in the initial state may lead to the same asymmetries that are considered even in the absence of dipole form factors (or any other genuine $CP$ breaking parameter.) Chapter 4 discusses this background to the asymmetries considered in Chapters 2 & 3.
$CP$-violating effects in the top quark production in $\gamma\gamma$-collider is discussed in Chapter 5. Chapter 6 describes $CP$ violation studies made in a leptoquark model. Finally, conclusions of the studies are given in Chapter 7.