Chapter 3

Using Mathematica

3.1 Introduction

A computer algebra system (CAS) is a software program that facilitates symbolic mathematics. This means it can do manipulation of mathematical expressions in symbolic form. It can handle polynomials in multiple variables, standard functions like trigonometric functions, hyperbolic functions, logarithmic functions etc; derivatives and integrals of such expressions and their algebraic combinations; matrix algebra and so on. Mathematica is a general purpose CAS initially developed by a theoretical physicist Stephen Wolfram from Caltech-founder CEO of Wolfram Research Incorporate. The first version was released in 1988 at present Mathematica 9.0.1 is available.

This chapter is based on our work using Mathematica for doing computations in general relativity, in usual four dimensions as well as higher dimensions. The initial work was done by Hasmani and Rathwa [28] to give a simple program to compute Ricci tensor and its spur, this program
was successfully used to compute these quantities for a most general orthogonal metric. This was further extended by Hasmani [29] to compute Weyl tensor and its complex scalars representations in Newman-Penrose (NP) formalism. In the another phase Hasmani and Andharia [30] gave a program for spin coefficients in NP formalism. Further Hasmani and Kambholja [40] modified the program to incorporate higher dimensional spacetimes. In the next sections basics of tensors used in relativity are given for ready reference. Also a brief introduction of Mathematica and its use for doing algebraic computations in general relativity is discussed. The program developed by us is given in section 3.3.

Einstein used non-Euclidean geometry developed by Riemann to describe his theory of gravitation. In his theory the physical space taken as a four dimensional Riemannian manifold is referred as spacetime. The geometry of the spacetime is characterized by the metric tensor and the physical content of the spacetime is described by the energy momentum tensor. The metric tensor and the energy momentum tensor are related to each other via Einsteins field equations in such a way that the spacetime geometry is determined by the energy tensor, and the motion of matter is determined by the metric tensor. The field equations are given in tensorial form and, in general, constitute a nonlinear system of partial differential equations. Hence, the study of general relativity involves a large number of problems requiring very tedious, time-consuming, and error-prone algebraic manipulations. For this reason, general relativity
was one of the earliest fields of application of CAS.
It can be noted that the quantities given in section 2.4 are computed by algebraic combination of components of metric tensor and their differentials with respect to spacetime coordinates. This fact is useful in writing programs for the computations.

### 3.2 Features of Mathematica Useful in Computational General Relativity

Mathematica is a general purpose program for doing mathematics. Using Mathematica it is easy to:
- Make numeric and symbolic calculations
- Simplify complicated mathematical expressions
- Plot the graphs of functions as well as curves and surfaces in 3-space
- Create sophisticated colour graphics
- Compute derivatives and integrals
- Solve equations, including differential equations
- Work with large data sets
- Create animations
- Write programs to carry out any algorithm
- Create slide show presentations.

Also Mathematica can effectively deal with special functions. So, for a highschool student or a Ph.D. mathematician, physicist or engineer, Mathematica is an ideal tool for meeting their computational needs.

Note that Mathematica can perform symbolic computations as well as it can handle very large numbers and large data too. This is how it is widely used for technical computations. One of the remarkable features of Mathematica is its user friendly commands. In short, Mathematica
represents a unique blend of major research breakthroughs, outstanding user-oriented design and world class software engineering. Mathematica has many features useful in mathematics as well as in all branch of science and technology. We will discuss only those features relevant to algebraic computations in relativity.

The work done in Mathematica composes a Mathematica notebook which displays all input statements and output statements and other messages form the package about errors if any etc. An input statement is typed and completed by pressing Shift and Enter keys simultaneously, the output is then displayed as soon as it is evaluated. Also the input statement, corresponding output are given serial numbers automatically. If the display of an output on the screen is not necessary then the input statement is terminated by a semicolon. A Mathematica command starts with a capital letter (e.g. the command for solving equations is Solve). Also we note that Mathematica is case sensitive. Many Mathematica commands are simple mathematical words as mentioned above. If a command is made of two (or more) words then each word starts with capital letters without leaving a space. List of Mathematica commands can be found from the Help which is provided with the package. Various kinds of brackets have specific meanings, for example, an operand to a command is enclosed within a pair of square brackets. If two letters (or groups of letters) are separated by a space then it understands as product of those variables for example $x \, y$ is taken as $x$ multiplied by $y$ but $x y$ is taken as a single variable. This makes typing simpler in
comparison with other packages. There is no restriction on size of a user defined symbol. Inbuilt help can be browsed at the moment it is required. Whole book on Mathematica of relevant version is made a part of the help. An entire notebook or a part of it can be converted to TeX form which can be then used for publication/reporting. This feature has two-fold advantage, no need to retype the work done; and on the other hand it makes the report authentic that the results are truly obtained using this software.

In the next section we discuss mathematica commands useful in computations in general relativity.

### 3.2.1 Mathematica Commands Useful in Computations in General Relativity

Various features and commands of Mathematica can be used for variety of purposes. In this section we will discuss some features and commands useful in computations in general relativity. The sequence followed is roughly the first occurrence of a command.

**For**

This command is used for making a loop its structure is, 

For[initiation,condition, increment, action],

i.e. the action is to be taken with initial value of a variable it is repeated for the given increment until the condition is satisfied.
Table

This command creates an array of desired depth. The structure of this command is,

\[
\text{Table}[\text{expression}, \{v_1, \text{minimum}, \text{maximum}\}, \{v_2, \text{minimum}, \text{maximum}\}, \ldots \{v_n, \text{minimum}, \text{maximum}\}]
\]

Here a table of depth n is generated with the expression involving n variables, \(v_1, v_2, \ldots, v_n\). Accordingly elements of such a table will be denoted by \(n\)-indices. Also we note that vector/list is regarded as a table of depth 1.

It is also possible to directly input an array with help of enclosing the elements in pair of braces. For example,

\[
\text{sample} = \{\{a, b, c, d\}, \{e, f, g, h\}, \{i, j, k, l\}, \{m, n, o, p\}\}
\]

can be used to mean

\[
\text{sample} = \{\{a, b, c, d\}, \{e, f, g, h\}, \{i, j, k, l\}, \{m, n, o, p\}\}
\]

This means that we have stored elements a,b,...p in a 4x4 table which is labeled as 'sample'.

The elements of a table/list can be extracted by typing the symbol used table/list enclosing the indices in two pairs of square brackets.

For example in the above example,

\[
\text{sample}[[2,4]] \text{ returns } h.
\]

Computations in general relativity are done for the given metric and a set of spacetime coordinates these form basic input. The coordinates are represented by a vector or a list having four entries. Metric tensor being of rank-2 is represented by a matrix or a compound list of depth-2
(which also can be regarded as table). Further since Christoffel symbols are having three indices they are stored in list or table of depth-3. Similarly for quantities having more indices can be stored in lists of higher depth.

**Inverse**

This command is used to determine inverse of a matrix. In relativity it useful for finding associated metric tensor $g^{ij}$ for the given metric $g_{ij}$.

This command works as,

Inverse[matrix]

For example the matrix earlier labeled as sample given by

$$\text{sample} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix}$$

can be obtained as

Inverse[sample]

the output is

$$\begin{pmatrix} \frac{-hkn + gln + hjo - flo - gjp + fkp}{dgjm - chjm - dfkm + bhkm + cflm - bglm - dgin + chin + dekn - ahkn - celn + agln + dfio - bhio - de jo + ahjo + belo - aflo - cfip + bgip + cejp - agjp - bekp + afkp}, \frac{dkn - cln - djo + blo + cj p - bkp}{dgjm - chjm - dfkm + bhkm + cflm - bglm - dgin + chin + dekn - ahkn - celn + agln + dfio - bhio - de jo + ahjo + belo - aflo - cfip + bgip + cejp - agjp - bekp + afkp}, \frac{-dgn + chn + dfo - bho - cf p + bgp}{dgjm - chjm - dfkm + bhkm + cflm - bglm - dgin + chin + dekn -} \end{pmatrix}$$
3.2. Features of Mathematica Useful in Computational General Relativity

\( \text{ahkn} - \text{celn} + \text{agln} + \text{dfio} - \text{bhio} - \text{de jo} + \text{ah jo} + \text{belo} - \text{aflo} - \text{cfip} + \text{bgip} + \text{cejp} - \text{agjp} - \text{bekp} + \text{afkp} \),

\( (\text{dgj} - \text{chj} - \text{dfk} + \text{bhk} + \text{clf} - \text{bgl})/(\text{dg jm} - \text{ch jm} - \text{df km} + \text{bhkm} + \text{clf} - \text{bgm} + \text{chin} + \text{dekn} - \text{ahkn} - \text{celn} + \text{agln} + \text{dfio} - \text{bhio} - \text{de jo} + \text{ah jo} + \text{belo} - \text{aflo} - \text{cfip} + \text{bgip} + \text{cejp} - \text{agjp} - \text{bekp} + \text{afkp}),(\text{dgm} - \text{chm} - \text{de o} + \text{aho} + \text{ce p} - \text{agp})/(\text{dg jm} - \text{ch jm} - \text{df km} + \text{bhkm} + \text{clf} - \text{bgm} - \text{dgin} + \text{chin} + \text{dekn} - \text{ahkn} - \text{celn} + \text{agln} + \text{dfio} - \text{bhio} - \text{de jo} + \text{ah jo} + \text{belo} - \text{aflo} - \text{cfip} + \text{bgip} + \text{cejp} - \text{agjp} - \text{bekp} + \text{afkp}), (-\text{dgi} + \text{chi} + \text{dekn} - \text{ahk} - \text{cel} + \text{agl})/(\text{dg jm} - \text{ch jm} - \text{df km} + \text{bhkm} + \text{clf} - \text{bgm} - \text{dgin} + \text{chin} + \text{dekn} - \text{ahkn} - \text{celn} + \text{agln} + \text{dfio} - \text{bhio} - \text{de jo} + \text{ah jo} + \text{belo} - \text{aflo} - \text{cfip} + \text{bgip} + \text{cejp} - \text{agjp} - \text{bekp} + \text{afkp}),(\text{djm} - \text{blm} - \text{din} + \text{aln} + \text{bip} - \text{ajp})/(\text{dg jm} - \text{ch jm} - \text{df km} + \text{bhkm} + \text{clf} - \text{bgm} - \text{dgin} + \text{chin} + \text{dekn} - \text{ahkn} - \text{celn} + \text{agln} + \text{dfio} - \text{bhio} - \text{de jo} + \text{ah jo} + \text{belo} - \text{aflo} - \text{cfip} + \text{bgip} + \text{cejp} - \text{agjp} - \text{bekp} + \text{afkp}), (-\text{dfm} + \text{bhm} + \text{den} - \text{ahn} - \text{bep} + \text{af p})/(\text{dg jm} - \text{ch jm} - \text{df km} + \text{bhkm} + \text{clf} - \text{bgm} - \text{dgin} + \text{chin} + \text{dekn} - \text{ahkn} - \text{celn} + \text{agln} + \text{dfio} - \text{bhio} - \text{de jo} + \text{ah jo} + \text{belo} - \text{aflo} - \text{cfip} + \text{bgip} + \text{cejp} - \text{agjp} - \text{bekp} + \text{afkp})\).
3.2. Features of Mathematica Useful in Computational General Relativity

If mat1 and mat2 denote two matrices then their product is obtained by,

mat1.mat2

All these matrix operations are performed only if they are valid, in case of non-validity Mathematica gives error message.

D

This command is used for finding partial differential of an expression with respect to a variable.
For example on evaluating
\[ D[Sin[x y], x] \]
returns
\[ y \cos[x y] \].

For higher order derivatives either this command is used repeatedly or more variables are put after the expression. For example,
\[ D[x^2 y z, x, z] \]
returns
\[ 2 x y \]

When an output generated in terms of derivatives of function of many variables, for example \( \frac{\partial^3 f}{\partial y \partial z} \) for a function \( f(x, y, z, t) \) is displayed as \( f^{[0,2,1,0]}[x, y, z, t] \) which is a clumsy notation, we in relativity denote the same by \( f_{223} \). It is possible to customize the output in a desired form but it again requires some programming.

Many times we need a simplified form of an output is desired, for this Mathematica commands `Simplify` and `FullSimplify` are used.

`Simplify[expr]` performs a sequence of algebraic and other transformations on expr, and returns the simplest form it finds.

For example,
\[ D[\text{Integrate}\left[\frac{1}{x^3 + 1}\right], x], x] \]
gives
\[ \frac{1}{3(1+x)} - \frac{-1+2x}{6(1-x+x^2)} + \frac{2}{3(1+\frac{1}{2}(-1+2x)^2)} \] and

`Simplify[%]` gives \( \frac{1}{1+x^3} \).
FullSimplify[expr] tries a wide range of transformations on expr involving elementary and special functions, and returns the simplest form it finds. For example,

\[ \text{Simplify}\left[-i \log\left(\frac{1+2i}{\sqrt{5}}\right)\right] \]

gives

\[-i \log \left(\frac{1+2i}{\sqrt{5}}\right)\]

while \text{FullSimplify}\left[-i \log\left(\frac{1+2i}{\sqrt{5}}\right)\right] \text{ gives}

\text{ArcTan}[2]

If

This is used as a conditional operator. It has usual meaning. We have used this command to display only non-zero components of various tensors.

\text{If}[\text{condition},\text{t},\text{f}] \text{ gives } \text{t} \text{ if } \text{condition} \text{ evaluates to True, and } \text{f} \text{ if it evaluates to False.}

\text{If}[\text{condition},\text{t},\text{f},\text{u}] \text{ gives } \text{u} \text{ if } \text{condition} \text{ evaluates to neither True nor False.}

\text{If} \text{ evaluates only the argument determined by the value of the condition.}

\text{If}[\text{condition},\text{t},\text{f}] \text{ is left unevaluated if } \text{condition} \text{ evaluates to neither True nor False.}

\text{If}[\text{condition},\text{t}] \text{ gives Null if } \text{condition} \text{ evaluates to False.}

Print

This command is used for displaying the output. It displays the value of variables listed as operand. A text enclosed within quotation marks is
displayed as it is. For example,

\[ \text{Print["My NAme"]} \]

returns

My NAme

\textbf{SessionTime[]}

This command gives the time (in seconds) since a Mathematica session is started. We have used this command in the beginning of the program and in the end of the program, so that their difference is the time taken for the computation by the program.

\section*{3.3 Computations for a 5-Dimensional Metric}

The geometry of a 5-dimensional Riemannian space is given by the metric

\[ ds^2 = \sum_{i=1}^{5} \sum_{j=1}^{5} g_{ij} dx^i dx^j , \]  

(3.1)

where \( g_{ij} \) are components of metric tensor and \( x^i \) \((i = 1 \ to \ 5)\) denote the space coordinates. The standard terminology is used for the following various quantities used in Riemannian geometry.

First we need Christoffel symbols of second kind. These are used to compute \( R_{hijk} \) - Riemann curvature tensor. These has 625 expressions as \( h, i, j, k = 1, 2, 3, 4, 5 \).
As it is anti-symmetric on the first and second pairs of indices and symmetric on the exchange of the two pairs. i.e. 

\[ R_{hijk} = -R_{ihjk} = -R_{hikj} = R_{jkhi} \]

with 

\[ R_{hijk} + R_{hki j} + R_{h jki} = 0, \]

it has 50 independent expressions [67].

If all 25 components of the metric tensors are non-zero and they are functions of all 5-coordinates, the computations become very complicated and too large. Hence they are very much time consuming. So, for such large quantity of computations to be error free, one has to opt a computer algebra system to do fast and accurate computations. Here, Mathematica has been chosen for this purpose.

We consider the following non-static spherically symmetric metric

\[
\begin{aligned}
ds^2 &= B(z, t)dz^2 - A(z, t)dt^2 - r^2(d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \sin^2 \theta_1 \sin^2 \theta_2 d\theta_3^2) \\
\end{aligned}
\]

where the metric components \( A(z, t) \) and \( B(z, t) \) are to be determined.

The coordinates are labeled as 

\[
x^1 = r, \ x^2 = \theta_1, \ x^3 = \theta_2, \ x^4 = \theta_3, \ x^5 = t. 
\]
Thus,

\[
(g_{ij}) = \begin{pmatrix}
-A(z, t) & 0 & 0 & 0 & 0 \\
0 & -t^2 & 0 & 0 & 0 \\
0 & 0 & -t^2 \sin^2 \theta_1 & 0 & 0 \\
0 & 0 & 0 & -t^2 \sin^2 \theta_1 \sin^2 \theta_2 & 0 \\
0 & 0 & 0 & 0 & B(z, t)
\end{pmatrix}
\]

In the next subsection the program for the algebraic compactions of various tensor quantities needed for the 5-dimensional interior black hole metric are given, the same program has been also extended to a program that can work for any finite dimensional spacetime.

### 3.3.1 Mathematica Notebook

(*Dimension of the spacetime*)

\n1 = 5;

(*Choosing coordinate labels*)

\ncoor = \{z, \theta_1, \theta_2, \theta_3, t\};

(*Giving metric components*)

\nFor[i = 1, i \leq n1, i++, xi[i] = coor[[i]]]

\nmetric = \{-A[z, t], 0, 0, 0, 0\}, \{0, -(t)^2, 0, 0, 0\},

\n\{0, 0, -(t \sin[\theta_1])^2, 0, 0\}, \{0, 0, 0, -(t \sin[\theta_1] \sin[\theta_2])^2, 0\},

\n\{0, 0, 0, 0, B[z, t]\};

(*Computing inverse metric tensors*)

\nFor[i = 1, i \leq n1, i++, For[j = 1, j \leq n1, j++, glowij[i, j] = metric[[i, j]]]]
gup = Inverse[metric];
For[i = 1, i ≤ n1, i++, For[j = 1, j ≤ n1, j++, gupij[i, j] = gup[[i, j]]];

(*Computing Christoffel symbols*)
gama = Table[(1/2)(D[metric[[i, k]], coor[[j]]] + D[metric[[k, j]], coor[[i]]] - D[metric[[i, j]], coor[[k]]]), {i, n1}, {j, n1}, {k, n1}];
For[i = 1, i ≤ n1, i++, For[j = 1, j ≤ n1, j++,
For[k = 1, k ≤ n1, k++, gamaijcomak[i, j, k] = gama[[i, j, k]]];
gamaup = Table[Sum[gupij[h, k]gamaijcomak[i, j, k], {k, n1}], {h, n1}, {i, n1}, {j, n1}];
For[h = 1, h ≤ n1, h++, For[i = 1, i ≤ n1, i++, For[j = 1, j ≤ n1, j++,
gamahij[h, i, j] = gamaup[[h, i, j]]];

(*Computing components of Riemann tensor*)
riemann = Table[FullSimplify[(1/2)(D[glowij[i, j], xi[h], xi[k]]
+ D[glowij[h, k], xi[i], xi[j]] - D[glowij[i, k], xi[h], xi[j]]
- D[glowij[h, j], xi[i], xi[k]]) + Sum[Sum[glowij[a, b]
(gamahij[a, i, j] gamahij[b, h, k] - gamahij[a, i, k]
gamahij[b, h, j]), {a, n1}], {b, n1}], {h, n1},
{i, n1}, {j, n1}, {k, n1}];
For[h = 1, h ≤ n1, h++, For[i = 1, i ≤ n1, i++,
For[j = 1, j ≤ n1, j++, For[k = 1, k ≤ n1, k++,
rhijk[h, i, j, k] = riemann[[h, i, j, k]]];

(*Computing components of Ricci tensor*)
riccilowij = Simplify[Table[Sum[Sum[gupij[h, k]
rhijk[h, i, j, k], {h, n1}], {k, n1}], {i, n1}, {j, n1}]]
3.3. Computations for a 5-Dimensional Metric

```
Print["The coordinates chosen are"];
For[i = 1, i ≤ n1, i++, Print[x, i, " = ", coor[[i]]]]
Print["The non-zero components of metric are"]; For[i = 1, i ≤ n1, i++, For[j = i, j ≤ n1, j++,
If[metric[[i, j]] != 0, Print[g, i, j, " = ", metric[[i, j]]]]]
Print["The non-zero components of inverse metric are"]; For[i = 1, i ≤ n1, i++, For[j = i, j ≤ n1, j++,
If[gup[[i, j]] != 0, Print[gup, i, j, " = ", gup[[i, j]]]]]
Print["The non-zero components of Riemann Tensor are"]; For[h = 1, h ≤ n1, h++, For[i = h, i ≤ n1, i++, For[j = 1, j ≤ n1, j++,
For[k = j, k ≤ n1, k++, If[rhijk[h, i, j, k] != 0,
Print[R, h, i, j, k, " = ", rhijk[h, i, j, k]]]]]]
Print["The non-zero components of Ricci Tensor are"]; For[i = 1, i ≤ n1, i++, For[j = i, j ≤ n1, j++,
If[ricciwij[[i, j]] != 0,
Print[R, " low", i, j, " = ", ricciwij[[i, j]]]]]
```

### 3.3.2 Output

The coordinates chosen are

- \(x_1 = z\)
- \(x_2 = \theta_1\)
- \(x_3 = \theta_2\)
x4 = θ3
x5 = t

The non-zero components of metric are

\[ g_{11} = -A[z, t] \]
\[ g_{22} = -t^2 \]
\[ g_{33} = -t^2 \sin[\theta_1]^2 \]
\[ g_{44} = -t^2 \sin[\theta_1]^2 \sin[\theta_2]^2 \]
\[ g_{55} = B[z, t] \]

The non-zero components of inverse metric are

\[ g_{\mu 11} = -\frac{1}{A[z, t]} \]
\[ g_{\mu 22} = -\frac{1}{t^2} \]
\[ g_{\mu 33} = -\frac{\csc[\theta_1]^2}{t^2} \]
\[ g_{\mu 44} = -\frac{\csc[\theta_1]^2 \csc[\theta_2]^2}{t^2} \]
\[ g_{\mu 55} = \frac{1}{B[z, t]} \]

The non-zero components of Reimann Tensor are

\[ R_{1212} = -\frac{t A^{(0,1)}[z, t]}{2B[z, t]} \]
\[ R_{1225} = \frac{t B^{(1,0)}[z, t]}{2B[z, t]} \]
\[ R_{1313} = -\frac{t \sin[\theta_1]^2 A^{(0,1)}[z, t]}{2B[z, t]} \]
3.3. Computations for a 5-Dimensional Metric 46

\[ R_{1335} = \frac{r \sin \theta \alpha^2 B^{(1,0)[z,t]}}{2B[z,t]} \]
\[ R_{1414} = -\frac{r \sin \theta \alpha B^{(0,1)[z,t]} A^{(0,1)[z,t]}}{2B[z,t]} \]
\[ R_{1445} = \frac{r \sin \theta \alpha^2 B^{(1,0)[z,t]}}{2B[z,t]} \]
\[ R_{1515} = \frac{1}{4} \left( -\frac{A^{(0,1)[z,t]} + A^{(1,0)[z,t]} B^{(1,0)[z,t]} + A^{(0,1)[z,t]} B^{(0,1)[z,t]} + B^{(1,0)[z,t]}^2}{B[z,t]} \right) + \frac{1}{2} \left( A^{(0,2)[z,t]} - B^{(2,0)[z,t]} \right) \]
\[ R_{2323} = -\frac{r^2 (1 + B[z,t]) \sin \theta \alpha^2}{B[z,t]} \]
\[ R_{2424} = -\frac{r^2 (1 + B[z,t]) \sin \theta \alpha B^{(0,1)[z,t]} \sin \theta \alpha^2}{B[z,t]} \]
\[ R_{2512} = \frac{I B^{(1,0)[z,t]}}{2B[z,t]} \]
\[ R_{2525} = -\frac{I B^{(0,1)[z,t]}}{2B[z,t]} \]
\[ R_{3434} = -\frac{r^2 (1 + B[z,t]) \sin \theta \alpha B^{(0,1)[z,t]} \sin \theta \alpha^2}{B[z,t]} \]
\[ R_{3513} = \frac{r \sin \theta \alpha^2 B^{(1,0)[z,t]}}{2B[z,t]} \]
\[ R_{3535} = -\frac{r \sin \theta \alpha B^{(0,1)[z,t]}}{2B[z,t]} \]
\[ R_{4514} = \frac{r \sin \theta \alpha B^{(0,1)[z,t]} B^{(1,0)[z,t]}}{2B[z,t]} \]
\[ R_{4545} = -\frac{r \sin \theta \alpha B^{(0,1)[z,t]} B^{(0,1)[z,t]}}{2B[z,t]} \]

The non-zero components of Ricci Tensor are

\[ R_{\text{low}11} = \frac{r B[z,t] (A^{(0,1)[z,t]} B^{(0,1)[z,t]} - B^{(1,0)[z,t]} B^{(0,1)[z,t]})}{4r A[z,t] B[z,t] B^{(0,1)[z,t]}^2} \]
\[ + \frac{A[z,t] (r (A^{(0,1)[z,t]} B^{(0,1)[z,t]} - B^{(1,0)[z,t]} B^{(0,1)[z,t]}) - 2B[z,t] (3A^{(0,1)[z,t]} + tA^{(0,2)[z,t]} - tB^{(2,0)[z,t]}))}{4r A[z,t] B[z,t] B^{(0,1)[z,t]}^2} \]
\[ R_{\text{low}15} = -\frac{3B^{(1,0)[z,t]}}{2r B[z,t]} \]
3.4 Conclusion

A simple program for algebraic computation of Riemann curvature tensor is given in Mathematica 9. It computes within seconds all the complicated expressions of Riemann curvature tensor, Ricci tensor. This program can compute the components of Riemann curvature tensor, Ricci tensor for any 5-dimensional Riemannian metric. Not only that, if number of coordinates are increased, that is, if a Riemannian metric in n-dimension is given, the same program can compute Riemann curvature tensor, Ricci tensor for it, too. It has also been further extended for the computations of Einstein tensor and mixed tensor expressions for Ricci tensor and Einstein tensor. The expressions obtained here for the non-static spherically symmetric metric in the subsection 3.3.2 are used to get interior black hole solution in 5-dimensions.