CHAPTER 8

GENERATION OF WH CODES USING
RECONFIGURABLE LATTICE FILTER
AND ITS APPLICATION

8.1 INTRODUCTION

A reconfigurable transceiver should support multiple air interfaces such as CDMA, OFDM, MC-DS-CDMA with dynamic reconfigurability in a single terminal as described by Chen et al (2000). Hence, reconfiguring the physical layer is essential if the same terminal needs to be used for diverse applications and in different networks as suggested by Mahmoud et al (2009). This is achieved in SDR by change in the software without any change in the hardware element. As mentioned by Alaus et al (2011), the size of the software to be downloaded should be small to reduce the reconfiguration time. Moreover, reconfigurability can be optimized by parameterization method in which the common aspects of different communication standards to be supported by the reconfigurable transceiver are identified and a common processing procedure is installed in the device. The two approaches in parameterisation method are Common Function technique (CF) and Common Operator approach (CO).

Thiagaraajan et al (2007) have applied CO technique to the two operators namely FFT and Reconfigurable Linear Feedback Shift Register(R-LFSR) in digital domain. In CO technique, common elements with similar structural aspects that can be described by a general equation and independent
of typical standards are identified. From the design and implementation point of view these selected CO functions should be reusable and reconfigurable so that they can be applied to any standard. In the present work, lattice filter module is used as a common element in realizing WH code generator, Matched filter and Gauss Markov model representing the slowly varying signals.

8.2 RELATION BETWEEN LINEAR FEEDBACK SHIFT REGISTER AND AR MODELLING

In general, maximal length sequences like pseudo noise sequences are generated by using a LFSR, suggested by Moon & Stirling (1999). For example, the block diagram to generate a pseudo noise sequence of length 15 using an LFSR of length 4 is shown in Figure 8.1 where ‘D’ and ‘+’ represents a delay element ($z^{-1}$) and exclusive OR operation respectively.

![Block Diagram of a Binary LFSR](image)

**Figure 8.1 Block Diagram of a Binary LFSR**

The problem of generating a given sequence $[c_0, c_1, \ldots, c_{N-1}]$, of length ‘N’, using an LFSR is as follows,

- The feedback connection polynomial $g(D)$ has to be determined.
- The initial register contents of the shortest LFSR that could produce the sequence $c(D)$ has to be determined.
Moon & Stirling (1999) provides a solution to this problem using Massey’s algorithm. This involves solving a set of system equations of the form shown in Equation (8.1). Assuming a LFSR of length 4, the set of equations are given by,

\[
\begin{bmatrix}
    c_3 & c_2 & c_1 & c_0 \\
    c_4 & c_3 & c_2 & c_1 \\
    c_5 & c_4 & c_3 & c_2 \\
    c_6 & c_5 & c_4 & c_3
\end{bmatrix}
\begin{bmatrix}
    g_1 \\
    g_2 \\
    g_3 \\
    g_4
\end{bmatrix}
= \begin{bmatrix}
    -c_4 \\
    -c_5 \\
    -c_6 \\
    -c_7
\end{bmatrix}
\tag{8.1}
\]

In Equation (8.1), the matrix on the left side is a Toeplitz matrix, and these equations represent the Yule-Walker equations of an AR process in signal modelling. This implies that it may be possible to generate the maximal length sequences from the impulse response of an AR deterministic signal model (Alaus et al 2011). In stochastic signal modelling, these equations can be solved by using the Levinson-Durbin algorithm. As indicated by Hayes (2002), Levinson-Durbin recursion algorithm is widely used to solve the Prony normal equations and autocorrelation normal equations. The Autocorrelation method, a modified form of Prony’s normal equations, is applied to a fixed length sequence (windowed sequence). The autocorrelation normal equations in matrix form for a real process \( X \) is a set of \( (p+1) \) linear equations with \( (p+1) \) unknowns \( a(1), a(2), \ldots, a(p) \) and \( \varepsilon_{p+1} \) given by,

\[
\begin{bmatrix}
    r(0) & r(1) & \ldots & r(p-1) & r(p) \\
    r(1) & r(0) & \ddots & r(p-2) & r(p-1) \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    r(p-1) & r(p-2) & \ldots & r(0) & r(p-2) \\
    r(p) & r(p-1) & \ldots & r(1) & r(0)
\end{bmatrix}
\begin{bmatrix}
    1 \\
    a(1) \\
    \vdots \\
    a(p)
\end{bmatrix}
= \begin{bmatrix}
    1 \\
    0 \\
    \vdots \\
    0
\end{bmatrix}
\tag{8.2}
\]

In matrix notation, Equation (8.2) can be written as,

\[
R_{p+1} a_{p+1} = \varepsilon_{p+1} U_{p+1}
\tag{8.3}
\]
The autocorrelation sequence $r(k)$ is given by,

$$r(k) = \sum_{n=k}^{N} x(n)x(n-k); k \geq 0$$  \hspace{1cm} (8.4)

‘N’ represents the length of the sequence $x(n)$ and the minimum modelling error $\varepsilon_{p+1}$ is given by,

$$\varepsilon_{p+1} = r(0) + \sum_{k=1}^{b} a(k)r(k)$$  \hspace{1cm} (8.5)

**Figure 8.2 Single Stage of an All Pole Lattice Filter**

**Figure 8.3 Order p All Pole Lattice Filter**

Equation (8.2) can be solved by using the Levinson Durbin Algorithm. It represents an all-pole model of the form given by,

$$H(z) = \frac{b_0}{1 + \sum_{k=1}^{p} a_p(k) z^{-k}}$$  \hspace{1cm} (8.6)
Equation (8.6) can be implemented using the direct form-II digital filter structure. The reflection coefficients, one of the by-products obtained by solving the autocorrelation normal equations can be used to implement an all-pole Infinite Impulse Response (IIR) lattice filter. Parhi (1999) have mentioned that the poles and zeros of the transfer function in direct form structures are very sensitive to quantisation effects since these structures are implemented directly using the quantised numerator and denominator coefficients. This drawback of sensitivity becomes worse as the number of crowded poles increase and its effect can be reduced by using cascade or parallel form of filter structures. However, filters realised with lattice structures possess good numerical properties since it can be realised as a cascade of regular modules. Figures 8.2 and 8.3 show the single stage and order 'p' all pole lattice filter respectively, where ‘$T_p$’ is referred as $p^{th}$ reflection coefficient and $f$, $g$ stands for all pole and all-pass response respectively. This modular structure makes it more suitable for VLSI implementation. In the present work, generation of WH codes using LF by solving the autocorrelation normal equations is discussed. The significance of these reflection coefficients is that they are all bounded by one in magnitude resulting in a stable all-pole model.

8.3 GENERATION OF WH CODES USING RECONFIGURABLE LATTICE FILTER

In this section, the proposed method of WH code generation from the impulse response of an AR model realized using lattice filter structure is described. A numerical example showing the generation of WH codes of length 8 is also discussed.

8.3.1 AR modeling of WH Code Generation

The algorithm to obtain the AR model of WH code is as follows:
a) Calculate the autocorrelation values for a WH code sequence of length ‘N’.

b) Apply Levinson Durbin algorithm to these autocorrelation values and obtain the reflection coefficients \( T_1, T_2, \ldots, T_{N/2} \).

c) Implement an Infinite Impulse Response (IIR) all pole lattice filter using these reflection coefficients. Increase the model order from \((N/2)\) to \((N/2)+1, \ldots, N\) to get highly precise results.

d) Obtain the all pole response \( \{f(k) : k=1,2,\ldots,N\} \) of this filter by giving a unit step sequence as input.

e) Initialise the first bit of the code as \( H(1,1)=1 \), since the first bit of all the WH code sequences start with one.

f) Calculate the remaining bits, \( H(1,i), \ i=2,3,\ldots,N \) using Equation (8.7),

\[
H_N(1,i)=\text{sgn}\{[f(i)-f(i-1)]\}; i=2,3,\ldots,N.
\]  

(8.7)

where \( \text{sgn} \) represents a signum function.

![Figure 8.4 WH Code Generation from All-pole Response of Lattice Filter](image.png)
8.3.2 Numerical Example of WH Code Generation from AR model of WH Codes

As an example, consider a Walsh Hadamard code set of size and length 8 in which each row represents a code sequence.

\[
H_8 = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 \\
\end{bmatrix}
\]  

(8.8)

Each row in the matrix \(H_8\) represents a code sequence. The following pattern can be observed from the code sequences. The code sequence 2 can be generated from code sequence 1 by changing the signs at bit positions 2, 4, 6, 8. In the same manner, code sequences 4, 6, 8 can be obtained from the code sequences 3, 5, 7 respectively. Reflection coefficients for a WH code of length 8 and 16 generated by the proposed algorithm are shown in Tables 8.1 and 8.2 respectively.

**Table 8.1 Reflection Coefficients of WH Code of Length 8**

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<tr>
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<th></th>
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<td>-0.875</td>
<td>0.875</td>
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<td>0.125</td>
<td>-0.625</td>
<td>0.625</td>
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<td>0.375</td>
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<td>0.0667</td>
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<td>0.7778</td>
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<td>0.2308</td>
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<td>0.4545</td>
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<td>-0.0714</td>
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<td>0.3571</td>
<td>0.3</td>
<td>-0.3</td>
<td>0.1</td>
<td>-0.1</td>
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<td>0.0769</td>
<td>0.0769</td>
<td>0.3684</td>
<td>0.3684</td>
<td>0.4286</td>
<td>0.4286</td>
<td>0.5556</td>
<td>0.5556</td>
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Table 8.2 Reflection Coefficients of WH Code of Length 16

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</tr>
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<td>-0.9375</td>
<td>0.9375</td>
<td>-0.0625</td>
<td>0.0625</td>
<td>-0.5625</td>
<td>0.5625</td>
<td>-0.4375</td>
<td>0.4375</td>
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<tr>
<td>0.0323</td>
<td>0.0323</td>
<td>0.8824</td>
<td>0.8824</td>
<td>0.28</td>
<td>0.28</td>
<td>0.3913</td>
<td>0.3913</td>
</tr>
<tr>
<td>0.0333</td>
<td>-0.0333</td>
<td>-0.4333</td>
<td>0.4333</td>
<td>0.3889</td>
<td>-0.3889</td>
<td>0.2778</td>
<td>-0.2778</td>
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<td>0.0345</td>
<td>0.0345</td>
<td>0.3488</td>
<td>0.3488</td>
<td>0.6364</td>
<td>0.6364</td>
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<td>0.6923</td>
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<tr>
<td>0.0357</td>
<td>-0.0357</td>
<td>-0.3116</td>
<td>0.3116</td>
<td>-0.6071</td>
<td>0.6071</td>
<td>-0.1981</td>
<td>0.1981</td>
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<td>0.037</td>
<td>0.037</td>
<td>0.1831</td>
<td>0.1831</td>
<td>0.1556</td>
<td>0.1556</td>
<td>0.4634</td>
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<tr>
<td>0.0385</td>
<td>-0.0385</td>
<td>-0.0897</td>
<td>0.0897</td>
<td>0.1842</td>
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<td>0.0649</td>
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<tr>
<td>0.04</td>
<td>0.04</td>
<td>0.1529</td>
<td>0.1529</td>
<td>0.2258</td>
<td>0.2258</td>
<td>0.4069</td>
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<td>0.1034</td>
<td>0.1034</td>
<td>0.6842</td>
<td>0.6842</td>
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<td>0.1852</td>
<td>0.5238</td>
<td>0.5238</td>
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<tr>
<td>0.1154</td>
<td>-0.1154</td>
<td>-0.2692</td>
<td>0.2692</td>
<td>0.2273</td>
<td>-0.2273</td>
<td>-0.0455</td>
<td>0.0455</td>
</tr>
<tr>
<td>0.1304</td>
<td>0.1304</td>
<td>0.3939</td>
<td>0.3939</td>
<td>0.2941</td>
<td>0.2941</td>
<td>0.4783</td>
<td>0.4783</td>
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<tr>
<td>0.15</td>
<td>-0.15</td>
<td>-0.4978</td>
<td>0.4978</td>
<td>-0.15</td>
<td>-0.15</td>
<td>-0.3912</td>
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<td>0.1765</td>
<td>0.1765</td>
<td>0.1321</td>
<td>0.1321</td>
<td>0.2174</td>
<td>0.2174</td>
<td>0.2093</td>
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<td>0.2143</td>
<td>-0.2143</td>
<td>0.2619</td>
<td>-0.2619</td>
<td>0.2778</td>
<td>-0.2778</td>
<td>0.2863</td>
<td>-0.2863</td>
</tr>
<tr>
<td>0.2727</td>
<td>0.2727</td>
<td>0.2258</td>
<td>0.2258</td>
<td>0.3846</td>
<td>0.3846</td>
<td>0.3772</td>
<td>0.3772</td>
</tr>
</tbody>
</table>

The following observations can be noted from the Tables 8.1 and 8.2.

a) It can be observed that the numerical values of reflection coefficients are the same for codes 1 and 2, 3 and 4, 5 and 6, 7 and 8, 9 and 10, 11 and 12, 13 and 14, 15 and 16 but with only a sign change in the odd bit positions. Hence, it can be
inferred that the two different code sequences can be generated from the same set of reflection coefficients by changing the sign in odd positions which means that the code generation architecture is reconfigurable.

b) To generate a code of length 8, the number of reflection coefficients to be stored in memory is only 4. Hence, to generate a code of length N, the number of memory elements needed is only N/2. This implies that the memory requirement is less.

c) The AR model order has to be increased to 2N to generate code sequences of length 4N. This can be accomplished by adding extra lattice filter modules and using the corresponding set of reflection coefficients.

Figure 8.5  Magnitude Spectrum of a Sample Sequence of WH-Code Set of Length 8, 16 and their Respective AR Models

Figure 8.5 shows the magnitude spectrum for a sample code sequence in 8, 16 length WH code sets and their respective impulse responses from the WH-AR model. It is observed that the magnitude spectrum distribution of the impulse response obtained from the AR model of WH
codes and the WH code are nearly the same. Hence, the proposed AR model can be used to generate the WH codes.

Table 8.3 shows the all pole response, the first order difference of the all pole response and the WH code generation from this first order difference for a sample WH code sequence of length 16 by using the algorithm described in section 8.3.1. It can be observed that the WH code sequence generated by the proposed method is the same as the original WH code.

Table 8.3 Proposed Method of WH Code Generation for a Sample Sequence of Length 16

<table>
<thead>
<tr>
<th>Reflection Coefficients</th>
<th>All pole response f(i)</th>
<th>difof = f(i)-f(i-1)</th>
<th>Signum (difof)</th>
<th>WH code</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3125</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<tr>
<td>0.5238</td>
<td>0.3772</td>
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<td>-1</td>
</tr>
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<td>0.0455</td>
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<td>0.4783</td>
<td>0.1802</td>
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<td>1</td>
</tr>
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<td>0.3912</td>
<td>-0.1223</td>
<td>-0.3025</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0.2093</td>
<td>0.2284</td>
<td>0.3506</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-0.2863</td>
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<td>1</td>
</tr>
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<td>0.3256</td>
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<td>-1</td>
<td>-1</td>
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<td>-0.0809</td>
<td>-0.4065</td>
<td>-1</td>
<td>-1</td>
<td></td>
</tr>
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<td>0.1096</td>
<td>0.1905</td>
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<td>1</td>
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<td></td>
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</table>
8.4  RLF AS A MATCHED FILTER IN DS-CDMA SYSTEM

In communication systems, binary messages transmitted across a noisy channel are detected by using a matched filter at the receiving end as described by Hayes (2002) and Moon & Stirling (1999). Matched filter is an optimum filter that maximizes the SNR of a signal in the presence of AWGN. A matched filter operates by correlating a known signal with an unknown signal and detects the presence of the known signal pattern. This is equivalent to convolving the incoming unknown signal with the known signal pattern.

![Figure 8.6 Conventional Method of Despreading](image)

![Figure 8.7 Proposed DS-CDMA System](image)
Figure 8.6 shows the conventional method of despreading. Figure 8.7 shows the DS-CDMA system employing the proposed reconfigurable matched LF. In the traditional method of DS-CDMA technique, spreading is accomplished at the transmitting end by multiplying the information bearing signal with the spreading codes. At the receiving end, the original message is recovered by correlating the received spreaded signals with a replica of the signature sequence used at the transmitting end. This process is called as despreading as mentioned by Haykins (1998) and Proakis (2001). In CDMA systems, despreading is achieved by sliding correlators, Surface Acoustic Wave (SAW) matched filters and Digital matched filters as described by Moon & Stirling (1999). As mentioned by Chapman et al (2000), the matched filter used in CDMA systems is tuned to match a code sequence. It is generally realized using a tapped delay line filter structure i.e, FIR structure or moving average model where the input data pattern is represented by digital words and the spreading code pattern of +1 or -1 are stored as the multiplication coefficients. As discussed previously, WH code generator can also be realized using the AR model. Here, despreading is done at the receiving end by the AR model of WH code generator realized by a lattice filter instead of the digital matched filter.

The modified algorithm of despreading is given below,

a) Replace the multiplier and code generator by the AR model WH (z) of the WH code.

b) Convert the received signal after demodulation into NRZ polar format represented as R(z), then pass it through the AR lattice filter with transfer function WH (z) and the filter output response f(n) is obtained which is given by,

\[
f(n) = F^{-1}[R(z)WH(z)]
\]  

(8.9)
where \( F^{-1} \) indicates inverse Fourier transform.

c) Pass this output \( f(n) \) through an integrator that integrates the lattice filter response over a period equal to the length of the spreading code and outputs one value during each period.

d) Pass the output of integrator through a decision device and an estimate \( b_e(n) \) of the transmitted data is obtained. It is given by,

\[
b_e = \text{sgn} \left( \sum_{k=1}^{N} f(k) \right)
\]

(8.10)

where \( N \) is the length of the spreading code and \( \text{sgn} \) represents the signum function.

e) Reconfigure the proposed matched filter receiver to match the users spreading codes by using their respective reflection coefficients stored in a look-up table.

### 8.4.1 Simulation Results

In this section, BER performance of a DS-CDMA system employing the conventional method and the proposed matched filter method of despreading is investigated. WH codes are used as signature codes to distinguish different users and spreading at the transmitting end is accomplished using the traditional approach. BPSK modulation is used. The signal at the receiving end is given by,

\[
r = \sum_{i=1}^{K} H_i S_i + W
\]

(8.11)
$i=1,2,\ldots,K$ represents the number of users,

$s_i$ is the transmitted spreaded signal of user $i$.

$W$ is the AWGN, $H_i$ represents the flat fading Rayleigh channel of user $i$.

**Figure 8.8** BER Performances Employing Conventional Method and Matched Filter AR Signal Model of 8, 16-Length Walsh Codes in an AWGN Plus Flat Fading Rayleigh Channel with Four User Scenario in a DS-CDMA System
Figure 8.9 Multi-user BER Performances Employing Conventional Method and Matched Filter AR Signal Model of 16-Length Walsh Codes in a DS-CDMA System

Figure 8.8 shows the BER performances in a DS-CDMA system employing conventional method and proposed matched filter method of despreading for 8 and 16 lengths Walsh codes respectively in an AWGN plus Rayleigh flat fading channel with four user scenario. The response of traditional method and lattice filter method are denoted as TR and LF
respectively in the graph. It can be observed that the outputs of the proposed and conventional method are marginally the same.

Figure 8.9 shows the multi-user BER performances employing the conventional and proposed matched filter method of despreading of 16-length Walsh codes in an AWGN plus flat fading Rayleigh channel for SNR=6dB and SNR=12dB as a function of the number of users in a DS-CDMA system. Simulation results show that the BER performance of the AR lattice model is slightly better than the conventional method of despreading. BER performance of lattice filter can be improved if the round off noise effects that occur in lattice filter structures are reduced. This round off noise effect can be reduced if the proposed LF structure is replaced by scaled normalised LF based on Schur algorithm which can be a further extension of this work. These filter structures provide the added advantages of pipelining, parallel processing, Givens rotation implementation using CORDIC algorithm and low power VLSI implementation.

8.5 RELATION BETWEEN AR MODEL OF WH CODE AND GAUSS- MARKOV PROCESS

In transform coding, slowly varying signals are represented by Gauss Markov model suggested by Dougherty (2003). In this type of modelling, if the distribution of observation of the current random variable is dependent only on one previous observation, it then represents an AR signal model. For a wide sense stationary input random vector \( \mathbf{X} = (X_1, X_2, \ldots, X_n)^T \), of unit variance, the covariance matrix for this single step correlation is given by,

\[
K_x = \begin{bmatrix}
1 & \rho & \rho^2 & \cdots & \rho^{n-1} \\
\rho & 1 & \rho & \cdots & \rho^{n-2} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & 1
\end{bmatrix}
\]  

(8.12)
where \( \rho \) represents the correlation coefficient.

Principal Component analysis of this covariance matrix (i.e.,) KLT gives the eigen values and their corresponding eigen vectors respectively. These eigen vectors are in general arranged in the order of increasing eigen values. The resulting eigen vectors represent unitary spreading codes. As described by Akansu & Poluri (2007a), KLT bases generated for low values of correlation coefficient from this covariance matrix can be used in the design of spreading codes. It has been shown that these varying power spreading codes, when applied in DS-CDMA communications, performs comparable to or marginally better than the widely used Gold codes and outperforms Walsh codes in an AWGN plus flat fading Rayleigh channel conditions.

By applying the procedure as discussed in Section 8.3.2 to these eigen vectors, a set of binary spreading codes can be generated. In this work, the autocorrelation values of these eigen vectors are calculated and used to find the reflection coefficients of the lattice filter structure. It is observed that the binary code set generated from the KLT bases of the Gauss Markov signal model realised by the proposed lattice filter structure are nearly the same as the WH codes.

8.5.1 Numerical Example of WH Code Generation from AR Model of First Order Gauss-Markov Process

Table 8.4 gives the reflection coefficients generated for a Gauss Markov process with a correlation coefficient of \( \rho=0.91 \). Table 8.5 shows the decimal equivalent of the WH code set of size 8, and KLT codes generated from a covariance matrix of size 8 by 8 for the correlation coefficients taking the values of \( \rho=0.2 \) and \( \rho=0.91 \).
Table 8.4 Reflection Coefficients of KLT Binary Code of Length 8

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.8925</td>
<td>0.1332</td>
<td>0.7464</td>
<td>-0.4871</td>
<td>-0.2001</td>
<td>0.9341</td>
<td>0.4653</td>
<td>-0.6992</td>
</tr>
<tr>
<td>0.1339</td>
<td>0.7814</td>
<td>0.8432</td>
<td>0.5773</td>
<td>0.7092</td>
<td>0.8544</td>
<td>0.8212</td>
<td>0.3616</td>
</tr>
<tr>
<td>0.1162</td>
<td>0.3586</td>
<td>0.5158</td>
<td>0.3036</td>
<td>0.321</td>
<td>0.5857</td>
<td>0.4296</td>
<td>0.2522</td>
</tr>
<tr>
<td>0.1004</td>
<td>0.3691</td>
<td>0.3948</td>
<td>0.1482</td>
<td>0.2234</td>
<td>0.2278</td>
<td>0.4555</td>
<td>0.1647</td>
</tr>
</tbody>
</table>

Table 8.5 Decimal values of Walsh Hadamard Codes and KLT Codes of Length 8

<table>
<thead>
<tr>
<th>Walsh-Hadamard code set</th>
<th>Proposed KLT code set for ( \rho=0.2 )</th>
<th>Proposed KLT code set for ( \rho=0.91 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>255</td>
<td>255</td>
<td>255</td>
</tr>
<tr>
<td>170</td>
<td>240</td>
<td>240</td>
</tr>
<tr>
<td>204</td>
<td>195</td>
<td>195</td>
</tr>
<tr>
<td>153</td>
<td>204</td>
<td>206</td>
</tr>
<tr>
<td>240</td>
<td>153</td>
<td>153</td>
</tr>
<tr>
<td>165</td>
<td><strong>178</strong></td>
<td><strong>178</strong></td>
</tr>
<tr>
<td>195</td>
<td>165</td>
<td>165</td>
</tr>
<tr>
<td><strong>150</strong></td>
<td>170</td>
<td>170</td>
</tr>
</tbody>
</table>

The order of KLT code sequence in the table corresponds to the code generated from the eigen vector that corresponds to the ascending eigen value. It can be noted from the table that only one code sequence is different between the WH codes and the KLT codes. It can be observed that by increasing the values of the correlation coefficient from 0.2 to 0.91, the number of bits in which they differ from the original WH code set is only 1 or 2 bits. Hence, the lattice filter structure representing an AR Gauss Markov process can be used to generate WH codes. Since the lattice filter transfer function represents a Gauss Markov process, this lattice filter structure can be
interpreted as matched with a slowly varying transmitted signal or slowly varying channels. Thus, this structure can be used as a matched filter receiver or as an equaliser at the receiving end.

8.6 CONCLUSION

It is shown that the lattice filter can be reconfigured to generate the WH codes by just changing the reflection coefficients. The results obtained show that the AR lattice filter structure can be reconfigured to function either as a WH code generator or as a matched filter. It is also shown that the AR model of Gauss Markov process can be used to generate WH codes. Hence, the same structure can be used as an equaliser to receive slowly varying transmitted signals or slow fading channels which is a further extension of the work. Thus, the proposed AR lattice filter structure satisfies the objective of using the same structure for multiple functions in SDR.