Chapter 4

MODIFIED FREQUENCY RESPONSE MASKING APPROACH

4.1 Introduction

One of the most difficult problems in filter synthesis is the design of narrow transition band FIR filters, which require high order for their implementation using conventional methods. Linear phase FIR digital filters require in applications demanding narrow transition bands considerably more arithmetic computations and hardware than their IIR equivalents [29]. The search of design and synthesis techniques which will produce efficient filters for the implementation of narrow transition width FIR filters has attracted much research efforts in the past few years.

Several methods have been proposed in the literature for reducing the complexity of very sharp FIR filters. One of the most successful techniques for synthesis of very narrow transition width filter is the Frequency Response Masking (FRM) technique because of reduced arithmetic complexity involved [7], [30]-[35]. FRM approach can be used to implement both linear phase and approximately linear phase high-speed recursive digital filters. When the frequency response masking technique is used to synthesis an FIR filter, resulting filter will comprise of a sparse coefficient filter and a pair of masking filters. The major advantages of FRM approach is that the filter has a very sparse coefficient vector so its arithmetic complexity is very low, though its length and delays are slightly longer than those in the conventional implementations. For a given frequency response specification its effective filter length including both zero and non-zero coefficients is only slightly longer than the infinite word length optimum minimax filter. These filters are suitable for VLSI implementations since hardware complexity is reduced. In addition to low pass designs the FRM approach can be extended to the design of highpass and bandpass filters [35].
Optimization techniques are being developed for superior filter performance with minimum filter hardware complexity and recently optimization techniques have been developed for reduction in passband ripple and increasing stopband attenuation for IIR filters using FRM approach which is dealt in [36].

Basic FRM technique [7] for implementing sharp linear phase FIR filters with arbitrary bandwidth was generalized in [30]. Very sharp transition filters are implemented using several simple subfilters with significant saving in arithmetic operations as high as 4:1 compared to conventional implementations was achieved. The overall FRM filter is designed by separately optimizing the model and masking filters in which case the filters can be designed easily and fast. The FRM filters obtained in the approach of separate subfilter optimization serve as good initial filters for further optimizations. A drawback is that the resulting overall filter is not optimal. It can therefore be beneficial to consider simultaneous optimization of the model and masking filters.

Design methods are presented for reducing the complexity of the masking filters by capitalizing on the fact that the frequency responses of the masking filters are similar. They are able to produce savings in terms of number of multipliers having different group delay and round off noise performances. Synthesis of very sharp half-band filter using frequency response masking technique is dealt in [37]. An important property of a half band filter is that half of its coefficient values are trivial. This yields significant advantages in terms of reduction in computational complexity.

The frequency response masking approach for high speed recursive infinite impulse response (IIR) digital filters is introduced in [33]. In this approach, the overall filter consists of a periodic IIR filter, its power complementary periodic filter and two linear phase FIR masking filters. High speed narrow band recursive filters are realized using an IIR filter for the model filter and an FIR filter for the masking filter. An advantage of using
techniques based on periodic and nonperiodic filters is that there is a large freedom to choose structures for the model and masking filters that are well suited for the specification and problem at hand. IIR model filters are realizable as a parallel connection of two all-pass filters, with a large freedom to choose structures with good properties since all pass filters can be realized in many different ways like lattice wave digital filters etc. In addition, the all pass filters can always be realized by cascading low order sections, which is attractive from an implementation point of view.

The maximal sample frequency for the overall filter is \( M \) times that of the corresponding conventional IIR filter. The maximal sample frequency can be increased to an arbitrary level for arbitrary bandwidths. The overall FRM filter can be designed by separately optimizing the model and masking filters with the aid of conventional approximation techniques. The obtained overall filter also serves as a good initial filter for further optimization. Further, robust filters under finite arithmetic conditions can always be obtained by using wave digital all pass filters and non-recursive FIR filters. Recursive infinite impulse response (IIR) digital filters have a drawback in that they restrict the sample frequency at which an implementation of the filters can operate. This may affect not only the speed but also the power consumption, since excess speed can be traded for low power consumption through the use of power supply voltage scaling techniques.

As an application of the FRM techniques, the design of a linear phase digital filter bank for audio system frequency response equalization is dealt in [38]. This filter bank uses frequency response masking and complementary filtering principles to achieve very sharp frequency response for each band, unity gain at all frequencies and extremely low hardware complexity. Several structures are discussed for implementing the overall FRM filter the two-branch structure being commonly used [7]. These structures are compared with each other and with equivalent direct-form minimax designs in terms of the number of distinct
multipliers, overall filter order, overall multiplication rate, number of delay elements, coefficient sensitivity and output noise variance.

In the FRM techniques [7], closed form expressions for impulse response coefficients were not obtained. We propose an analytical approach to the design of sharp transition filters with least arithmetic complexity. The approach for the design of subfilters is simple, analytical, without extensive computations and can be extended to design sharp cutoff, highpass, bandpass and multiband filters with arbitrary passband. This chapter deals with a modified Frequency Response Masking (MFRM) technique for the synthesis of linear phase, sharp transition and low arithmetic complexity FIR filter. The structure is composed of proposed linear phase, sharp transition FIR lowpass and bandpass filters used as subfilters to obtain the MFRM filter. The frequency response of the subfilters are modeled using trigonometric functions of frequency and the design yields closed form expressions for the impulse response coefficients of the subfilters and the final MFRM. The bandpass filter eliminates one masking filter and a model filter from the basic FRM approach thereby simplifying the synthesis of the proposed MFRM filter. Expressions for impulse response coefficients are derived, coefficients obtained and simulation of the lowpass and bandpass filters required for realization of MFRM filter is carried out.

4.2 Review of Frequency Response Masking Approach

4.2.1 Basic Frequency Response Masking Approach

The basic idea behind the frequency response masking technique [7] is to compose the FRM filter using several sub-filters. The first sub-filter is known as model filter or edge shaping filter is upsampled to form the sharp edge needed in the FRM filter. The second sub-filter is the complement of the model filter, which helps to form the arbitrary bandwidth of the FRM filter response. The other two subfilters are masking filters, which
extract one or several passbands of the periodic model filter and periodic complementary model filter and remove unwanted frequency components to form the stop band of the FRM filter response [39], [40].

In the FRM approach, the transfer function of the FRM filter is expressed as

$$H(Z) = G(Z^M)F_0(Z) + G_c(Z^M)F_1(Z) \quad (4.1)$$

The filters $G(z)$ and $G_c(z)$ are prototype linear phase model and a complementary model filter respectively. The filters $F_0(z)$ and $F_1(z)$ are masking filters which extract one or several pass bands of the periodic model filter $G(z^M)$ and periodic complementary model filter $G_c(z^M)$ respectively. The frequency responses of $G(z^M)$ and $G_c(z^M)$ are those of $G(z)$ and $G_c(z)$ compressed in frequency scale by a factor $M$, that is replacing every delay in $G(z)$ and $G_c(z)$ by $M$ delays each. In particular, the transition band of the filters $G(z^M)$ and $G_c(z^M)$ is $M$ times as sharp as that of $G(z)$ and $G_c(z)$.

Also

$$G_c(Z^M) = Z^{-M\left(\frac{N-1}{2}\right)} - G(Z^M) \quad (4.2)$$

where $N$ is the model filter length and $M$ is some positive integer. The corresponding structure is as shown in Fig. 4.1. Typical magnitude responses for the model, masking and overall FRM filter $H(z)$ is as shown in Fig. 4.2 where $k$ is some positive integer. The transition band of overall FRM filter $H(z)$ can be selected to be equal to one of the transition region of one of the periodic filters $G(z^M)$ or $G_c(z^M)$. In order that the two branches should be added in phase at the output the delays along with the two parallel paths must be same in Fig. 4.1.

The transitions of the frequency masking filter $F_0(z)$ and $F_1(z)$ shown in the Fig. 4.2 are governed by the width of the individual tooth in $G(z^M)$ and by the separation between neighboring teeth. When the desired transition is very sharp, $M$ can be quite large, making
Fig. 4.1. Block diagram of structure used in basic FRM approach.
Fig. 4.2. Illustration of basic FRM approach
the tooth width very narrow. In this case, the transition width of the two masking \( F_0(z) \) and \( F_1(z) \) is also very narrow, leading to high order masking filters. To overcome this difficulty, the idea of frequency compression is applied to the design of these masking filters [30]. Two prototype masking filters \( F_0(z) \) and \( F_1(z) \) are designed and then by applying frequency compression to these filters i.e. \( F'_0(z) = F_0(z^N) \) and \( F'_1(z) = F_1(z^N) \) for some integer \( N \). The transition band of \( F'_0(z) \) and \( F'_1(z) \) are now \( N \) times as sharp as those of \( F_0(z) \) and \( F_1(z) \) respectively. However this frequency compression introduces spurious teeth in \( F'_0(z) \) and \( F'_1(z) \) at high frequencies. These teeth can be removed by a low pass filter \( E(z) \) which has a rather gentle transition hence will not be of high order. The basic frequency response masking approach can be regarded as a special case where \( N \) is one and \( E(z) \) is not needed in this case. Also a two stage FRM structure is used when the desired transition band is extremely narrow [30]. Therefore in this design, \( M \) can be very large making the transition width of the model filter very narrow. Hence transition width of masking filters will also be sharp and hence require a high order. Therefore, in this case, a second stage of frequency masking can be added with masking filters also compressed in suitable frequency scale.

4.3 Modified Frequency Response Masking Approach

Linear phase, equiripple passband and stopband, sharp transition lowpass and bandpass FIR filters designed using our proposed approach are used as various subfilters for the realization of the sharp transition modified FRM (MFRM) lowpass FIR filter [27]. Let \( \varphi, \theta \) be the stopband and passband edges of model filter \( F_a(z) \) respectively and \( \omega_s, \omega_c \) be the stopband and passband edges of modified FRM filter \( F(z) \). The transition bandwidth \( \Delta \omega_m \) of model filter of length \( N_m \) is

\[
\Delta \omega_m = \varphi - \theta = M(\omega_s - \omega_c) = M \Delta \omega
\]

(4.3)
where $\Delta \omega$ is the transition bandwidth of the MFRM filter.

The complementary model filter $F_c(z)$ has a transfer function

$$F_c(z) = z^{-\frac{(N_m-1)}{2}} F_a(z)$$

(4.4)

In our approach, the model and complementary model filters need not be symmetrical around $\omega = \frac{\pi}{2}$ as in the conventional FRM filter. This makes the design more flexible and universal such that a wide range of narrow and wide passbands can be achieved. Fig. 4.3 illustrates the proposed MFRM approach with band edges and slopes of various subfilters shown. The frequency domain responses of the periodic model and periodic complementary model filters are related as

$$F_a(e^{j\omega}) + F_c(e^{j\omega}) = 1$$

(4.5)

Masking filters $F_{ma}(z)$ and $F_{mc}(z)$ used to mask $F_a(z)$ and $F_c(z)$ respectively have frequency responses with equal transition bandwidth and are parallel in the respective transition region. The transition bandwidth of the masking filters depends on the values of $\theta$ and $\varphi$ which are functions of $M$ and $k$ which are positive integers. The frequency domain synthesis of this scheme with typical magnitude responses of model, masking and bandpass filters is as shown in Fig.4.3. The frequency response of the two masking filters are related by

$$F_{ma}(e^{j\omega}) = F_{mc}(e^{j\omega}) + F_{bp}(e^{j\omega})$$

(4.6)

where $F_{bp}(e^{j\omega})$ is a bandpass filter designed with center frequency, passband edges and passband width as shown in Fig.4.3.

Referring to Fig.4.3, Gain of bandpass filter, $A = \frac{2(\pi - \varphi)}{M} \cdot \frac{M}{2\pi - (\varphi + \theta)}$

(4.7)
Fig. 4.3 Illustration of the proposed modified frequency response masking approach
Simplifying, \( A = \frac{2(\pi - \phi)}{2\pi - (\phi + \theta)} \)  

(4.8)

A is chosen to give a passband gain of unity for the MFRM filter. We have for the basic FRM approach,

\[ F(e^{j\omega}) = F_n(e^{jM\omega}) F_m(e^{j\omega}) + F_c(e^{jM\omega}) F_{nc}(e^{j\omega}) \]  

(4.9)

Substituting (4.5) and (4.6) in (4.9), we obtain,

\[ F(e^{j\omega}) = F_{nc}(e^{j\omega}) + F_n(e^{jM\omega}) F_{bp}(e^{j\omega}) \]  

(4.10)

Hence the proposed transfer function for the sharp transition lowpass MFRM filter realization derived from the basic FRM approach is

\[ F(z) = F_{nc}(z) + F_n(z^M) F_{bp}(z) \]  

(4.11)

The proposed structure shown in Fig.4.4 is a two branch realization where the delays of the two parallel paths must be same in order to be added in phase at the output. In Fig 4.4, it is assumed that the delay in the two parallel branches are equalized. In our approach the bandpass filter eliminates one of the masking filter and one periodic model filter greatly simplifying the synthesis of FRM filter. The slopes of the various sub-filters and their filter edges have been designed for the MFRM approach as shown in Fig.4.3. Our proposed lowpass and bandpass filters designed in earlier sections have been used as subfilters in the MFRM approach. The expressions for the impulse response coefficients \( h(n) \) of the proposed linear phase lowpass and bandpass FIR filter are as in Section (2.8.3) and (3.2.2) respectively.
Fig. 4.4. Block diagram of structure for low pass Modified FRM filter realization.
4.3.1 Filter Synthesis Results

Design Example: Lowpass linear phase sharp transition FIR filters are designed for the desired filter specifications: cutoff edge \( \omega_c \) is 0.7332\( \pi \), stopband edge \( \omega_s \) is 0.7432\( \pi \), maximum passband ripple \( \delta_p \) is \( \pm 0.1 \)dB (0.2 dB) and minimum stopband attenuation \( \delta_S \) is 40dB using the proposed Modified FRM approach.

The filter is designed using MATLAB with program MMF and measurement of various filter specifications is done using MATLAB’s SP toolbox. Results are tabulated in Table 4.1. The Modified FRM approach, composed of proposed lowpass and bandpass filters as subfilters (refer Sections 2.8 and 3.2). The MFRM filter is synthesized with various band edges for the lowpass and bandpass subfilters obtained using equations in Fig.4.3. The specifications of subfilters band edges measured using SP toolbox meet the design specifications and the final MFRM filter passband edge is 0.7332\( \pi \) and stopband edge is 0.7432\( \pi \) with a transition width of 0.01\( \pi \) with passband ripple \( \delta_p \) of 0.13dB and stopband attenuation \( \delta_S \) of 42.2dB are obtained for a ‘sum of subfilter order’ of 273. The magnitude response of various subfilters, final MFRM filter, impulse response sequence and phase response is as shown in Fig. 4.5. The actual MFRM filter order is higher than ‘sum of subfilter order’ but it does not contribute to filter complexity because it has zero value coefficients which only increases delays in the MFRM filter. Table 4.1 gives filter order of various subfilters and final MFRM filter for a transition bandwidths of 0.01\( \pi \) and 0.004\( \pi \).
Fig. 4.5 (a) Magnitude response of various subfilters used in proposed MFRM approach with Model filter (light green), Masking filter (red), bandpass filter (green) and final MFRM filter (blue) (b) Linear plot (c) Final MFRM filter magnitude response (d) MFRM impulse response sequence (e) MFRM phase response.

Fig. 4.6. Magnitude response of masking subfilters (using proposed filters) employed in basic FRM approach and the magnitude response of final FRM filter (red colour).
### Table 4.1

Filter order required for various proposed subfilters for realization of MFRM filter with transition bandwidths of 0.01\( \pi \) and 0.004\( \pi \) for maximum passband ripple of 0.15dB and minimum stopband attenuation of 40dB.

<table>
<thead>
<tr>
<th>M</th>
<th>k</th>
<th>Model filter order</th>
<th>Masking filter order</th>
<th>Bandpass filter order</th>
<th>Sum of Subfilter order</th>
<th>Passband Ripple in dB</th>
<th>Stopband Attn. in dB</th>
<th>Transition bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>3</td>
<td>71</td>
<td>161</td>
<td>101</td>
<td>333</td>
<td>0.076</td>
<td>36.8</td>
<td>0.004( \pi )</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>66</td>
<td>86</td>
<td>121</td>
<td>273</td>
<td>0.13</td>
<td>42.2</td>
<td>0.01( \pi )</td>
</tr>
</tbody>
</table>

### Table 4.2

Filter order required for various proposed subfilters for realization of basic FRM filter with transition bandwidths of 0.01\( \pi \) and 0.005\( \pi \) for maximum passband ripple of 0.15dB and minimum stopband attenuation of 40dB.

<table>
<thead>
<tr>
<th>M</th>
<th>k</th>
<th>Model filter order</th>
<th>Masking filter1 order</th>
<th>Masking filter2 order</th>
<th>Sum of Subfilter order</th>
<th>Passband ripple in dB</th>
<th>Stopband Attn. in dB</th>
<th>Transition width</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3</td>
<td>67</td>
<td>67</td>
<td>67</td>
<td>201</td>
<td>0.14</td>
<td>40.4</td>
<td>0.01( \pi )</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>40</td>
<td>120</td>
<td>150</td>
<td>310</td>
<td>0.15</td>
<td>42.1</td>
<td>0.005( \pi )</td>
</tr>
</tbody>
</table>
4.4 Proposed filters as subfilters in FRM approach

Using our proposed lowpass filters as subfilters for the basic FRM approach [7] in Fig. 4.2 the impulse response coefficients can be obtained for the component subfilters as well as the overall FRM filter. The filter is designed using MATLAB with program MFRM. The FRM filter order is higher in our case since the subfilters i.e. proposed lowpass filters are not optimized to obtain the desired filter specifications with least filter order as done for subfilters designed by Lim [7]. Magnitude response of masking subfilters and final FRM filter using proposed filters is shown in Fig. 4.6.

4.5 Conclusions

We have proposed a novel technique for a sharp transition, linear phase, lowpass FIR filter with low arithmetic complexity obtained by modifying the basic frequency response masking approach. The subfilters required i.e. lowpass and bandpass filters are designed using our approach which is a simple direct design and possesses closed form expressions for impulse response coefficients. In our approach, only one masking filter need to be synthesized instead of the two masking filters required in [7]. The bandpass filter has wider transition response which decreases arithmetic complexity of the subfilter. Unlike in frequency response masking approach, the transfer function for subfilters and final MFRM filter in our approach is evolved both in frequency and time domain. The accuracy of the filter approximation can be improved by including a larger number of terms in the impulse response sequence. The lowpass realization can be extended to the realization highpass, bandstop, bandpass and multiband filters.

Basic FRM approach is a graphical approach. In the proposed approach, the same philosophy is adopted as in basic FRM approach, while greatly simplifying the design of subfilters majority of which do not have a steep transition. Lim’s filter [7] uses subfilter optimization hence FRM filter is of lower order. Our subfilters are without optimizations
and of higher order. Hence the resulting final MFRM filter possesses higher order for the same filter specifications of passband ripple and stopband attenuation compared to FRM approach [7]. With MFRM approach sharp to very sharp transition filters can be synthesized.