Chapter 3

SYNTHESIS OF SHARP TRANSITION LINEAR PHASE DIGITAL BANDPASS FIR FILTERS

3.1 Introduction

Various approaches for the design of sharp transition FIR bandpass filters have been dealt in literature. In many of these bandpass realization schemes, the location of the passband as well as the passband width are critical factors affecting the design procedure and the resulting filter implementations. Sharp transition bandpass filters reported in literature generally involve design of component lowpass and highpass filters to realize them. An approximate analytical characterization for a wide band sharp transition bandpass filter realization using FRM approach is dealt in [25]. The component narrow band lowpass and highpass filters of the bandpass filter are implemented using the Interpolated FIR technique. In this case, the frequency specifications for the two component filters depend on the edge frequencies of the desired band pass filter. The two component filters are approximately of the same length as they have the same transition bandwidths. But they have to be designed separately because their passband bandwidths are different depending upon the location and bandwidth of the desired passband of the bandpass filter. The two component filters may have to be redesigned if the center frequency of the bandpass filter is changed. Approximate expressions for the value of interpolating factor and filter hardware required are derived which minimizes the total arithmetic hardware used in the overall bandpass realization. Two branch structure realization [15] is used which is computationally more efficient than the conventional direct form realization with a moderate increase in the number of delays. The value of interpolation factor depends on the center frequency of the bandpass filters. As the
interpolating factor is increased, the computational complexity of the shaping filter decreases
and that of the interpolator increases. Theory and approximate design is developed in [26]
along with formulas that enables to design the overall bandpass filter with minimal
computational complexity. The filter structure utilizes the symmetry of the coefficients to
reduce the number of multipliers in the linear phase case and shares the same set of delays
between the two branches in a two branch structure. As each of the two branches is essentially
an IFIR filter, the overall filter has good roundoff noise and coefficient sensitivity properties.
The two branch method can be used for the design of symmetrical and nonsymmetrical
bandpass filters.

Another approach for the design and implementation of computationally efficient
symmetric bandpass FIR filters is described in [26]. The implementation takes the form of two
parallel, real, quadrature filter branches with each branch derived from a complex modulation
of a lowpass Interpolated FIR filter prototype by complex exponentials. The input data stream
is explicitly modulated with a sine /cosine sequence in order to achieve the desired frequency
shift in the frequency response. Practical design rules for design of optimal FIR digital
bandpass filters using a program is dealt in [26]. A problem of practical interest is to find the
lowest order filter, which satisfies certain maximum ripple requirements in pass and stopbands.
To study the effect of the center frequency on the bandpass filter order, a large number of
filters were designed to meet different ripple requirements. The filter order was observed to be
nearly independent of passband width but is largely dependent on the transition bandwidth.

A new direct technique is proposed for synthesis of a sharp transition, equiripple
passband, and linear phase bandpass FIR filter with low arithmetic complexity from a model of
pseudo-magnitude bandpass response with equiripple passband, equiripple stopband and linear
transition region. Sharp transition bandpass filters reported in literature generally involve design of component lowpass and highpass filters to realize bandpass filters. Also specifications of component filters i.e. lowpass and highpass filters are to be redesigned for any change in center frequency and passband width of the desired bandpass filter. The proposed technique radically departs from this approach. The frequency response of the proposed filter with narrow transition width is modeled using trigonometric functions of frequency. Slopes of the response are matched at the edges of the transition region, which makes the proposed function continuous and hence reduces the effects due to Gibb's phenomenon thereby reducing passband ripple of the filter. Expressions for impulse response coefficients are derived, coefficients obtained and simulation of the bandpass filter is carried out. The bandpass design is adaptable to any change in the center frequency and passband width.

3.2 Digital Bandpass Filter Model I and Design

In this section, the formulation and design of a linear phase, sharp transition bandpass FIR filter model with equiripple passband, stopband and linear transition region is presented [27]. The filter is designed for arbitrary center frequency and passband width. In the proposed bandpass I filter model the various regions of the filter are modeled using trigonometric functions of frequency as follows. The filter model magnitude response \( H_{pm}(\omega) \) is as shown in Fig. 3.1.

In the passband region the frequency response is given by,

\[
H_{pm}(\omega) = A_0 \left( 1 + \frac{\delta_p}{2} \cos k_{pb} (\omega - \omega_b) \right), \quad (\omega_b - \omega_p) \leq \omega \leq (\omega_b + \omega_p) \tag{3.1}
\]
where $\omega$ is the frequency variable, $H_{pm}(\omega)$ is the pseudo-magnitude of the bandpass filter response, $A_0$ is the filter gain parameter, $\delta_p$ is the passband ripple, $k_{pb}$ is a filter design parameter, $\omega_b$ is the center frequency and $\omega_p$ is half passband width of the bandpass filter, $H_{pm}(\omega_b \pm \omega_p) = A_0$, and $(\omega_b \pm \omega_p)$ are the passband edges of the bandpass filter.

In the transition region the frequency response is given by,

$$H_{pm}(\omega) = A_0 \left[ 1 - \frac{\omega - (\omega_b + \omega_p)}{(\omega_z - \omega_p)} \right], \quad (\omega_b + \omega_p) \leq \omega \leq (\omega_b + \omega_z)$$

(3.2)

and $H_{pm}(\omega) = A_0 \left[ 1 - \frac{(\omega_b - \omega_p) - \omega}{(\omega_z - \omega_p)} \right], \quad (\omega_b - \omega_z) \leq \omega \leq (\omega_b - \omega_p)$

(3.3)

In the stopband regions the frequency response is given by,

$$H_{pm}(\omega) = -\frac{\delta_s}{2} A_0 \sin k_{pb} (\omega - (\omega_b + \omega_z)) \quad , \quad (\omega_b + \omega_z) \leq \omega \leq \pi$$

(3.4)

$$H_{pm}(\omega) = -\frac{\delta_s}{2} A_0 \sin k_{pb} (\omega_b - \omega_z - \omega) \quad , \quad 0 \leq \omega \leq (\omega_b - \omega_z)$$

(3.5)

where $\delta_s$ is the stopband attenuation and $(\omega_b \pm \omega_z)$ are the frequencies at which $H_{pm}(\omega)$ is zero which lie close to but beyond the edge of the stopband. Similarly $(\omega_b \pm \omega_z)$ are cut-off edges at which $H_{pm}(\omega) = A_0 \left( 1 - \frac{\delta_s}{2} \right)$.

At cut-off edge $(\omega_b - \omega_z)$, $H_{pm}(\omega) = A_0 \left[ 1 - \frac{\delta_s}{2} \right]$.

(3.6)

Using (3.3), $H_{pm}(\omega_b - \omega_z) = A_0 \left[ 1 - \frac{(\omega_b - \omega_p) - (\omega_b - \omega_z)}{\omega_z - \omega_p} \right]$.

(3.6a)

Equating (3.6) and (3.6a) and simplifying we get,
Fig. 3.1. Illustration of proposed bandpass filter model I with equiripple magnitude response and linear transition region.
\[ \omega_c = \omega_z - \left(1 - \frac{\delta_p}{2}\right)(\omega_z - \omega_p) \]  

(3.7)

Using (3.3), \( H_{pm}(\omega_b - \omega_z) = \frac{\delta_s}{2} = A_0 \left[ 1 - \frac{(\omega_b - \omega_p) - (\omega_b - \omega_z)}{\omega_z - \omega_p} \right] \)  

(3.8)

Simplifying, \( \omega_s = \omega_z - \frac{(\delta_s/2)(\omega_z - \omega_p)}{A_0} \)  

(3.9)

(\( \omega_b \pm \omega_s \)) are the stopband edges at which \( H_{pm}(\omega) = \frac{\delta_s}{2} \)

Using (3.7) and (3.9), we obtain the transition region width

\[ (\omega_s - \omega_c) = (\omega_z - \omega_p) \left[ 1 - \frac{\delta_p}{2} - \frac{(\delta_s/2)}{A_0} \right] \]  

(3.10)

3.2.1 Slope Equalization

The parameters of the model are evaluated by equalizing the slopes of the pseudo-magnitude response function at \((\omega_b \pm \omega_s)\) and \((\omega_b \pm \omega_z)\). This allows the proposed function to be continuous at the extremes of the transition region thus reducing the effects due to Gibb’s phenomenon.

Using (3.1), we obtain,

Slope at \((\omega_b + \omega_p)\) = \( \frac{d}{d\omega} H_{pm}(\omega) \) \((\omega=(\omega_b + \omega_p))\)

\[ = \frac{d}{d\omega} \left[ A_0 \left( 1 + \frac{\delta_p}{2} \cos k_{pb}(\omega - \omega_b) \right) \right] \]

\[ = -\frac{A_0 \delta_p}{2} k_{pb} \sin k_{pb} \omega_p \]  

(3.11)

Using (3.1) and also from the filter model we have,
\[ H_{pn}(\omega_b + \omega_p) = A_0 \left[ 1 + \frac{\delta_p}{2} \cos k_{pb} \omega_p \right] = A_0 \]

Simplifying, \( \cos k_{pb} \omega_p = 0 \) and \( \sin k_{pb} \omega_p = \pm 1 \). Positive sign is used to yield the negative slope, i.e., \( \sin k_{pb} \omega_p = 1 \) \hspace{1cm} (3.12)

Substituting (3.12) in (3.11) we have,

\[ \text{Slope at } (\omega_b + \omega_p) = \frac{-k_{pb} \delta_p A_0}{2} \] \hspace{1cm} (3.13)

Since the transition region \([ (\omega_b + \omega_p), (\omega_b + \omega_z) ] \) is linear, the slope of the frequency response at \( (\omega_b + \omega_z) \) is

\[ \text{At } (\omega_b + \omega_z) = -\frac{A_0}{(\omega_z - \omega_p)} \] \hspace{1cm} (3.14)

Equating the slopes at \( (\omega_b + \omega_p) \) and \( (\omega_b + \omega_z) \) we obtain from (3.13) and (3.14) we obtain,

\[ k_{pb} = \frac{2}{\delta_p} \left[ \frac{1}{\omega_z - \omega_p} \right] \] \hspace{1cm} (3.15)

Substituting (3.10) in (3.15) we obtain

\[ k_{pb} = \frac{2}{\delta_p (\omega_z - \omega_p)} \left[ \left( 1 - \frac{\delta_p}{2} \right) - \frac{(\delta_z / 2)}{A_0} \right] \] \hspace{1cm} (3.16)

From (3.15) we obtain,

\[ \omega_z = \omega_p + \frac{2}{\delta_p k_{pb}} \] \hspace{1cm} (3.17)

### 3.2.2 Expressions for Impulse Response Coefficients

Referring to filter design theory of section 2.4, the impulse response coefficients \( h(n) \) for the bandpass filter are obtained by evaluating the integral below.
h(n) = \frac{1}{\pi} \int_{0}^{\omega_h-\omega_p} H_{pm}(\omega) \cos k\omega \, d\omega
\int_{\omega_h+\omega_p}^{\omega_h+\omega_z} H_{pm}(\omega) \cos k\omega \, d\omega
\int_{\omega_h-\omega_p}^{\omega_h-\omega_z} H_{pm}(\omega) \cos k\omega \, d\omega
(3.18)

h(n) = \frac{1}{\pi} \int_{0}^{\omega_h-\omega_z} \left[ \frac{A_0}{2} \sin k_{pb} ((\omega_b - \omega_z) - \omega) \cos k\omega \, d\omega \right.
\int_{\omega_h+\omega_p}^{\omega_h+\omega_z} \left[ 1 - \frac{(\omega_b - \omega_p) - \omega}{(\omega_z - \omega_p)} \right] \cos k\omega \, d\omega + \int_{\omega_h-\omega_p}^{\omega_h-\omega_z} \left[ 1 + \frac{\delta_p}{2} \cos k_{pb} (\omega - \omega_b) \right] \cos k\omega \, d\omega
\left. + \int_{\omega_h+\omega_p}^{\omega_h+\omega_z} \left[ 1 - \frac{\omega - (\omega_b + \omega_p)}{(\omega_z - \omega_p)} \right] \cos k\omega \, d\omega \right]
(3.19)

where \( n=0,1, \ldots, \frac{N-1}{2} \) for \( N \) odd
\( \frac{N}{2} - 1 \) for \( N \) even and \( k = \frac{(N-1)}{2} - n \)

Evaluating (3.20), the expressions obtained for the impulse response coefficients \( h(n) \) for the bandpass filter are,

\[ h(n) = \frac{A_0 \delta_p \cos k_{pb}}{\pi(k_{pb}^2 - k^2)} \left[ k_{pb} \sin k_{pb} \omega_{pb} \cos k\omega_{pb} - k \cos k_{pb} \omega_{pb} \sin k\omega_{pb} \right] \]

91
\[
+ \frac{A_0 \delta_s k_{pb}}{2\pi(k_{pb}^2 - k^2)} \left[ \cos k_{pb}(\omega_b - \omega_x) - \cos k(\omega_b - \omega_z) \right] \\
+ \frac{A_0}{k^2 \pi(\omega_z - \omega_p)} \left[ \cos k(\omega_b - \omega_p) - \cos k(\omega_b - \omega_z) + k(\omega_z - \omega_p) \sin k(\omega_b - \omega_p) \right] \\
+ \frac{A_0}{k^2 \pi(\omega_z - \omega_p)} \left[ \cos k(\omega_b + \omega_p) - \cos k(\omega_b + \omega_z) - k(\omega_z - \omega_p) \sin k(\omega_b + \omega_p) \right] \\
+ \frac{A_0 \delta_s}{2\pi(k_{pb}^2 - k^2)} \left[ k_{pb} \cos k_{pb}(\pi - (\omega_b + \omega_z)) \cos k\pi - k_{pb} \cos k(\omega_b + \omega_z) \right] \\
+ k \sin k_{pb}(\pi - (\omega_b + \omega_z)) \sin k\pi + \frac{2A_0}{k\pi} \cos k \omega_b \sin k \omega_p
\]

Eq. (3.21) is valid for \( N \) even where \( k \) is a non-integer, for \( N \) odd (3.21) is valid except for \( k = 0 \) and \( k = k_{pb} \).

For \( N \) odd, \( k = 0 \),

\[
h\left(\frac{N-1}{2}\right) = \frac{A_0 \delta_s}{2\pi k_{pb}} \left[ \cos k_{pb}(\pi - (\omega_b + \omega_z)) + \cos k_{pb}(\omega_b - \omega_z) - 2 \right] \\
+ \frac{A_0 (\omega_p + \omega_z)}{\pi} + \frac{A_0 \delta_s \sin k_{pb} \omega_p}{\pi k_{pb}}
\]

(3.22)

For \( N \) odd, \( k = k_{pb} \)

\[
h(n) = h\left(\frac{N-1}{2} - k_{pb}\right) = h\left(\frac{N-1}{2} + k_{pb}\right) = -\frac{A_0 \delta_s}{4\pi} \left[ (\omega_b - \omega_z) \sin k_{pb}(\omega_b - \omega_z) \right] \\
+ \frac{A_0}{k_{pb}^2 \pi(\omega_z - \omega_p)} \left[ \cos k_{pb}(\omega_b - \omega_p) - \cos k_{pb}(\omega_b - \omega_z) + k_{pb}(\omega_z - \omega_p) \sin k_{pb}(\omega_b - \omega_p) \right] \\
+ \frac{A_0 \delta_p}{4\pi} \left[ \frac{\sin 2k_{pb} \omega_p \cos k_{pb} \omega_b}{k_{pb}} + 2\omega_p \cos k_{pb} \omega_b \right] + \frac{2A_0}{k_{pb} \pi} \cos k_{pb} \omega_b \sin k_{pb} \omega_p
\]

92
\[
\frac{A_0}{k_p^2 \pi (\omega_z - \omega_p)} \left[ \cos k_p (\omega_b + \omega_p) - \cos k_p (\omega_b + \omega_z) - k_p (\omega_z - \omega_p) \sin k_p (\omega_b + \omega_p) \right] \\
+ \frac{A_0 \delta_p}{4\pi} \left[ \frac{\cos 2k_p \pi - k_p (\omega_b + \omega_z) - \cos k_p (\omega_b + \omega_z)}{2k_p} \right] + \left[ (\omega_b + \omega_z) \sin k_p (\omega_b + \omega_z) \right]
\]

(3.23)

3.2.3 Filter Synthesis Results

Design Example: A bandpass linear phase sharp transition FIR filter is designed for the desired specifications of center frequencies, passband widths and transition bandwidths as specified in Table 3.1. The maximum passband ripple is ±0.1dB (0.2 dB) and the minimum stopband attenuation is 40dB.

The filter is designed using MATLAB with program MBP-1 and measurement of various filter specifications is done using MATLAB's Signal Processing toolbox. Results approximate the desired filter specifications closely with low filter order with the desired passband widths, center frequencies, passband ripple and stopband attenuation obtained as shown in Table 3.1. The magnitude, impulse and phase response of the proposed bandpass filter with center frequency \( \omega_b = 0.444 \pi \), passband width \( \omega_p = 0.6667 \pi \), transition bandwidth \( (\omega_z - \omega_z) = 0.011 \pi \), passband ripple \( \delta_p = 0.144 \text{dB} \) and stopband attenuation \( \delta_s = 39.4 \text{dB} \) obtained for a filter order of 701 as shown in Fig.3.2. Table 3.1 depicts the performance of the bandpass filter. From Table 3.1, it is observed that the bandpass filter design is adaptable to any change in passband width (wideband to narrowband) and arbitrary center frequencies. Filter synthesis and design steps are given in Appendix A6.1.6.
Table 3.1

Proposed bandpass I filter performance for various passband widths, center frequencies and transition bandwidths with required order for filter realization.

<table>
<thead>
<tr>
<th>Filter Order</th>
<th>Transition bandwidth</th>
<th>Center Frequency</th>
<th>Passband width</th>
<th>Passband ripple in dB</th>
<th>Stopband attenuation in dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>451</td>
<td>0.0215 π</td>
<td>0.6667 π</td>
<td>0.5556 π</td>
<td>0.113</td>
<td>41.40</td>
</tr>
<tr>
<td>225</td>
<td>0.0418 π</td>
<td>0.6667 π</td>
<td>0.5556 π</td>
<td>0.114</td>
<td>41.57</td>
</tr>
<tr>
<td>701</td>
<td>0.0110 π</td>
<td>0.4444 π</td>
<td>0.6666 π</td>
<td>0.144</td>
<td>39.40</td>
</tr>
<tr>
<td>451</td>
<td>0.0209 π</td>
<td>0.3333 π</td>
<td>0.2778 π</td>
<td>0.108</td>
<td>41.59</td>
</tr>
</tbody>
</table>
Fig. 3.2 (a) Magnitude response of the proposed bandpass I filter (b) Magnified view of the passband (c) Impulse response sequence (d) Phase response.
3.3 Digital Bandpass Filter Model II and Design

In this section, the formulation and design of a linear phase, sharp transition bandpass FIR filter model with equiripple passband, equiripple stopband and variable density of ripple cycles in passband and stopband regions of the filter is presented. Filter model is such that large density of ripples are introduced in the regions where discontinuities are present to increase the sharpness of the transition and a non-ideal frequency response modeled without any abrupt discontinuities is used to reduce Gibb's phenomenon. The filter model magnitude response $H_{pm}(\omega)$ is as shown in Fig. 3.3. The filter is designed for arbitrary center frequency and passband width. In the proposed filter model, the various regions of the filter are modeled using trigonometric functions of frequency as follows.

In the stopband region, $0 \leq \omega \leq (\omega_b - \omega_{s0})$ the frequency response is given by

$$H_{pm}(\omega) = \frac{\delta}{2} \cos \left[ k_p 0 \left( (\omega_b - \omega_{s0}) - \omega \right) \right], \quad 0 \leq \omega \leq (\omega_b - \omega_{s0})$$  \hspace{1cm} (3.24)

where $\omega$ the frequency variable, $H_{pm}(\omega)$ is the pseudo-magnitude of the bandpass filter response, $\delta$ is stopband attenuation, $\omega_b$ is the band center frequency, $\omega_{s0}$ is an intermediate frequency of the stopband such that $0 < (\omega_b - \omega_{s0}) < (\omega_b - \omega_z)$ and $k_p 0$ is the filter design parameter in this region. At frequencies $(\omega_b \pm \omega_{s0})$, the filter design parameter $k_p$ changes and $H_{pm}(\omega) = \frac{\delta}{2}$.

In the stopband region, preceding the passband i.e. $(\omega_b - \omega_{s0}) \leq \omega \leq (\omega_b - \omega_z)$ the frequency response is,
Fig. 3.3 Illustration of proposed bandpass filter model II with equiripple magnitude response, linear transition and variable density ripple cycles.
\[ H_{pm}(\omega) = \frac{\delta}{2} \cos \left[ kp \left( \omega - (\omega_b - \omega_z) \right) \right], \quad (\omega_b - \omega_z) \leq \omega \leq (\omega_b - \omega_z) \]  
(3.25)

In the transition region, \((\omega_b - \omega_z) \leq \omega \leq (\omega_b - \omega_p)\) the frequency response is,

\[ H_{pm}(\omega) = \left[ \frac{\omega - (\omega_b - \omega_z)}{(\omega_z - \omega_p)} \right], \quad (\omega_b - \omega_z) \leq \omega \leq (\omega_b - \omega_p) \]  
(3.26)

\(\omega_p\) is half passband width of bandpass filter, \((\omega_b \pm \omega_z)\) are the frequencies at which \(H_{pm}(\omega)\) is zero which lie close to but beyond the edge of the stopband and \((\omega_b \pm \omega_p)\) are the passband edges at which \(H_{pm}(\omega) = 1\).

In the passband region, \((\omega_b - \omega_p) \leq \omega \leq (\omega_b - \omega_{p0})\) the frequency response is,

\[ H_{pm}(\omega) = 1 + \frac{\delta}{2} \sin \left[ kp \left( \omega - (\omega_b - \omega_p) \right) \right], \quad (\omega_b - \omega_p) \leq \omega \leq (\omega_b - \omega_{p0}) \]  
(3.27)

\(\delta_p\) is passband ripple, \(k_p\) is the filter design parameter and \((\omega_b \pm \omega_{p0})\) are intermediate frequencies in the passband at which filter design parameter \(k_p\) changes.

In the passband region, \((\omega_b - \omega_{p0}) \leq \omega \leq \omega_b\) the frequency response is

\[ H_{pm}(\omega) = 1 + \frac{\delta}{2} \cos \left[ k_{p0} \left( \omega - (\omega_b - \omega_{p0}) \right) \right], \quad (\omega_b - \omega_{p0}) \leq \omega \leq \omega_b \]  
(3.28)

In the passband region, \(\omega_b \leq \omega \leq (\omega_b + \omega_{p0})\) the frequency response is

\[ H_{pm}(\omega) = 1 + \frac{\delta}{2} \cos \left[ k_{p0} (\omega - \omega_b) \right], \quad \omega_b \leq \omega \leq (\omega_b + \omega_{p0}) \]  
(3.29)

In the passband region, \((\omega_b + \omega_{p0}) \leq \omega \leq (\omega_b + \omega_p)\) the frequency response is

\[ H_{pm}(\omega) = 1 + \frac{\delta}{2} \cos \left[ k_p (\omega - (\omega_b + \omega_{p0}) \right], \quad (\omega_b + \omega_{p0}) \leq \omega \leq (\omega_b + \omega_p) \]  
(3.30)
In the transition region, \((\omega_b + \omega_p) \leq \omega \leq (\omega_b + \omega_z)\) from the frequency response is

\[
H_{pm}(\omega) = 1 - \frac{\omega - (\omega_b + \omega_p)}{(\omega_z - \omega_p)}, \quad (\omega_b + \omega_p) \leq \omega \leq (\omega_b + \omega_z) \quad (3.31)
\]

In the stopband region, \((\omega_b + \omega_z) \leq \omega \leq (\omega_b + \omega_\alpha)\) the frequency response is

\[
H_{pm}(\omega) = -\frac{\delta}{2} \sin \left[ k_p \left( \omega - (\omega_b + \omega_z) \right) \right], \quad (\omega_b + \omega_z) \leq \omega \leq (\omega_b + \omega_\alpha) \quad (3.32)
\]

In the stopband region, \((\omega_b + \omega_\alpha) \leq \omega \leq \pi\) the frequency response is

\[
H_{pm}(\omega) = -\frac{\delta}{2} \cos \left[ k_{p0} \left( \omega - (\omega_b + \omega_\alpha) \right) \right], \quad (\omega_b + \omega_\alpha) \leq \omega \leq \pi \quad (3.33)
\]

\((\omega_b + \omega_\alpha)\) is an intermediate frequency of the stopband succeeding the passband such that \((\omega_b + \omega_z) \leq \omega_\alpha \leq \pi\).

The bandpass filter design, has three regions in the passband and possesses a total of \((n+1/2)\) cycles of ripple in the entire passband. In the proposed model, the first region possesses \((m+1/4)\) cycles of ripple where \(m\) is an integer and ranges from \((\omega_b - \omega_p)\) to \((\omega_b - \omega_{p0})\). It is characterized by filter design parameter \(k_p\). The second region possesses \((n-2m)\) cycles of ripple where \(n\) is an even integer and this region spans the frequency range from \((\omega_b - \omega_{p0})\) to \((\omega_b + \omega_{p0})\). It is characterized by filter design parameter \(k_{p0}\). The third region possesses \((m+1/4)\) cycles of ripple where \(m\) is an integer and ranges from \((\omega_b + \omega_{p0})\) to \((\omega_b + \omega_p)\). It is characterized by filter design parameter \(k_p\).

For the bandpass design we have,
\[ k_p(\omega_{s0} - \omega_z) = 2\pi(p + 3/4) \quad \text{where } p \text{ is a positive integer} \]  

(3.34)

From (3.34), \( \omega_{s0} = \omega_z + \frac{2\pi(p + 3/4)}{k_p} \)

Also, \( k_p(\omega_p - \omega_{p0}) = 2\pi(m + 1/4) \)  

(3.35)

From (3.35), \( \omega_{p0} = \omega_p - \frac{2\pi}{k_p}(m + 1/4) \)  

(3.36)

Also, \( k_{p0}(\omega_{p0} = 2\pi(n/2 - m) \)

(3.37)

From (3.37), \( k_{p0} = \frac{2\pi(n/2 - m)}{\omega_{p0}} \)  

(3.38)

From (3.34), (3.35) and (3.37)

\[ \cos k_p(\omega_{s0} - \omega_z) = 0 \quad \text{i.e. } \sin k_p(\omega_{s0} - \omega_z) = -1 \]

\[ \cos k_p(\omega_p - \omega_{p0}) = 0 \quad \text{i.e. } \sin k_p(\omega_p - \omega_{p0}) = 1 \]

\[ \cos k_{p0}\omega_{p0} = 1 \quad \text{i.e. } \sin k_{p0}\omega_{p0} = 0 \]  

(3.38a)

where \( p, n, m \) are integers and assumed where \( n \) must be even.

In the bandpass filter model we have,

\[ H_{pm}(\omega_b) = \left(1 + \frac{\delta_p}{2}\right), \quad H_{pm}(\omega_b \pm \omega_p) = 1, \]

\[ H_{pm}(\omega_b \pm \omega_{p0}) = \left(1 + \frac{\delta_{p0}}{2}\right), \quad H_{pm}(\omega_b \pm \omega_{s0}) = 0, \quad H_{pm}(\omega_b \pm \omega_{s}) = \frac{\delta_c}{2}, \quad H_{pm}(\omega_b \pm \omega_{c}) = 1 - \frac{\delta_c}{2} \quad \text{and} \]

\[ H_{pm}(\omega_b \pm \omega_{s}) = \frac{\delta_c}{2} \]  

(3.38b)
3.3.1 Slope Equalization

The parameters of the model are evaluated by equalizing the slopes of the pseudo-magnitude response function at \((\omega_b + \omega_p)\) and \((\omega_b + \omega_z)\). This allows the proposed function to be continuous at the extremes of the transition region thus reducing the effects due to Gibb's phenomenon.

\[
\text{Slope at } (\omega_b + \omega_p) = \frac{d}{d\omega} \left[ H_{pm}(\omega) \right]_{\omega=(\omega_b + \omega_p)} = \frac{d}{d\omega} \left[ \frac{\delta}{2} \cos k_p \left( \omega - (\omega_b + \omega_0) \right) \right]_{\omega=(\omega_b + \omega_p)}
\]

\[
= -\frac{k_p \delta p}{2} \sin k_p (\omega_p - \omega_0)
\]

Using \((3.38a)\) we get, \(\text{slope at } (\omega_b + \omega_p) = \left( -\frac{k_p \delta p}{2} \right) \) \hspace{1cm} (3.39)

Since the transition region \([ (\omega_b + \omega_p), (\omega_b + \omega_z) \] \) is linear,

\[
\text{slope at } (\omega_b + \omega_z) = -\frac{1}{(\omega_z - \omega_p)} \hspace{1cm} (3.40)
\]

Equating the above slopes at \((\omega_b + \omega_p)\) and \((\omega_b + \omega_z)\), using (3.39) and (3.40),

\[
\left( -\frac{k_p \delta p}{2} \right) = -\frac{1}{(\omega_z - \omega_p)}
\]

\hspace{1cm} (3.41)

Simplifying (3.41), \(\omega_z = \omega_p + \frac{2}{k_p \delta p} \hspace{1cm} (3.42)\)

\[
\text{Slope at } (\omega_b - \omega_p) = \frac{d}{d\omega} \left[ H_{pm}(\omega) \right]_{\omega=(\omega_b - \omega_p)} = \frac{d}{d\omega} \left[ \frac{\delta}{2} \sin k_p \left( \omega - (\omega_b - \omega_0) \right) \right]_{\omega=(\omega_b - \omega_p)}
\]

\[
= \frac{k_p \delta p}{2} \cos k_p \left( \omega - (\omega_b - \omega_p) \right) \hspace{1cm} \omega = (\omega_b - \omega_p)
\]
\[ \left( \frac{k_p \delta_p}{2} \right) \]

In the linear region, \((\omega_b - \omega_z) \leq \omega \leq (\omega_b + \omega_z)\),

Slope at \((\omega_b - \omega_z)\) \(= \frac{1}{(\omega_z - \omega_p)}\) \hspace{1cm} (3.44)

Equating the slopes at \((\omega_b - \omega_p)\) and \((\omega_b - \omega_z)\) we obtain,

\[ \left( \frac{k_p \delta_p}{2} \right) = \frac{1}{(\omega_z - \omega_p)} \] \hspace{1cm} (3.45)

Using (3.32) slope at \((\omega_b + \omega_z)\),

\[ = \frac{d}{d\omega} \left[ H_{pm}(\omega) \right]_{\omega=\omega_b+\omega_z} = \frac{d}{d\omega} \left[ \frac{\delta_s}{2} \sin k_p \left( \omega - (\omega_b + \omega_z) \right) \right]_{\omega=\omega_b+\omega_z} \]

\[ = -\frac{k_p \delta_s}{2} \cos \left( \omega - (\omega_b + \omega_z) \right) \]

\[ = \left( \frac{k_p \delta_s}{2} \right) \] \hspace{1cm} (3.46)

Using (3.25) slope at \((\omega_b - \omega_z)\)

\[ = \frac{d}{d\omega} \left[ H_{pm}(\omega) \right]_{\omega=\omega_b-\omega_z} = \frac{d}{d\omega} \left[ \frac{\delta_s}{2} \cos k_p \left( \omega - (\omega_b - \omega_z) \right) \right]_{\omega=\omega_b-\omega_z} \]

\[ = -\frac{k_p \delta_s}{2} \sin k_p \left( \omega_0 - \omega_z \right) \]

\[ = \left( \frac{k_p \delta_s}{2} \right) \] \hspace{1cm} (3.47)

The magnitudes of the slopes at \((\omega_b \pm \omega_z)\) are equal as seen in (3.46) and (3.47).
Also, \( H_{pm}(\omega_b + \omega_c) = 1 - \frac{\delta_p}{2} \)  \( (3.48) \)

From the linear region relation, using (3.31)

\[
H_{pm}(\omega_b + \omega_c) = 1 - \frac{(\omega_b + \omega_c) - (\omega_b + \omega_p)}{(\omega_z - \omega_p)} = 1 - \frac{\omega_c - \omega_p}{(\omega_z - \omega_p)}
\]

\( (3.49) \)

Equating (3.48) and (3.49) and using (3.41) we get,

\[
\omega_c = \omega_p + \frac{1}{k_p}
\]

\( (3.50) \)

Also, as stated earlier, \( H_{pm}(\omega_b + \omega_s) = \frac{\delta_p}{2} \)

\( (3.50a) \)

Using the linear region (falling edge), we obtain from (3.31) and (3.50a),

\[
H_{pm}(\omega_b + \omega_s) = 1 - \frac{(\omega_b + \omega_s) - (\omega_b + \omega_p)}{(\omega_z - \omega_p)} = \frac{\delta_p}{2}
\]

\( (3.51) \)

Simplifying (3.51) and using (3.42) we get,

\[
\omega_z = \omega_s + \frac{1}{k_p} = \omega_p + \frac{2}{k_p \delta_p}
\]

\( (3.52) \)

From (3.52), \( \omega_s = \omega_p - \frac{1}{k_p} + \frac{2}{k_p \delta_p} \)

\( (3.53) \)

From (3.50) and (3.53) we get,

\[
(\omega_s - \omega_c) = \frac{2}{k_p} \left( \frac{1}{\delta_p} - 1 \right)
\]

\( (3.54) \)

In the bandpass filter model, \( (\omega_s - \omega_c) \) is the transition region width.

From (3.54), \[ k_p = \frac{2}{\delta_p} - \frac{1}{(\omega_s - \omega_c)} \]

\( (3.55) \)
3.3.2 Expressions for Impulse Response Coefficients

Referring to filter design theory of section 2.4, the impulse response coefficients \( h(n) \) for the bandpass filter are obtained by evaluating the integral below.

\[
h(n) = \frac{1}{\pi} \left[ \int_{0}^{\omega_b - \omega_0} H_{pm}(\omega) \cos k\omega d\omega + \int_{\omega_b - \omega_0}^{\omega_h - \omega_p} H_{pm}(\omega) \cos k\omega d\omega + \int_{\omega_h - \omega_p}^{\omega_b + \omega_0} H_{pm}(\omega) \cos k\omega d\omega + \int_{\omega_b + \omega_0}^{\omega_b + \omega_0} H_{pm}(\omega) \cos k\omega d\omega \right]
\]  

\[
+ \int_{\omega_b + \omega_0}^{\omega_h + \omega_0} H_{pm}(\omega) \cos k\omega d\omega + \int_{\omega_h + \omega_0}^{\omega_h + \omega_0} H_{pm}(\omega) \cos k\omega d\omega
\]  

\[
+ \int_{\omega_h + \omega_0}^{\omega_b + \omega_0} H_{pm}(\omega) \cos k\omega d\omega + \int_{\omega_h + \omega_0}^{\omega_h + \omega_0} H_{pm}(\omega) \cos k\omega d\omega
\]  

\[
+ \int_{\omega_h + \omega_0}^{\omega_b + \omega_0} H_{pm}(\omega) \cos k\omega d\omega + \int_{\omega_h + \omega_0}^{\omega_h + \omega_0} H_{pm}(\omega) \cos k\omega d\omega
\]  

\[
(3.56)
\]  

\[
h(n) = \frac{1}{\pi} \left[ \int_{0}^{\omega_b - \omega_0} \frac{8}{2} \cos k_0 (\omega_b - \omega_0) \omega \cos k\omega d\omega + \int_{\omega_b - \omega_0}^{\omega_h - \omega_p} \frac{8}{2} \cos k_0 (\omega_b - \omega_0) \omega \cos k\omega d\omega + \int_{\omega_h - \omega_p}^{\omega_b + \omega_0} \left( \frac{8}{2} \cos k_0 (\omega_b - \omega_0) \omega \right) \cos k\omega d\omega + \int_{\omega_b + \omega_0}^{\omega_b + \omega_0} \left( \frac{8}{2} \cos k_0 (\omega_b - \omega_0) \omega \right) \cos k\omega d\omega \right]
\]  

\[
+ \int_{\omega_b - \omega_0}^{\omega_b - \omega_p} \left[ \frac{\omega - (\omega_b - \omega_0)}{(\omega - \omega_p)} \right] \cos k\omega d\omega + \int_{\omega_b - \omega_0}^{\omega_b - \omega_p} \left[ \frac{\delta}{2} \sin k_0 \left( \omega - (\omega_b - \omega_0) \right) \right] \cos k\omega d\omega
\]  

\[
+ \int_{\omega_b - \omega_p}^{\omega_h + \omega_0} \left[ \frac{\omega - (\omega_b - \omega_0)}{(\omega_0 - \omega_0)} \right] \cos k\omega d\omega + \int_{\omega_b - \omega_p}^{\omega_b + \omega_0} \left[ \frac{\delta}{2} \cos k_0 \left( \omega - (\omega_b - \omega_0) \right) \right] \cos k\omega d\omega
\]  

\[
+ \int_{\omega_h + \omega_0}^{\omega_h + \omega_0} \left[ \frac{\omega - (\omega_b + \omega_0)}{(\omega - \omega_p)} \right] \cos k\omega d\omega + \int_{\omega_h + \omega_0}^{\omega_h + \omega_0} \left[ \frac{\delta}{2} \cos k_0 \left( \omega - (\omega_b + \omega_0) \right) \right] \cos k\omega d\omega
\]  

\[
+ \int_{\omega_h + \omega_0}^{\omega_h + \omega_0} \left[ \frac{\omega - (\omega_b + \omega_0)}{(\omega - \omega_p)} \right] \cos k\omega d\omega
\]  

\[
(3.57)
\]
\[ \int_{\omega_b + \omega_s}^{\omega_b + \omega_0} - \frac{s}{2} \sin \left( \frac{\omega - (\omega_b + \omega_b)}{\omega_b + \omega_s} \right) \cos k\omega \, d\omega + \int_{\omega_b + \omega_s}^{\omega_b + \omega_0} \frac{s}{2} \cos \left( \frac{\omega - (\omega_b + \omega_s)}{\omega_b + \omega_s} \right) \cos k\omega \, d\omega \]

(3.58)

where \[ n = 0, 1, \ldots, \frac{N-1}{2} \] for N odd

\[ n = 0, 1, 2, \ldots, \frac{N}{2} - 1 \] for N even \[ k = \frac{(N-1)}{2} - n \]

Evaluating (3.58), the expressions obtained for the impulse response coefficients \( h(n) \) for the lowpass filter

\[ h(n) = \frac{k \cos(k\omega_b)}{\pi} \left[ \frac{1}{\left(k_p^2 - k^2\right)} - \frac{1}{(k_p^2 - k^2)} \right] \left[ \delta_p \sin(k\omega_p) - \delta_s \sin(k\omega_s) \right] \]

\[ + \frac{\cos(k\omega_b)[\cos(k\omega_p) - \cos(k\omega_s)]}{\pi} \left[ \frac{2}{k^2(\omega_s - \omega_b)} \right] + \frac{k_p \cos(k\omega_b)}{\pi(k_p^2 - k^2)} \left[ \delta_p \cos(k\omega_p) - \delta_s \cos(k\omega_s) \right] \]

\[ + \frac{\delta_s k_p^2}{2\pi(k_p^2 - k^2)} \left[ \sin k_p^0 (\omega_b - \omega_{s0}) + \cos(k\pi)\sin k_p^0 (\pi - (\omega_b + \omega_{s0})) \right] \]

(3.59)

(3.59) is valid for N even where k is a non-integer. For N odd (3.59) is valid except for k=0 and \( k = k_p^0 \).

For N odd, k=0 we obtain,

\[ h\left( \frac{N-1}{2} \right) = \frac{\omega_z + \omega_p}{\pi} + \frac{\delta_s}{\pi k_p^0} \sin k_p^0 \left( \frac{\pi - \omega_{s0}}{2} - \omega_b \right) \cos k_p^0 \left( \frac{\pi - \omega_b}{2} \right) + \left( \frac{\delta_p - \delta_s}{\pi^2} \right) \]

(3.60)

For N odd, k = k_p^0 we obtain,
\[
\begin{align*}
& h\left(\frac{N-1}{2} - k_p\right) = h\left(\frac{N-1}{2} + k_p\right) = -\frac{\delta_s \cos(k_\omega_b)}{\pi (k_p^2 - k^2)} \left[k \sin(k_\omega_{z0}) + k_p \cos(k_\omega_z)\right] \\
& - \frac{\delta_s}{2k_p\pi} \left[\cos(k_p\omega_b)\sin(k_p\omega_{z0})\right] + \frac{\delta_p \cos(k_\omega_p)}{\pi (k_p^2 - k^2)} \left[k_p \cos(k_\omega_p) + k \sin(k_\omega_{p0})\right] \\
& + \frac{\delta_s}{4\pi} \left[\cos(k_p\omega_b)\cos(k_p\omega_{z0}) + (\pi - (\omega_b + \omega_{z0}))\sin(k_p\omega_b + \omega_{z0})\right] \\
& - \frac{2}{k_\pi} \left[\cos(k_\omega_b)\sin(k_\omega_{z0})\right] + \frac{2\cos(k_\omega_p)}{\pi k^2 (\omega_z - \omega_p)}
\end{align*}
\] (3.61)

3.3.3 Filter Synthesis Results

Design Example: A bandpass linear phase sharp transition FIR filter is designed for the desired specifications of center frequencies, passband widths and transition bandwidths as specified in Table 3.2. The maximum passband ripple is ±0.1dB (0.2 dB) and the minimum stopband attenuation is 40dB.

The filter is designed using MATLAB with program MBP-2 and measurement of various filter specifications is done using MATLAB's Signal Processing toolbox. Results approximate the desired filter specifications closely with low filter order with the desired passband widths, center frequencies, passband ripple and stopband attenuation obtained as shown in Table 3.2. The magnitude, impulse and phase response of the proposed bandpass filter with center frequency \( \omega_b = 0.6667\pi \), passband width \( \omega_p = 0.5556\pi \), transition bandwidth \((\omega_z - \omega_z) = 0.011\pi\), passband ripple \( \delta_p = 0.14\text{dB} \) and stopband attenuation \( \delta_s = 40.2\text{dB} \) obtained for a filter order of 701 as shown in Fig.3.4. Table 3.2 depicts the performance of the bandpass filter. From Table 3.2, it is observed that the bandpass filter
design is adaptable to any change in passband width (wideband to narrowband) and arbitrary center frequencies.

It is observed that there is a marginal improvement in filter performance in terms of decrease in passband ripple and increase in stopband attenuation because of further reduction in Gibb's phenomenon by employing variable ripple cycles density technique compared to previous bandpass model. The bandpass filter design is adaptable to any change in passband width and arbitrary center frequencies. Filter synthesis and design steps are given in Appendix A6.1.7.
Table 3.2
Proposed bandpass II filter performance for various passband widths, center frequencies and transition bandwidths with required order for filter realization.

<table>
<thead>
<tr>
<th>Filter Order</th>
<th>Transition bandwidth</th>
<th>Center Frequency</th>
<th>Passband width</th>
<th>Passband ripple in dB</th>
<th>Stopband attenuation in dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>451</td>
<td>0.021 $\pi$</td>
<td>0.6667 $\pi$</td>
<td>0.5556 $\pi$</td>
<td>0.11</td>
<td>42.15</td>
</tr>
<tr>
<td>225</td>
<td>0.010 $\pi$</td>
<td>0.6667 $\pi$</td>
<td>0.5556 $\pi$</td>
<td>0.11</td>
<td>41.69</td>
</tr>
<tr>
<td>701</td>
<td>0.011 $\pi$</td>
<td>0.6667 $\pi$</td>
<td>0.5556 $\pi$</td>
<td>0.14</td>
<td>40.20</td>
</tr>
<tr>
<td>451</td>
<td>0.021 $\pi$</td>
<td>0.3333 $\pi$</td>
<td>0.2778 $\pi$</td>
<td>0.10</td>
<td>40.10</td>
</tr>
</tbody>
</table>
(a)

(b)
Fig. 3.4 (a) Magnitude response of the proposed bandpass II filter (b) Magnified view of the passband (c) Impulse response sequence (d) Phase response.
3.4 Lowpass Cascade Filter Model and Design

In this section, a cascade of two lowpass linear phase FIR filters is modeled with out of phase passband ripples, parallel skirts in the transition regions and equiripple stopband ripples [28]. The magnitude responses $H_{pm1}(\omega)$ and $H_{pm2}(\omega)$ of the filters are as shown in Fig. 3.5. The pseudo-magnitude responses of the two filters sections of the proposed filters to be cascaded are formulated as follows.

In the passband region, the frequency responses of component filters are given by

$$H_{pm1}(\omega) = 1 + \frac{\delta_p}{2} \sin k_p \omega \quad 0 \leq \omega \leq \omega_{p1}$$  \hspace{1cm} (3.62)

and

$$H_{pm2}(\omega) = 1 - \frac{\delta_p}{2} \sin k_p \omega \quad 0 \leq \omega \leq \omega_{p2}$$  \hspace{1cm} (3.63)

where $\omega$ the frequency variable, $H_{pm1}(\omega)$, $H_{pm2}(\omega)$ are the pseudo-magnitudes of the individual filter responses of the component filters constituting the cascade, $\delta_p$ is passband ripple, $k_p$ is a filter design parameter in the passband , $\omega_{p1}$, $\omega_{p2}$ are the passband edges.

We have, $H_{pm1}(\omega_{p1}) = H_{pm2}(\omega_{p2}) = 1.0$

In this design for a specified $\omega_{p1}$,

$$\omega_{p2} = \omega_{p1} + \frac{\pi}{k_p}$$  \hspace{1cm} (3.64)

Since the component filter responses are shifted by $\frac{\pi}{k_p}$ along the frequency axis, this leads to parallel skirts in the transition region of the two responses.

In the transition region, frequency responses of component filters are given by

$$H_{pm1}(\omega) = A \cos k_t (\omega - \omega_0) \quad \omega_{p1} \leq \omega \leq \omega_{z1}$$  \hspace{1cm} (3.65)
and \( H_{pm2}(\omega) = A \cos k_t \left( \omega - \left( \omega_0 + \frac{\pi}{k_p} \right) \right) \omega_{p2} \leq \omega \leq \omega_{z2} \) \hspace{1cm} (3.66)

where \( k_t \) is a filter design parameter in the transition region, \( A \) is amplitude parameter and is chosen greater than 1, \( \omega_0 \) is the frequency at which \( H_{pm1}(\omega) \) equals \( A \) and \( \left( \omega_0 + \frac{\pi}{k_p} \right) \) is the frequency at which \( H_{pm2}(\omega) \) equals \( A \). Note that \( \omega_0 \) is a fictitious frequency used to shape the transition regions as was done in proposed Class III filter. Also,

\( H_{pm1}(\omega_{c1}) = H_{pm2}(\omega_{c2}) = 1 - \frac{\delta_s}{2} \) \hspace{1cm} and \hspace{1cm} \( H_{pm1}(\omega_{z1}) = H_{pm2}(\omega_{z2}) = 0 \).

Finally, the stopband region of the component frequency responses are given by

\[
H_{pm1}(\omega) = -\frac{\delta_s}{2} \sin k_s (\omega - \omega_{z1}) \hspace{1cm} \omega_{z1} \leq \omega \leq \pi \hspace{1cm} (3.67)
\]

\[
H_{pm2}(\omega) = -\frac{\delta_s}{2} \sin k_s (\omega - \omega_{z2}) \hspace{1cm} \omega_{z2} \leq \omega \leq \pi \hspace{1cm} , \text{respectively} \hspace{1cm} (3.68)
\]

where \( \delta_s \) is the stopband attenuation, \( k_s \) is the filter design parameter in the stopband region and 

\( H_{pm1}(\omega_{c1}) = H_{pm2}(\omega_{c2}) = \frac{\delta_s}{2} \).

In the stopband region, we have

\[
\omega_{z2} = \left( \omega_{z1} + \frac{\pi}{k_p} \right) \hspace{1cm} (3.69)
\]

Also, \( H_{pm1}(\omega_{p1}) = 1.0 = 1 + \frac{\delta_p}{2} \sin k_p \omega_{p1} \hspace{1cm} (3.70) \)
Fig. 3.5: Magnitude responses of component filters $H_{pm1}(\omega)$ and $H_{pm2}(\omega)$ of proposed cascade lowpass filter model.
Simplifying (3.70) \( \sin k_p \omega_{p1} = 0 \) i.e. \( \cos k_p \omega_{p1} = \pm 1 \). Since the slope shall be negative, we have \( \cos k_p \omega_{p1} = -1 \) \hspace{1cm} (3.71)

From (3.65), \( H_{pm1}(\omega_{p1}) = 1.0 = A \cos k_t (\omega_{p1} - \omega_0) \) \hspace{1cm} (3.72)

yielding, \( \cos k_t (\omega_{p1} - \omega_0) = \frac{1}{A} \) and \( \sin k_t (\omega_{p1} - \omega_0) = \sqrt{1 - \frac{1}{A^2}} \) \hspace{1cm} (3.73)

Positive sign is chosen for \( \sin k_t (\omega_{p1} - \omega_0) \) from the consideration of the slope of the response at \( \omega = \omega_{p1} \).

From (3.65), \( H_{pm1}(\omega_{s1}) = 0 = A \cos k_t (\omega_{s1} - \omega_0) \) \hspace{1cm} (3.74)

yielding, \( \cos k_t (\omega_{s1} - \omega_0) = 0 \) or \( \sin k_t (\omega_{s1} - \omega_0) = \pm 1 \) \hspace{1cm} (3.75)

where \( \omega_{s2} \) and \( \omega_{s1} \) are respective stopband edges and \( \omega_{c2}, \omega_{c1} \) are the respective cutoff edges. Positive sign is chosen for \( \sin k_t (\omega_{s1} - \omega_0) \) from the consideration of the slope of the response at \( \omega = \omega_{s1} \).

From (3.65), \( H_{pm1}(\omega_{c1}) = 1 - \frac{\delta_p}{2} = A \cos k_t (\omega_{c1} - \omega_0) \)

Simplifying, \( \omega_{c1} = \frac{1}{k_t} \cos^{-1} \left( \frac{1 - \frac{\delta_p}{2}}{A} \right) + \omega_0 \) \hspace{1cm} (3.76)

From (3.65), \( H_{pm1}(\omega_{s1}) = \frac{\delta_s}{2} = A \cos k_t (\omega_{s1} - \omega_0) \)

Simplifying, we obtain, \( \omega_{s1} = \frac{1}{k_t} \cos^{-1} \left( \frac{\delta_s}{2} \right) + \omega_0 \) \hspace{1cm} (3.77)
Using (3.76) and (3.77) we have,

\[
k_t = \left( \frac{1}{(\omega_{s1} - \omega_{c1})} \right) \left[ \cos^{-1} \left( \frac{\delta_s}{2} \right) - \cos^{-1} \left( \frac{1 - \delta_p}{2} \right) \right]
\]

(3.78)

**Slope Equalization**

The various filter design parameters of the cascade model are evaluated by equalizing the slopes of the pseudo-magnitude response function at \(\omega_{p1}, \omega_{p2}\) and \(\omega_{z1}, \omega_{z2}\). This allows the proposed function to be continuous thus reducing the effects due to Gibb’s phenomenon.

The slope of the response given by (3.62) at \(\omega = \omega_{p1}\) is

\[
\text{Slope at } \omega_{p1} = \frac{d}{d\omega} \left[ H_{pm1}(\omega) \right]_{\omega=\omega_{p1}} = \frac{k_p \delta_p}{2} \cos(k_p \omega_{p1})
\]

(3.79)

Substituting (3.71) in (3.79),

\[
\text{Slope at } \omega_{p1} = -\frac{k_p \delta_p}{2}
\]

(3.80)

Using (3.65) and (3.73) we obtain the slope at \(\omega_{p1} = -A k_s \sin k_t (\omega_{p1} - \omega_0)\)

\[
= -k_t (\sqrt{A^2 - 1})
\]

(3.81)

**Equalization of slopes at \(\omega_{p1}\), using (3.80) and (3.81) we obtain,**

\[
k_t = \frac{k_p \delta_p}{2(\sqrt{A^2 - 1})}
\]

(3.82)

**Equalization of slope at \(\omega_{p2}\)** also gives identical expression for \(k_t\) as above.

Using (3.67) we obtain,

\[
\text{Slope at } \omega_{z1} = \frac{d}{d\omega} \left[ H_{pm1}(\omega) \right]_{\omega = \omega_{z1}} = -\frac{k_s \delta_s}{2} \cos k_s (\omega_{z1} - \omega_{z1})
\]
\[
= \frac{k_s \delta_s}{2}
\]  

(3.83)

Using (3.65) and (3.75), we obtain,

Slope at \( \omega_{z1} = -Ak_t \sin k_t (\omega_{z1} - \omega_0) = -Ak_t \)  

(3.84)

From (3.83) and (3.84), equalization of slopes at \( \omega_{z1} \) yields

\[
k_s = \frac{2A_k_t}{\delta_s}
\]  

(3.85)

Equalization of slope at \( \omega_{z2} \) also gives an identical expression for \( k_s \).

The expressions for impulse response coefficients \( h(n) \) for the cascade filter are obtained by evaluating the integrals below.

\[
h(n) = \frac{1}{\pi} \left[ \int_0^{\omega_{z1}} H_{pm1}(\omega) H_{pm2}(\omega) \cos k\omega \, d\omega + \int_{\omega_{z1}}^{\omega_{z2}} H_{pm1}(\omega) H_{pm2}(\omega) \cos k\omega \, d\omega 

+ \int_{\omega_{z2}}^{\omega_{z1}} H_{pm1}(\omega) H_{pm2}(\omega) \cos k\omega \, d\omega 

+ \int_{\omega_{z1}}^{\omega_{z2}} H_{pm1}(\omega) H_{pm2}(\omega) \cos k\omega \, d\omega \right]
\]  

(3.86)

where \( n = 0, 1, 2, \ldots, \frac{N-1}{2} \) for \( N \) odd

\[
n = 0, 1, 2, \ldots, \frac{N}{2} - 1 \quad \text{for} \quad N \quad \text{even} \quad \text{and} \quad k = \left( \frac{N-1}{2} - n \right)
\]

3.4.1 Filter Synthesis Results

Design Example: Consider the design of a proposed cascade lowpass FIR filter, where a pair of Class III lowpass linear phase FIR filters are cascaded, one component lowpass filter with
passband edge specified at \(0.558\pi\) and stopband edge at \(0.572\pi\) and another component filter shifted with passband and stopband edge to \(0.558\pi + \frac{\pi}{k_p}\) and \(0.572\pi + \frac{\pi}{k_p}\) respectively, each with passband ripple of \(\pm 0.1\text{dB}\) and stopband attenuation of \(35\text{dB}\).

The filter is designed using MATLAB with program MCS. The integrals in (3.86) is evaluated to obtain the coefficients of the resultant filter. The filter obtained after cascading the two component filters has specifications of cutoff edge \(\omega_c = 0.5587\pi\), stopband edge \(\omega_s = 0.5724\pi\), passband ripple of \(0.14\text{dB}\), stopband attenuation of \(38\text{dB}\) and transition bandwidth of \(0.01\pi\) was obtained for filter order of 701. Stopband attenuation of the resultant filter improves however there is imperceptible change in transition region width. Passband ripple of the resultant filter increases slightly from those of the component filters. Hence it is found that there is no much improvement in filter specifications by cascading proposed lowpass sharp transition filters.

3.5 Conclusions

Linear phase, sharp transition, equiripple response, low arithmetic complexity, bandpass FIR filter designs are proposed. Various regions of the filter are approximated with trigonometric functions of frequency, making it convenient to evaluate the impulse response coefficients in closed form and thus its transfer function is evolved in frequency and time domain. The novel slope equalizing technique is applied to bandpass filter design which makes the proposed function continuous across transition band and hence reduces the effects due to Gibb's phenomenon thereby reducing ripples at the edges of the transition region of the filter. The synthesized filter proves to be a good alternative to filters of the same class reported.
in the literature with added advantage of ease of computation of the impulse response and
simplicity of design since the filter is without any subfilters i.e. direct design compared to
sharp transition bandpass filters realized using highpass and lowpass filters in literature. Also
the proposed filters are designed for arbitrary center frequency and passband width with no
separate design required. In another proposed bandpass approach, the filter model is such that
large density of ripple cycles are introduced in the regions where discontinuities are present in
the filter magnitude response, to increase the sharpness of the transition and remove any
abrupt discontinuities, which further reduces Gibb’s phenomenon. In the proposed
approximation technique impulse response coefficients are obtained for facilitating the direct
synthesis of the filter. The approach can be extended to design of sharp cutoff bandstop and
multiband filters with arbitrary passband and center frequencies.