CHAPTER 5

CHANNEL ESTIMATION FOR CHAOS BASED
UWB SYSTEM

5.1 PREAMBLE

In the recent years, UWB communication technology has generated interest due to the promise of multi-path diversity and energy conservation. UWB propagation depends on the geometry of the indoor environment, the architecture of the buildings and the existence of observed clusters (Li et al 2005). Due to the high resolution capability of the UWB signal, the channel estimation process is particularly cumbersome, when no a-priori information is available on the nature of the channel. The channel estimation efforts are concentrated around the nominal parameters to reduce the dimensionality of the estimation of the parameters associated with the multi-path components (Lottici et al 2002). This chapter addresses the channel estimation technique for the chaos based UWB system.

5.2 CHANNEL ESTIMATION TECHNIQUES FOR UWB SYSTEMS

Channel estimation based on maximum-likelihood criterion has recently been proposed for ultra-wideband communications (Huang and Ko 2004). Lei et al (2004) has introduced a channel estimation method for multi-band ultra-wideband communication system in which RAKE receiver is applied in analog domain in order to make use of multi-path channel property.
This channel estimation method based on maximum-likelihood criterion gets the path delay and attenuation (Yang and Giannakis 2004).

A two-stage clustered channel estimation scheme for UWB was proposed by Carbonelli and Mitra (2004). In the first stage, coarse channel estimation was conducted to estimate the location and dispersion of a cluster. Then, channel estimation efforts are concentrated around the channel parameters to reduce the dimensionality of the estimation of the parameters associated with the multipath components. Different strategies based on maximum likelihood, least squares and expectation maximization are presented and compared in respect of estimation accuracy and complexity (Cai et al 2006).

A Power-of-R (POR) technique to blindly estimate multipath parameters of the desired user in a Multiple-Access (MA) UWB system, where $R$ represents the data covariance matrix was proposed (Liu and Xu 2006). The POR technique is related to the subspace technique in that it directly estimates the unknown noise-subspace component in the subspace method but avoids rank estimation where errors may be incurred by perturbations in practical conditions.

A method based on Orthogonal Sinusoidal Correlation Receiver (OSCR) for UWB communications to estimate channel parameters has been proposed (Ke et al 2005). Matching Pursuit (MP) based tap selection technique was proposed and applied to UWB indoor channel equalization in the presence of Inter Symbol Interference (ISI) and Multiple Access Interference (MAI) (Muller and Elmirghani 1999). Given the limited training sample support, quadratic constraint is incorporated into MP algorithm to ensure the robustness for tap selection (Zhiwei et al 2005).
Impulse radio UWB communication systems employ a time-hopping technique to enable multiple access. Each user is assigned a distinct time-hopping sequence, either periodic or aperiodic, to suppress multiuser interferences. Aperiodic codes have many promising features compared with periodic codes, such as smoothed spectrum and enhanced security (Ciblat et al 2008). The channel estimation issues in aperiodic time-hopping systems were also studied using least squares technique and a correlation matching technique (Xu and Tang 2006).

Sheng and Haimovich (2007) investigated the impact of imperfect estimates on ultra-wideband system performance was investigated when path delays and path amplitudes are jointly estimated. The Cramer–Rao Lower Bound (CRLB) of the path delay estimates was presented as a function of the signal-to-noise ratio (SNR) and signal bandwidth. The performance of a UWB system employing a Rake receiver and maximum ratio combining was analyzed taking into account estimation errors as predicted by the CRB (Yang et al 2007).

Based on a survey on indoor wireless channels for UWB radio, IEEE 802.15.3a model for UWB channel was introduced by Song et al (2008). The Least Square (LS) channel estimation based on comb type pilot and linearity insertion were described. Further, a Pulse Compression (PC) method for Data Aided (DA) and Non-Data Aided (NDA) channel estimation has been proposed (Alizad et al 2008).

A non coherent UWB demodulation and detection algorithm has been proposed that circumvents the problems of timing synchronization and channel estimation for impulse based UWB systems (Skelton and Fu 2009).
5.3 CHANNEL ESTIMATION TECHNIQUE FOR CHAOS BASED UWB SYSTEM

Owing to the wideband nature of chaotic signals, the received response has multiple reflections and hence multipath reception. The algorithms for estimating channel parameters, namely attenuation and delay incurred by signal echoes along the propagation paths are usually carried out using suboptimal methods (Muller et al 1999). As the chaotic signals are weakly correlated, the Maximum Likelihood (ML) method based optimal channel estimation algorithm will not estimate the attenuation parameters exactly (Yang et al 2007).

In this chapter, a ML methods and matching pursuit based algorithms are proposed with suitable modifications for estimating UWB channel parameters of a chaos based UWB communication system.

5.4 CHANNEL ESTIMATION USING ML METHOD

5.4.1 Signal Model

A discrete time model of chaos based communication system is shown in Figure 5.1.

The transmitted signal for a chaos based UWB system is modeled as

\[ s(t) = \sum_{i=0}^{M-1} \sum_{j=0}^{N_f-1} a_i ch(j) g(t - jT_f - iN_f T_f) \]  

(5.1)

where \( a_i \) is the data symbols belongs to the set \{+1, -1\}, \( N_f \) is the number of frames in a frame duration \( T_f \), \( ch(j) \) is the \( k^{th} \) chaotic sample generated by a chaotic signal generator and \( g(t) \) is the pulse shape.
The second derivative Gaussian pulse $g(t)$ is used as UWB pulse shape which is expressed as

$$g(t) = A \left[ 1 - 4\pi \left( \frac{t}{T_w} \right)^2 \right] e^{-2\pi \left( \frac{t}{T_w} \right)^2}, \quad 0 \leq t \leq T_0 \tag{5.2}$$

Assuming a $L$ path environment, the received signal $r(t)$ is written as

$$r(t) = \sum_{l=1}^{L} \gamma_l s(t - \tau_l) + n(t), \quad 0 \leq t \leq T_0 \tag{5.3}$$

where $\gamma_p$ for $1 \leq p \leq L$ and $\tau_p$ for $1 \leq p \leq L$ are the attenuation and the delay parameters of the channel. The channel is assumed to be static. It implies that the channel parameters $\gamma = [\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_L]$ and $\tau = [\tau_1, \tau_2, \ldots, \tau_L]$ are either fixed or vary so slowly that they are practically constant over several data symbols. $T_0$ is the observation interval and it is assumed to be an integer.
multiple of symbol period \( N_j T_j \) and \( n(t) \) is assumed to be zero mean additive white Gaussian noise.

Substituting (5.1) in (5.3), the received signal \( r(t) \) is written as

\[
r(t) = \sum_{i=0}^{M-1} \sum_{j=0}^{N_j - 1} \sum_{l=1}^{L} c_{l, i, j} g(t - jT_j - iN_j T_j) + w(t) \quad 0 \leq t \leq T_0
\]

(5.4)

The likelihood function for estimating the channel parameters \( \gamma = [\gamma_1, \gamma_2, \ldots, \gamma_L] \) and \( \tau = [\tau_1, \tau_2, \ldots, \tau_L] \) is formulated as,

\[
\min_{\gamma, \tau} \left( \int_0^{T_0} \left[ r(t) - \sum_{p=1}^{L} \bar{v}_p s(t - \bar{\tau}_p) \right]^2 dt \right)
\]

(5.5)

where \( \sum_{p=1}^{L} \bar{v}_p s(t - \bar{\tau}_p) \) is a possible realization of the signal component corresponding to the channel parameters \( \bar{v} \) and \( \bar{\tau} \).

By expanding the above integral, the first term, \( \int_0^{T_0} r^2(t) dt \) is independent of the channel parameters \( \bar{v} \) and \( \bar{\tau} \). Hence neglecting this term, the likelihood function is written as

\[
\max_{\gamma, \tau} \left( \sum_{p=1}^{L} \bar{v}_p \int_0^{T_0} r(t) s(t - \bar{\tau}_p) dt - \frac{1}{2} \sum_{p=1}^{L} \bar{v}_p^2 \int_0^{T_0} s^2(t - \bar{\tau}_p) dt \right)
\]

(5.6)
Substituting (5.4) in (5.6), then the likelihood function is written as

$$
\max_{\gamma, \tau^*} \left\{ 2 \sum_{i=0}^{M-1} a_i \sum_{j=0}^{N_f-1} \sum_{l=1}^{L} \gamma_{il} ch(j) g_t \left( t - jT_c - jN_f T_f - \tau_l \right) \bar{s}(t) dt - \int_0^{T_c} \bar{s}^2(t) dt \right\} (5.7)
$$

If the pulses appearing in $s(t)$ are widely separated from each other, the correlation between the signal echoes can be assumed as

$$
\int_0^{T_c} s(t-\bar{\tau}_{l_1}) s(t-\bar{\tau}_{l_2}) dt \approx 0 \quad l_1 \neq l_2 (5.8)
$$

It implies that even a small time misalignment makes the echoes virtually orthogonal.

Then, the likelihood function can be simplified as

$$
\max_{\gamma, \tau^*} \left\{ 2 \sum_{p=1}^{L} \bar{r}_p z(p, \bar{\tau}_p) - \sum_{p=1}^{L} \gamma_p^2 p_s(\bar{\tau}_p) dt \right\} (5.9)
$$

where the matched filter output at time $\bar{\tau}_p$ for $1 \leq p \leq L$ is

$$
z(\bar{\tau}_p) = \int_0^{T_c} r(t) s(t-\bar{\tau}_p) dt \quad 1 \leq p \leq L (5.10)
$$

and $p_s(\bar{\tau}_p) = \int_0^{T_c} s^2(t-\bar{\tau}_p) dt$. 
The estimates of attenuation constants $\bar{p}_p, 1 \leq p \leq L$, are computed by taking derivative of the likelihood function equation (5.9) with respect to $\bar{p}_p, 1 \leq p \leq L$ and equating them to zero. It results in

$$
\bar{p}_p = \frac{z(\bar{p}_p)}{p_s(\bar{p}_p)} \quad 1 \leq p \leq L
$$

(5.11)

$z(\bar{p}_p)$ for $1 \leq p \leq L$ are the sufficient statistics to estimate the delay parameters $\tau_p$ for $1 \leq p \leq L$. Hence attenuation and delay parameters can be estimated by plotting the ratio $\frac{z(\tau)}{p_s(\tau)}$ in the interval $\tau \in [0, T_d]$ where $T_d$ is the channel delay spread. The estimates of delay parameters $\bar{p}_p, 1 \leq p \leq L$ are obtained by observing the peaks in the plot of $\frac{z(\tau)}{p_s(\tau)}$ and the attenuation parameters $\bar{p}_p, 1 \leq p \leq L$ are the respective amplitudes at the peaks.

If the correlation function of chaotic spreading sequence $\{ch(n)\}$ is approximated as an impulse function, the plot of $\frac{z(\tau)}{p_s(\tau)}$ will give exact values for both delay and attenuation parameters $\gamma$ and $\tau$. However the chaotic spreading sequence does not satisfy the property. $E[ch(n)ch(k)] = 0$ if $k \neq n$, where $ch(n)$ and $ch(k)$ are chaotic spreading sequences at time ‘n’ and ‘k’ respectively Figure 5.2 shows the typical autocorrelation property of the chaotic binary sequence of length 127. This affects the estimation of the attenuation parameters.
Hence it is essential to make suitable correction to the estimates obtained ML method and it is carried out as follows:

Consider a $L$ path environment with the channel parameters $\gamma$ and $\tau$. The received waveform for $L$ path channel environment is written as

$$ r(t) = \sum_{i=1}^{L} \gamma_i s(t - \tau_i) + i(t) + n(t) \quad 0 \leq t \leq T_0 $$  \hspace{1cm} (5.12)

The delay parameters are perfectly estimated using the ML estimate. Hence the matched filter outputs at the delays $\tau_1, \tau_2, \ldots, \tau_L$ are given by

$$ z(\tau_i) = \int_0^{\tau_i} r(t) s(t - \tau_i) dt \quad \text{for} \quad i = 1, 2, \ldots, L $$ \hspace{1cm} (5.13)
It is calculated as

\[ z(\tau_i) = \sum_{i=1}^{L} \gamma_i r_s(\tau_j - \tau_i) + \xi_i \quad \text{for} \quad i = 1, 2, \ldots, L \]  \hspace{1cm} (5.14)

where

\[ r_s(\tau_j, \tau_i) = \int_0^{\tau_0} s(t - \tau_i) s(t - \tau_j) \, dt \quad \text{and} \quad \xi_i = \int_0^{\tau_0} (i(t) + n(t)) s(t - \tau_i) \, dt \quad \text{for} \quad i = 1, 2, \ldots, L. \]

For simplicity, let \( z_i = z(\tau_i) \) then the matched filter outputs is represented in matrix form as

\[ z = R\gamma + \xi \]  \hspace{1cm} (5.15)

where

\[ z = [z_1, z_2, \ldots, z_L]^T, \xi = [\xi_1, \xi_2, \ldots, \xi_L]^T, \gamma = [\gamma_1, \gamma_2, \ldots, \gamma_L]^T \]

and

\[ R = \begin{bmatrix}
\gamma_s(0) & \gamma_s(\tau_2 - \tau_1) & \cdots & \cdots & \gamma_s(\tau_L - \tau_1) \\
\gamma_s(\tau_2 - \tau_1) & \gamma_s(0) & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
\gamma_s(\tau_L - \tau_1) & \cdots & \cdots & \cdots & \gamma_s(0)
\end{bmatrix} \]

Let \( P \) be \( L \times L \) diagonal matrix with its elements

\[ \begin{bmatrix}
\frac{1}{p_s(\tau_1)} & 1 & \cdots & 1 \\
1 & \frac{1}{p_s(\tau_2)} & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
1 & \cdots & \cdots & \frac{1}{p_s(\tau_L)}
\end{bmatrix}. \]  \hspace{1cm} (5.10)

The ML estimates of \( \gamma_i \)'s are given by \( \frac{z(\tau)}{p_s(\tau)} \) as in (5.10).

Premultiplying (5.15) by \( P \), then

\[ Pz = PR\gamma + P\xi \]  \hspace{1cm} (5.16)
The left hand side term is \( Pz = \hat{\gamma}_{ML} \) as in (5.11). Hence (5.16) can be written as

\[
\tilde{\gamma}_{ML} = PR\gamma + P\xi \tag{5.17}
\]

where \( \tilde{\gamma}_{ML} \) is the ML estimate of \( \gamma \). The effects of noise and narrowband interference at times \( \tau_1, \tau_2, \ldots, \tau_L \) are greatly reduced by the desired signal energy terms \( p_1(\tau_1), \ldots, p_L(\tau_L) \) respectively. Hence the omission of second term in (5.17) will not affect the estimate of \( \gamma \). Finally, the estimate of attenuation parameters are given by

\[
\tilde{\gamma}_c = (PR)^{-1} \hat{\gamma}_{ML} \tag{5.18}
\]

5.5 MATCHING PURSUIT BASED ML FOR CHAOS BASED UWB

Matching pursuit algorithm is an algorithm that decomposes any signal into a linear expansion of waveforms that belong to a redundant dictionary of functions. These waveforms are selected in order to best match the signal structures. Although a matching pursuit is nonlinear, like an orthogonal expansion, it maintains an energy conservation which guaranties its convergence (Mallat and Zhang 1993).

Matching pursuit decomposition provides an interpretation of the signal structures. The signal is decomposed into waveforms selected among a dictionary of time-frequency representations that are the dilations, translations, and modulations of a single window function. If a structure does not correlate well with any particular dictionary element, it is sub decomposed into several elements and its information in dilated. Matching pursuit is a greedy algorithm that chooses at each iteration a waveform that is best adapted to approximate part of the signal (Krstulovic et al 2005).
Let $H$ be a Hilbert space. It is assumed that a dictionary $J$ contains orthonormal vectors in $H$. Further, the received vector $r$ also belongs to $H$. In matching pursuit algorithm, a linear expansion of $r$ over a set of vectors selected from $J$ is computed in order best match its inner structures. This is done by successive approximation approach as follows.

Let $g_{\gamma_0} \in J$. The signal $r$ can be represented as

$$r = \langle r, g_{\gamma_0} \rangle g_{\gamma_0} + R'r$$

(5.19)

where $\langle r, g_{\gamma_0} \rangle$ represents inner product between $r$ and $g_{\gamma_0}$ and $R'$ referred to as first order residual signal after approximating $r$ in the direction of $g_{\gamma_0}$. Since $g_{\gamma_0}$ is orthogonal to $R'r$, the energy of the received signal can be represented as

$$\|r\|^2 = \langle r, g_{\gamma_0} \rangle^2 + \|R'r\|^2$$

(5.20)

The vector $g_{\gamma_0} \in J$ is chosen such that $\langle r, g_{\gamma_0} \rangle$ is maximized and $\|R'r\|$ is minimized.

The residue $R'r$ is decomposed into

$$R'r = \langle R'r, g_{\gamma_1} \rangle g_{\gamma_1} + R'^2r$$

(5.21)

In general, $n$th residue of the received signal $r$ is represented as

$$R^n'r = \langle R^n'r, g_{\gamma_n} \rangle g_{\gamma_n} + R'^{n+1}r$$

(5.22)
Since $R^{n+1} r$ is orthogonal to $g_{\gamma_n}$

$$\|R^n r\|^2 = \left|\langle R^n r, g_{\gamma_n} \rangle \right|^2 + \|R^{n+1} r\|^2$$ \hspace{1cm} (5.23)

This decomposition is performed up to the order $L$ which represents the number of paths in the UWB channel. Decomposing $r$ into the concatenated sum

$$r = \sum_{n=0}^{L-1} (R^n r - R^{n+1} r) + R^L r$$ \hspace{1cm} (5.24)

Using (5.21), (5.23) can be as

$$r = \sum_{n=0}^{L-1} \langle R^n r, g_{\gamma_n} \rangle g_{\gamma_n} + R^L r$$ \hspace{1cm} (5.25)

Similarly, $\|r\|^2$ is decomposed in a concatenated sum

$$\|r\|^2 = \sum_{n=0}^{L-1} \left( \|R^n r\|^2 + \|R^{n+1} r\|^2 \right) + \|R^L r\|^2$$ \hspace{1cm} (5.26)

Thus, the received signal $r$ is decomposed into a sum of dictionary elements that are chosen to best match its residues. Although this decomposition is nonlinear, energy conservation is maintained as in a linear orthogonal decomposition. When the $L$ strongest paths are obtained, the iterations are stopped. Figures (5.3) and (5.4) show the original and estimated attenuations and delays using matching pursuit based ML method respectively. It is observed that most of the path delays are exactly matched with original and estimated values.
Figure 5.3  Original delays and attenuations of CM2 for 10 strongest paths

Figure 5.4  Estimated delays and attenuations of CM2 for 10 strongest paths
5.6 RESULTS AND DISCUSSION

In this section, the performance of the proposed method is analyzed using a five path channel. The channel attenuation parameters are modeled as independent random variables and the delays are taken in such a way that phase information is uniformly distributed in the interval $[0, 2\pi]$. The channel parameters $\gamma = [0.35, 0.7, 0.5, 0.7, 0.9]$ and $\tau = [20, 80, 100, 150, 175]$ are assumed fixed during the observation interval $t \in [0, T_o]$. Even though the number of paths may be much higher in a dense multipath environment, to keep the simulation time within tolerable limits, we have considered up to the five path channel. Further, this has been sufficient for analysis and extending it to more number of paths is straightforward as shown in (5.18). The MSE between the actual and estimated values of the attenuation parameter for $L$ paths is defined as

$$MSE = \frac{1}{L} \sum_{p=1}^{L} (\gamma_p - \overline{\gamma}_{c,p})^2$$  \hspace{1cm} (5.27)

Figure 5.5 illustrates the relationship between the ML estimate of attenuation parameters and delay at the input signal to noise ratio of 10 dB, with observation time is 100 symbol duration and spreading factor is 100. It is observed that the attenuation parameters require corrections as the ML estimate of $\overline{\gamma}_{ML}$ deviates more from $\gamma$.

Figure 5.6 shows the MSE performance on the estimation of attenuation parameter $\gamma$ using the proposed method for the observed times $T_0 = 25T_s$, $T_0 = 50T_s$ and $T_0 = 100T_s$. As the ratio of $T_0/T$ increases, the accuracy of the estimation of channel parameter $\gamma$ also improves. However, with a time varying channel, the ratio $T_0/T$ must be much smaller than the channel decorrelation time. Hence, a tradeoff between the accuracy and the condition for decorrelation time is to be considered for analysis.
Figure 5.5  Delay Vs Attenuation at 10 dB SNR for ML method

Figure 5.6  Performance of the proposed ML method for To/T = 25, 50, 100
The MSE performance of the proposed method for the estimation of $\gamma$ is shown in Figure 5.7 for the input SNR varying from -20dB to +20dB, at the ratio between the observation time $T_0$ and symbol duration $T$ is 100.

![MSE performance analysis for ML and ML for chaos](image)

**Figure 5.7 Performance analysis for ML and ML for chaos**

The BER performance of the proposed Matching pursuit technique based channel estimation algorithm for chaos based UWB system are analyzed using the channel model of IEEE 802.15.3a working group for short range high data rate wireless communication. A second derivative of the Gaussian function with unit energy and duration $T_p=1.67$ns is used as UWB pulse shape. The chaotic sequence for single user setup is generated by assuming seed value of 0.7. The map used to generate the chaotic sequence is given by $x_{k+1} = 4x_k (1 - x_k)$. The values less than 0.5 are treated as ‘0’; otherwise it is treated as ‘1’.
It is assumed the values of number of frames $N=12$, number of symbols $M=100$ and number of resolvable paths $= 10$. The simulations are done for all the four channel models CM1, CM2, CM3 and CM4. Figures 5.8 to 5.11 shows the BER performance of second derivative Gaussian repetitive frame for all the channel models respectively. The BER, when the channel parameters are known, is compared with the BER, when the channel parameters are estimated. The BER of the estimated channel almost equals to that of a known channel for the CM1, CM2, CM3 and CM4 models. The performance of CM1 is better then CM2, CM3 and CM4 as it has the shortest delay spread among the four models.

![BER for CM1 using Gaussian II derivative as UWB pulse shape](image)

**Figure 5.8** BER for CM1 using Gaussian II derivative as UWB pulse shape
Figure 5.9  BER for CM2 using Gaussian II derivative as UWB pulse shape

Figure 5.10  BER for CM3 using Gaussian II derivative as UWB pulse shape
5.7 CONCLUSION

In this chapter, ML based channel estimation algorithm is proposed for chaos signal based UWB system. It is found that the direct application of ML Method does not give the accurate estimates of the attenuation parameters and hence, a novel method is developed for correcting the ML estimates of attenuation parameters. Simulation results show that the proposed method is well suitable for estimating the channel parameters of the proposed system. Further, a matching pursuit based maximum likelihood algorithm for channel estimation in chaos based UWB system is proposed. The estimation has been done for all the four channel models using Gaussian II derivative as the UWB pulse shape. The performance analysis of the algorithm is also presented through BER simulations.